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GORDON SYKES

A TEXTBOOK

ON

MECHANICAL AND ELECTRICAL ENGINEERING

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

V.6

ANSWERS TO QUESTIONS

11893

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A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

CONTAINED IN THE

QUESTION PAPERS

INCLUDED IN THE PRECEDING VOLUMES.

It will be noticed that the Key is divided into sections corresponding to the various sections of the questions and examples at the end of the preceding volumes. The answers are so numbered as to be similar to the numbers before the questions to which they refer.

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ARITHMETIC.

(QUESTIONS 1-75.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.

605 = Six hundred five.

28,284 = Twenty-eight thousand, two hundred eighty-four.

9,006,042 = Nine million, six thousand and forty-two.

850,317,002 = Eight hundred fifty million, three hundred seventeen thousand and two.

700,004 = Seven hundred thousand and four.

(6) Seven thousand six hundred = 7,600.

Eighty-one thousand four hundred two = 81,402.

Five million, four thousand and seven = 5,004,007.

One hundred and eight million, ten thousand and one = 108,010,001.

Eighteen million and six = 18,000,006.

Thirty thousand and ten = 30,010.

(7) In adding whole numbers, place the numbers to be added directly under each other so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column, and reserve 1 ten for the column

```
  3290
   504
865403
  2074
    81
     7
------
871359   Ans.
```

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of tens. $1+8+7+9=25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $2+4+5+2=13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $1+2+5+3=11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten-thousand for the column of ten-thousands. $1+6=7$ ten-thousands. Place this seven ten-thousands in the ten-thousands column. There is but one figure 8 in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

A simpler (though less scientific) explanation of the same problem is the following: $7+1+4+3+4+0=19$; write the nine and reserve the 1. $1+8+7+0+0+9=25$; write the 5 and reserve the 2. $2+0+4+5+2=13$; write the 3 and reserve the 1. $1+2+5+3=11$; write the 1 and reserve 1. $1+6=7$; write the 7. Bring down the 8 to its place in the sum.

(8)

709
8304725
391
100302
300
909

8407336 Ans.

(9) (*a*) In subtracting whole numbers, place the subtrahend or smaller number under the minuend or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract. We can not subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. As 1 *ten* = 10 *units*, we have 10 units + 2 units = 12 units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so

only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We can not subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the ten-thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

$$\begin{array}{r} 50962 \\ 3338 \\ \hline 47624 \end{array}$$ Ans.

(*b*) $$\begin{array}{r} 15339 \\ 10001 \\ \hline 5338 \end{array}$$ Ans.

(10) (*a*) $$\begin{array}{r} 70968 \\ 32975 \\ \hline 37993 \end{array}$$ Ans.

(*b*) $$\begin{array}{r} 100000 \\ 98735 \\ \hline 1265 \end{array}$$ Ans.

(11) We have given the minuend or greater number (1,004) and the difference or remainder (49). Placing these in the usual form of subtraction we have $\frac{1004}{\text{——}}$ in which $\overline{49}$ the dash (——) represents the number sought. This number is evidently *less* than 1,004 by the difference 49, hence, 1,004 − 49 = 955, the smaller number. For the sum of the two numbers we then have

$$\begin{array}{rl} 1004 & \textit{larger} \\ 955 & \textit{smaller} \\ \hline 1959 & \textit{sum.} \end{array}$$ Ans.

Or, this problem may be solved as follows: If the greater of two numbers is 1,004, and the difference between them is 49, then it is evident that the smaller number must be equal to the difference between the greater number (1,004)

and the difference (49); or, $1,004 - 49 = 955$, the smaller number. Since the greater number equals 1,004 and the smaller number equals 955, their sum equals $1,004 + 955 = 1,959$ sum. Ans.

(12) The numbers connected by the plus (+) sign must first be added. Performing these operations we have

5962	3874
8471	2039
9023	5913 *sum.*
23456 *sum.*	

Subtracting the smaller number (5,913) from the greater (23,456) we have

23456
5913
17543 *difference.* Ans.

(13) \$44675 = amount willed to his son.
26380 = amount willed to his daughter.
\$71055 = amount willed to his two children.
\$125000 = amount willed to his wife and two children.
71055 = amount willed to his two children.
\$53945 = amount willed to his wife. Ans.

(14) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

(*a*)
526387
7
3684709 Ans.

7×7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 tens + 4 tens reserved = 60 tens or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7×3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve

the 2 thousands. 7×6 thousands $= 42$ thousands; $42 + 2$ thousands reserved $= 44$ thousands or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten-thousands. 7×2 ten-thousands $= 14$ ten-thousands; $14 + 4$ ten-thousands reserved $= 18$ ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7×5 hundred-thousands $= 35$ hundred-thousands; $35 + 1$ hundred-thousand reserved $= 36$ hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times $7 = 49$; write the 9 and reserve the 4. 7 times $8 = 56$; $56 + 4$ reserved $= 60$; write the 0 and reserve the 6. 7 times $3 = 21$; $21 + 6$ reserved $= 27$; write the 7 and reserve the 2. $7 \times 6 = 42$; $42 + 2$ reserved $= 44$; write the 4 and reserve 4. $7 \times 2 = 14$; $14 + 4$ reserved $= 18$; write the 8 and reserve the 1. $7 \times 5 = 35$; $35 + 1$ reserved $= 36$; write the 36.

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, $7 \times$ 700,298 and $10 \times$ 700,298. The actual operation is performed as follows:

(*b*)

700298	
17	
4902086	
700298	
11905066	Ans.

7 times 8 = 56; write the 6 and reserve the 5. 7 times 9 = 63; 63 + 5 reserved = 68; write the 8 and reserve the 6. 7 times 2 = 14; 14 + 6 reserved = 20; write the 0 and reserve the 2. 7 times 0 = 0; 0 + 2 reserved = 2; write the 2. 7 times 0 = 0; 0 + 0 reserved = 0; write the 0. 7 times 7 = 49; 49 + 0 reserved = 49; write the 49.

To multiply by the 1 ten we say 1 times 700298 = 700298, and write 700298 under the first partial product, as shown, with the right-hand figure 8 under the multiplier 1. Add the two partial products; their sum equals the entire product.

(*c*) 217 Multiply any two of the numbers together
103 and multiply their product by the third
651 number.

2170
22351
67
156457
134106
1497517 Ans.

(15) If your watch ticks every second, then to find how many times it ticks in one week it is necessary to find the number of seconds in 1 week.

60 seconds = 1 minute.
60 minutes = 1 hour.
3600 seconds = 1 hour.
24 hours = 1 day.
14400
7200
86400 seconds = 1 day.
7 days = 1 week.
604800 seconds in 1 week or the number of times that
Ans. your watch ticks in 1 week.

(16) If a monthly publication contains 24 pages, a yearly
24 volume will contain 12×24 or 288 pages, since
12 there are 12 months in one year; and eight
288 yearly volumes will contain 8×288, or 2,304
8 pages.
2304 Ans.

(17) If an engine and boiler are worth \$3,246, and the building is worth 3 times as much, plus \$1,200, then the building is worth

\$3246
3
9738
plus 1200
\$10938 = value of building.

If the tools are worth twice as much as the building, plus $1,875, then the tools are worth

$10938
2
21876
plus 1875
$23751 = value of tools.

Value of building = $10938
Value of tools = 23751
$34689 = value of the building and tools. (*a*) Ans.

Value of engine and boiler = $ 3246
Value of building and tools = 34689
$37935 = value of the whole plant. (*b*) Ans.

(18) (*a*) $(72 \times 48 \times 28 \times 5) \div (96 \times 15 \times 7 \times 6)$.

Placing the numerator over the denominator the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$$

The 5 in the *dividend* and 15 in the *divisor* are both *divisible* by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. *Cross off* the 5 and write the 1 *over* it; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \overset{1}{\cancel{5}}}{96 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

The 5 and 15 are *not* to be considered any longer, and, in fact, may be erased entirely and the 1 and 3 placed in their stead, and treated as if the 5 and 15 *never* existed. Thus,

$$\frac{72 \times 48 \times 28 \times 1}{96 \times 3 \times 7 \times 6} =$$

72 in the *dividend* and 96 in the *divisor* are *divisible* by 12, since 72 divided by 12 equals 6, and 96 divided by 12 equals 8. *Cross off* the 72 and write the 6 *over* it; also, *cross off* the 96 and write the 8 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times 1}{\underset{8}{\cancel{96}} \times 3 \times 7 \times 6} =$$

The 72 and 96 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 6 and 8 placed in their stead, and treated as if the 72 and 96 *never* existed. Thus,

$$\frac{6 \times 48 \times 28 \times 1}{8 \times 3 \times 7 \times 6} =$$

Again, 28 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{6 \times 48 \times \overset{4}{\cancel{28}} \times 1}{8 \times 3 \times \underset{1}{\cancel{7}} \times 6} =$$

The 28 and 7 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 4 and 1 placed in their stead, and treated as if the 28 and 7 *never* existed. Thus,

$$\frac{6 \times 48 \times 4 \times 1}{8 \times 3 \times 1 \times 6} =$$

Again, 48 in the *dividend* and 6 in the *divisor* are *divisible* by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, cross off the 6 and write the 1 *under* it. Thus,

$$\frac{6 \times \overset{8}{\cancel{48}} \times 4 \times 1}{8 \times 3 \times 1 \times \underset{1}{\cancel{6}}} =$$

The 48 and 6 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 8 and 1 placed in their stead, and treated as if the 48 and 6 *never* existed. Thus,

$$\frac{6\times 8\times 4\times 1}{8\times 3\times 1\times 1}=$$

Again, 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, cross off the 3 and write the 1 *under* it. Thus,

$$\frac{\overset{2}{\cancel{6}}\times 8\times 4\times 1}{8\times \underset{1}{\cancel{3}}\times 1\times 1}=$$

The 6 and 3 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{2\times 8\times 4\times 1}{8\times 1\times 1\times 1}=$$

Canceling the 8 in the dividend and the 8 in the divisor, the result is

$$\frac{2\times \overset{1}{\cancel{8}}\times 4\times 1}{\underset{1}{\cancel{8}}\times 1\times 1\times 1}=\frac{2\times 1\times 4\times 1}{1\times 1\times 1\times 1}.$$

Since there are *no two remaining numbers* (one in the dividend and one in the divisor) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further.

Multiply all the *uncanceled numbers* in the *dividend* together, and divide their *product* by the *product* of all the *uncanceled* numbers in the divisor. The *result* will be the *quotient*. The *product* of all the *uncanceled numbers* in the *dividend* equals $2\times 1\times 4\times 1=8$; the product of all the *uncanceled* numbers in the *divisor* equals $1\times 1\times 1\times 1=1$.

Hence, $$\frac{2\times 1\times 4\times 1}{1\times 1\times 1\times 1}=\frac{8}{1}=8.$$ Ans.

Or, $$\frac{\overset{2}{\cancel{6}}\quad 8\quad 4\quad 1}{\cancel{72}\times\cancel{48}\times\cancel{28}\times\cancel{5}}=\frac{8}{1}=8.$$ Ans.
$$\underset{1}{\cancel{96}}\times\underset{1}{\cancel{15}}\times 7\times\underset{1}{\cancel{6}}$$
$$\underset{1}{\cancel{8}}\quad\underset{1}{\cancel{3}}$$

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the *dividend* and 70 in the *divisor* are both *divisible* by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

The 50 and 70 are not to be considered any longer, and, in fact, may be erased entirely and the 5 and 7 placed in their stead, and treated as if the 50 and 70 *never* existed. Thus,

$$\frac{80 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 20} =$$

Also, 80 in the *dividend* and 20 in the *divisor* are *divisible* by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. Cross off the 80 and write the 4 *over* it; also, cross off the 20 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times \underset{1}{\cancel{20}}} =$$

The 80 and 20 are *not* to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 80 and 20 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 1} =$$

Again, 16 in the *dividend* and 24 in the *divisor* are *divisible* by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also cross off the 24 and write the 3 *under* it. Thus,

$$\frac{4 \times 60 \times 5 \times \overset{2}{\cancel{16}} \times 14}{7 \times 50 \times \underset{3}{\cancel{24}} \times 1} =$$

The 16 and 24 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 3 placed in their stead, and treated as if the 16 and 24 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 2 \times 14}{7 \times 50 \times 3 \times 1} =$$

Again, 60 in the *dividend* and 50 in the *divisor* are *divisible* by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{4 \times \overset{6}{\cancel{60}} \times 5 \times 2 \times 14}{7 \times \underset{5}{\cancel{50}} \times 3 \times 1} =$$

The 60 and 50 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 5 placed in their stead, and treated as if the 60 and 50 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 14}{7 \times 5 \times 3 \times 1} =$$

The 14 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}} \times 5 \times 3 \times 1} =$$

The 14 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 14 and 7 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 2}{1 \times 5 \times 3 \times 1} =$$

The 5 in the *dividend* and 5 in the *divisor* are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the 5 of the *dividend* and write the 1 *over* it; also, cross off the 5 of the *divisor* and write the 1 *under* it. Thus,

$$\frac{4\times 6\times \overset{1}{\cancel{5}}\times 2\times 2}{1\times \underset{1}{\cancel{5}}\times 3\times 1}=$$

The 5 in the *dividend* and 5 in the *divisor* are not to be considered any longer, and, in fact, may be erased entirely and 1 and 1 placed in their stead, and treated as if the 5 and 5 *never* existed. Thus,

$$\frac{4\times 6\times 1\times 2\times 2}{1\times 1\times 3\times 1}=$$

The 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3 and place 1 *under* it. Thus,

$$\frac{4\times \overset{2}{\cancel{6}}\times 1\times 2\times 2}{1\times 1\times \underset{1}{\cancel{3}}\times 1}=$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{4\times 2\times 1\times 2\times 2}{1\times 1\times 1\times 1}=\frac{32}{1}=32.\quad \text{Ans.}$$

Hence, $\dfrac{\overset{4}{\cancel{80}}\times \overset{2}{\cancel{60}}\times \overset{1}{\cancel{50}}\times \overset{2}{\cancel{16}}\times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{70}}\times \underset{1}{\cancel{50}}\times \underset{1}{\cancel{24}}\times \underset{1}{\cancel{20}}}=\dfrac{4\times 2\times 1\times 2\times 2}{1\times 1\times 1\times 1}=\dfrac{32}{1}=32.$ Ans.

(19) 28 acres of land at $133 an acre would cost $28\times \$133=\$3{,}724$.

$$\begin{array}{r} 28 \\ \hline 1064 \\ 266 \\ \hline \$3724 \end{array}$$

If a mechanic earns $1,500 a year and his expenses are $968 per year, then he would save $1500—$968, or $532 per year.

$$\begin{array}{r} 968 \\ \hline \$532 \end{array}$$

If he saves $532 in 1 year, to save $3,724 it would take as many years as $532 is contained times in $3,724, or 7 years.

$$\begin{array}{l} 532)3724(7 \text{ years. Ans.} \\ \quad 3724 \\ \quad \overline{} \end{array}$$

(20) If the freight train ran 365 miles in one week, and 3 times as far lacking 246 miles the next week, then it ran (3 × 365 miles) — 246 miles, or 849 miles the second week. Thus,

$$\begin{array}{rl} & 365 \\ & \quad 3 \\ \hline & 1095 \\ & \quad 246 \\ \hline \textit{difference} & 849 \text{ miles. Ans.} \end{array}$$

(21) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in one mile, in 354 miles there are 354 × 5,280 feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, then in going from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make 1,869,120 ÷ 16, or 116,820 revolutions.

$$\begin{array}{l} 16)1869120(116820 \text{ rev. Ans.} \\ \quad 16 \\ \quad \overline{} \\ \quad 26 \\ \quad 16 \\ \quad \overline{} \\ \quad 109 \\ \quad 96 \\ \quad \overline{} \\ \quad 131 \\ \quad 128 \\ \quad \overline{} \\ \quad 32 \\ \quad 32 \\ \quad \overline{} \\ \quad 0 \end{array}$$

(22) (*a*) 576) 589824 (1024 Ans.

576

1382

1152

2304

2304

(*b*) 43911) 369730620 (8420 Ans.

351288

184426

175644

87822

87822

0

(*c*) 505) 2527525 (5005 Ans.

2525

2525

2525

(*d*) 1234) 4961794302 (4020903 Ans.

4936

2579

2468

11143

11106

3702

3702

(23) The harness evidently cost the difference between $444 and the amount which he paid for the horse and wagon.

Since $264 + $153 = $417, the amount paid for the horse and wagon, $444 − $417 = $27, the cost of the harness.

$264 $444

153 417

$417 $27 Ans.

(24) (*a*)

$$\begin{array}{r} 1024 \\ 576 \\ \hline 6144 \\ 7168 \\ 5120 \\ \hline 589824 \end{array}$$ Ans.

(*b*)

$$\begin{array}{r} 5005 \\ 505 \\ \hline 25025 \\ 250250 \\ \hline 2527525 \end{array}$$ Ans.

(*c*)

$$\begin{array}{r} 43911 \\ 8420 \\ \hline 878220 \\ 175644 \\ 351288 \\ \hline 369730620 \end{array}$$ Ans.

(25) Since there are 12 months in a year, the number of days the man works is $25 \times 12 = 300$ days. As he works 10 hours each day, the number of hours that he works in one year is $300 \times 10 = 3{,}000$ hours. Hence, he receives for his work $3{,}000 \times 30 = 90{,}000$ cents, or $90{,}000 \div 100 = \$900$. Ans.

(26) See Art. **71.**

(27) See Art. **77.**

(28) See Art. **73.**

(29) See Art. **73.**

(30) See Art. **75.**

(31) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(32) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(33) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms we divide both numerator and denominator by 4, and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$. Ans.

(34) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6. To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since this denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4 we shall have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form*. Ans.

(35) In order to reduce a mixed number to an improper fraction, we must *multiply the whole number by the denominator of the fraction* and *add the numerator of the fraction to that product.* This *result* is the *numerator of the improper fraction*, of which the *denominator is the denominator of the fractional part of the mixed number.*

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$, hence in 7 there are $7 \times \frac{8}{8} = \frac{56}{8}$; $\frac{56}{8}$ plus the $\frac{7}{8}$ of the mixed number $= \frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\frac{5}{16} = \frac{(13 \times 16) + 5}{16} = \frac{213}{16}; \quad 10\frac{3}{4} = \frac{(10 \times 4) + 3}{4} = \frac{43}{4}.$$

(36) The value of a fraction is obtained by dividing the numerator by the denominator.

To obtain the value of the fraction $\frac{13}{2}$ we divide the numerator 13 by the denominator 2. 2 is contained in 13 six times, with 1 remaining. This 1 remaining is written over the denominator 2, thereby making the fraction $\frac{1}{2}$, which is annexed to the whole number 6, and we obtain $6\frac{1}{2}$ as the mixed number. The reason for performing this operation is the following: In 1 there are $\frac{2}{2}$ (two halves), and in $\frac{13}{2}$ (thirteen halves) there are as many units (1) as 2 is contained times in 13, which is 6, and $\frac{1}{2}$ (one-half) unit remaining. Hence, $\frac{13}{12} = 6 + \frac{1}{2} = 6\frac{1}{2}$, the required mixed number. Ans. $\frac{17}{4} = 4\frac{1}{4}$. Ans. $\frac{69}{16} = 4\frac{5}{16}$. Ans. $\frac{16}{8} = 2$. Ans. $\frac{67}{64} = 1\frac{3}{64}$. Ans.

(37) In division of fractions, *invert the divisor* (or, in other words, turn it upside down) *and proceed as in multiplication.*

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112$. Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}$. Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}$. Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1,808}{448} = \frac{452}{112} =$

$$\begin{array}{r} \frac{113}{28}\overline{)113}(4\frac{1}{28}. \text{ Ans.} \\ \underline{112} \\ 1 \end{array}$$

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4)+3}{4} = \frac{60+3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8)+3}{8} = \frac{32+3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}$.

$$\frac{18}{5})\,18\,(3\frac{3}{5} \quad \text{Ans.}$$
$$\underline{15}$$
$$3$$

(38) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{1+2+5}{8} = \frac{8}{8} = 1$. Ans.

When the *denominators* of the fractions to be added *are alike*, we know that the units are divided into the *same number of parts* (in this case *eighths*); we, therefore, *add the numerators* of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$ or 1 as the sum.

(39) When the *denominators* are *not alike* we know that the units are divided into *unequal parts*, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to sixteenths and then add their numerators.

$\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms

of the fraction by some number which will make the denominator 16. This number evidently is 4, hence, $\frac{1}{4} \times \frac{4}{4} = \frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3}{8} \times \frac{2}{2} = \frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16} + \frac{6}{16} + \frac{5}{16} = \frac{4+6+5}{16} = \frac{15}{16}$. Ans.

(40) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5}{8} \times \frac{2}{2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers we have $\frac{10}{16} + \frac{7}{16} = \frac{10+7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

42
31
9
$1\frac{1}{16}$
$83\frac{1}{16}$ Ans.

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

(41) $29\frac{3}{4} + 50\frac{5}{8} + 41 + 69\frac{3}{16} = ?$ $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$.

$\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. $\frac{12}{16} + \frac{10}{16} + \frac{3}{16} = \frac{12 + 10 + 3}{16} = \frac{25}{16} = 1\frac{9}{16}$

The problem now becomes $29 + 50 + 41 + 69 + 1\frac{9}{16} = ?$

$$\begin{array}{r} 29 \text{ square inches.} \\ 50 \text{ square inches.} \\ 41 \text{ square inches.} \\ 69 \text{ square inches.} \\ 1\frac{9}{16} \text{ square inches.} \\ \hline 190\frac{9}{16} \text{ square inches.} \quad \text{Ans.} \end{array}$$

(42) (*a*) $\dfrac{\frac{7}{3}}{\frac{16}{}} = 7 \div \frac{3}{16} = 7 \times \frac{16}{3} = \frac{7 \times 16}{3} = \frac{112}{3} = 37\frac{1}{3}$. Ans.

The line between 7 and $\frac{3}{16}$ means that 7 is to be divided by $\frac{3}{16}$.

(*b*) $\dfrac{\frac{15}{32}}{\frac{5}{8}} = \frac{15}{32} \div \frac{5}{8} = \frac{15}{32} \times \frac{8}{5} = \frac{\overset{3}{\cancel{15}} \times \overset{1}{\cancel{8}}}{\underset{4}{\cancel{32}} \times \cancel{5}} = \frac{3}{4}$. Ans.

(*c*) $\dfrac{\frac{4+3}{2+6}}{5} = \dfrac{\frac{7}{8}}{5} = \frac{7}{8 \times 5} = \frac{7}{40}$. (See Art. **131.**) Ans.

(43) $\frac{7}{8}$ = value of the fraction, and 28 = the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32} = \frac{7}{8}$. Hence, 32 is the required denominator. Ans.

(44) (*a*) $\frac{7}{8} - \frac{7}{16} = ?$ When the *denominators* of fractions are *not alike* it is evident that the units are divided into *unequal parts*, therefore, before subtracting, *reduce the*

fractions to fractions having a common denominator. Then, *subtract the numerators, and place the remainder over the common denominator.*

$$\frac{7\times 2}{8\times 2}=\frac{14}{16}. \quad \frac{14}{16}-\frac{7}{16}=\frac{14-7}{16}=\frac{7}{16}. \quad \text{Ans.}$$

(*b*) $13-7\frac{7}{16}=?$ This problem may be solved in two ways:

First: $13=12\frac{16}{16}$, since $\frac{16}{16}=1$, and $12\frac{16}{16}=12+\frac{16}{16}=12+1=13$.

$12\frac{16}{16}$
$7\frac{7}{16}$
$5\frac{9}{16}$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12-7=5$ and $\frac{16}{16}-\frac{7}{16}=\frac{16-7}{16}=\frac{9}{16}$. $5+\frac{9}{16}=5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$$13=\frac{13}{1}=\frac{13\times 16}{1\times 16}=\frac{208}{16}. \quad 7\frac{7}{16}=\frac{(7\times 16)+7}{16}=\frac{112+7}{16}=\frac{119}{16}.$$

Subtracting, we have $\frac{208}{16}-\frac{119}{16}=\frac{208-119}{16}=\frac{89}{16}$ and the same result that was obtained by the first method.

$\frac{89}{16}=16)89(5\frac{9}{16}$
$\underline{80}$
$\frac{9}{16}$

(*c*) $312\frac{9}{16}-229\frac{5}{32}=?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this we have $\frac{9}{16}=\frac{9\times 2}{16\times 2}=\frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312-229=83$ and $\frac{18}{32}-\frac{5}{32}=\frac{18-5}{32}=\frac{13}{32}$.

$312\frac{18}{32}$
$229\frac{5}{32}$
$83\frac{13}{32}$

$83+\frac{13}{32}=83\frac{13}{32}$. Ans.

(45) The man evidently traveled $85\frac{5}{12}+78\frac{9}{15}+125\frac{17}{35}$ miles.

Adding the fractions separately in this case,

$$\frac{5}{12}+\frac{9}{15}+\frac{17}{35}=\frac{5}{12}+\frac{3}{5}+\frac{17}{35}=\frac{175+252+204}{420}=\frac{631}{420}=1\frac{211}{420}.$$

Adding the whole numbers and the mixed number representing the sum of the fractions, the sum is $289\frac{211}{420}$ miles. Ans.

$$\begin{array}{r} 85 \\ 78 \\ 125 \\ 1\frac{211}{420} \\ \hline 289\frac{211}{420} \end{array}$$

To find the least common denominator, we have

$$\begin{array}{r|l} 5 & 12,\ 5,\ 35 \\ 7 & 12,\ 1,\ 7 \\ \hline & 12,\ 1,\ 1, \end{array} \quad \text{or } 5\times 7\times 12=420.$$

(46)

$573\frac{4}{5}$ tons. $\quad \frac{4}{5}=\frac{32}{40}$

$216\frac{5}{8}$ tons. $\quad \frac{5}{8}=\frac{25}{40}$

difference $357\frac{7}{40}$ tons. Ans. $\quad \frac{7}{40}=$ *difference.*

(47) Reducing $9\frac{1}{4}$ to an improper fraction, it becomes $\frac{37}{4}$. Multiplying $\frac{37}{4}$ by $\frac{3}{8}$, $\frac{37}{4}\times\frac{3}{8}=\frac{111}{32}=3\frac{15}{32}$ dollars. Ans

(48) Referring to Arts. **114** and **116,**

$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 multiplied by $\frac{7}{8}$ of $\frac{5}{6}$ of $45=$

$$\frac{\overset{1}{\cancel{2}}\times\overset{1}{\cancel{3}}\times 7\times 19\times\overset{1}{\cancel{11}}\times 7\times 5\times\overset{3}{\cancel{45}}^{15}}{\underset{1}{\cancel{3}}\times 4\times\overset{1}{\cancel{11}}\times\underset{4}{\cancel{20}}\times 1\times 8\times\underset{1}{\cancel{6}}\times 1}=\frac{7\times 19\times 7\times 5\times 3}{4\times 4\times 8}=\frac{13{,}965}{128}=$$

$109\frac{13}{128}$. Ans.

(49) $\frac{3}{4}$ of $16=\frac{3}{\cancel{4}}\times\frac{\overset{4}{\cancel{16}}}{1}=12.$ $\quad 12\div\frac{2}{3}=\frac{\overset{6}{\cancel{12}}}{1}\times\frac{3}{\cancel{2}}=18.$ Ans.

(50) $211\frac{1}{4}\times 1\frac{7}{8}=\frac{845}{4}\times\frac{15}{8}$, reducing the mixed numbers

to improper fractions. $\frac{845}{4} \times \frac{15}{8} = \frac{12,675}{32}$ cents = amount paid for the lead. The number of pounds sold is evidently $\frac{12,675}{32} \div 2\frac{1}{2} = \frac{\overset{2,535}{\cancel{12,675}}}{\underset{16}{\cancel{32}}} \times \frac{2}{\cancel{5}} = \frac{2,535}{16} = 158\frac{7}{16}$ pounds. The amount remaining is $211\frac{1}{4} - 158\frac{7}{16} = \frac{845}{4} - \frac{2,535}{16} = \frac{3,380}{16} - \frac{2,535}{16} = \frac{845}{16} = 52\frac{13}{16}$ pounds. Ans.

(51)

tenths.	hundredths.	thousandths.	ten-thousandths.	millionths.	hundred-thousandths.	ten-millionths.	
. 0	8						= *Eight* **hundredths.**
. 1	3	1					= *One hundred thirty-one* **thousandths.**
0	0	0	1				= *One* **ten-thousandth.**
0	0	0	0	0	2	7	= *Twenty-seven* **millionths.**
0	1	0	8				= *One hundred eight* **ten-thousandths.**

tenths.
hundredths.
thousandths.
ten-thousandths.

93.0 1 0 1 = Ninety-three, and *one hundred one* ***ten-thousandths***

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the *name* of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example above, the simple reading of the figure is *eight*, and the name of its position in the decimal scale is **hundredths,** so that the decimal reading is *eight* **hundredths.** Similarly, the figures in the fourth example are ordinarily read *twenty-seven;* the name of the position of the figure 7 in the decimal scale is **millionths,** giving, therefore, the decimal reading as *twenty-seven* **millionths.**

If there should be a whole number before the decimal point, read it as you would read any whole number, and read the decimal as you would if the whole number were not there; or, read the whole number and then say, "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three, *and* one hundred one ten thousandths."

(52) See Art. **139.**

(53) See Art. **153.**

(54) See Art. **160.**

(55) A fraction is one or more of the equal parts of a unit, and is expressed by a numerator and a denominator, while a decimal fraction is a number of *tenths*, *hundredths*, *thousandths*, etc., of a unit, and is expressed by placing a period (.), called a decimal point, to the left of the figures of the number, and omitting the denominator.

(56) See Art. **165.**

(57) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the quotient, or .5, since only *one* cipher was annexed to the numerator. Ans.

$\frac{7}{8}$)7.000
.875 Ans.

$\frac{5}{32}$)5.00000(.15625 Ans.
32
180
160
200
192
80
64
160
160

Since $.65 = \frac{65}{100}$, then, $\frac{65}{100}$ must equal .65. Or, when the denominator is 10, 100, 1000, etc., point off as many places in the numerator as there are ciphers in the denominator. Doing so, $\frac{65}{100} = .65$. Ans.

$\frac{125}{1000} = .125$. Ans.

(58) (*a*) This example, written in the form of a fraction, means that the numerator $(32.5 + .29 + 1.5)$ is to be divided by the denominator $(4.7 + 9)$. The operation is as follows:

$$\frac{32.5 + .29 + 1.5}{4.7 + 9} = ?$$

32.5
\+ .29
\+ 1.5

13.7)34.29000(2.5029 Ans.
274
689
685
400
274
1260
1233
27

4.7
\+ 9.0
13.7

Since there are 5 decimal places in the dividend and 1 in the divisor, there are 5 − 1 or 4 places to be pointed off in the quotient. The fifth figure of the decimal is evidently less than 5.

(*b*) Here again the problem is to divide the numerator, which is $(1.283 \times \overline{8+5})$, by the denominator, which is 2.63. The operation is as follows:

$$\frac{1.283 \times \overline{8+5}}{2.63} = ? \quad \overline{8+5} = 13.$$

$$\begin{array}{r} 1.283 \\ \times \quad 13 \\ \hline 3849 \\ 1283 \\ \hline 2.63)16.679000(6.3418 \quad \text{Ans.} \\ 1578 \\ \hline 899 \\ 789 \\ \hline 1100 \\ 1052 \\ \hline 480 \end{array} \qquad \begin{array}{r} 480 \\ 263 \\ \hline 2170 \\ 2104 \\ \hline 66 \end{array}$$

(*c*) $\dfrac{\overline{589+27} \times \overline{163-8}}{25+39} = ?$

$$\begin{array}{r} 589 \\ +\ 27 \\ \hline 616 \end{array} \qquad \begin{array}{r} 25 \\ +39 \\ \hline 64 \end{array}$$

$$\begin{array}{r} 163 \\ -\ \ 8 \\ \hline 155 \\ \times 616 \\ \hline 930 \\ 155 \\ 930 \\ \hline 64)95480.000(1491.875 \quad \text{Ans.} \\ 64 \\ \hline 314 \\ 256 \\ \hline 588 \\ 576 \\ \hline 120 \\ 64 \\ \hline 560 \\ 512 \\ \hline 480 \\ 448 \\ \hline 320 \\ 320 \end{array}$$

There are three decimal places in the quotient, since three ciphers were annexed to the dividend.

(*d*) $\dfrac{\overline{40.6+7.1}\times(3.029-1.874)}{6.27+8.53-8.01}=?$

$$\begin{array}{rr}
40.6 & 3.029 \\
+\ 7.1 & -\ 1.874 \\
\hline
47.7 & 1.155 \\
 & \times\ 47.7 \\
\hline
6.27 & 8085 \\
+\ 8.53 & 8085 \\
\hline
14.80 & 4620 \\
-\ 8.01 & \\
\hline
6.79 & 6.79)55.093500(8.1139.\ \text{Ans} \\
 & 5432 \\
\hline
 & 773 \\
 & 679 \\
\hline
 & 945 \\
 & 679 \\
\hline
 & 2660 \\
 & 2037 \\
\hline
 & 6230 \\
 & 6111 \\
\hline
 & 119
\end{array}$$

6 decimal places in the dividend − 2 decimal places in the divisor = 4 decimal places to be pointed off in the quotient.

(59) $.875=\dfrac{875}{1{,}000}=\dfrac{175}{200}=\dfrac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\frac{7}{8}\text{ of 1 foot}=\frac{7}{\cancel{8}_{2}}\times\frac{\overset{3}{\cancel{12}}}{1}=\frac{21}{2}=10\frac{1}{2}\text{ inches.}\quad\text{Ans.}$$

(60) 12 inches = 1 foot.

$$\frac{3}{16}\text{ of an inch}=\frac{3}{16}\div 12=\frac{\cancel{3}}{16}\times\frac{1}{\cancel{12}_{4}}=\frac{1}{64}\text{ of a foot.}$$

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, 6 − 0 = 6 places to be pointed off.

$$\frac{1}{64}) 1.000000 (.015625 \text{ Ans.}$$

64
360
320
400
384
160
128
320
320

(61) If 1 cubic inch of water weighs .03617 of a pound, the weight of 1,500 cubic inches will be $.03617 \times 1{,}500 = 54.255$ lb.

.03617 lb.
1500
1808500
3617
54.25500 lb. Ans.

(62) 72.6 feet of fencing at $.50 a foot would cost

72.6 × .50, or $36.30.
.50
$36.300

If, by selling a carload of coal at a profit of $1.65 per ton, I make $36.30, then there must be as many tons of coal in the car as 1.65 is contained times in 36.30, or 22 tons.

1.65) 36.30 (22 tons. Ans.
330
330
330

(63) 231) 17892.00000 (77.454454, or 77.4545 to four decimal places. Ans.

1617

1722
1617

1050
924

1260
1155

1050
924

1260
1155

1050

(64) $\dfrac{37.13 \times 24 \times 3.1416 \times 19 \times 19 \times 350}{33,000 \times 12 \times 4} =$

$\dfrac{37.13 \times .0952 \times 19 \times 19 \times 350}{1,000} = \dfrac{446,618.947600}{1,000} =$

446.619 to three decimal places. Ans.

37.13 × .0952 = 7426, 18565, 33417 = 3.534776

19 × 19 = 171, 19 = 361

361 × 350 = 18050, 1083 = 126350

3.534776 × 126350 = 176738800, 10604328, 21208656, 7069552, 3534776 = 446618.947600

(65) See Art. **174.** Applying rule in Art. **175,**

(*a*) $.7928 \times \frac{64}{64} = \frac{50.7392}{64} = \frac{51}{64}$. Ans.

(*b*) $.1416 \times \frac{32}{32} = \frac{4.5312}{32} = \frac{5}{32}$. Ans.

(*c*) $.47915 \times \frac{16}{16} = \frac{7.6664}{16} = \frac{8}{16} = \frac{1}{2}$. Ans

(66) In subtraction of decimals, *place the decimal points directly under each other*, and proceed as in the subtraction of whole numbers, placing the *decimal point in the remainder directly under the decimal points above.*

(*a*) 709.6300 − .8514 = 708.7786 Ans.

In the above example we proceed as follows: We can not subtract 4 ten-thousandths from 0 ten-thousandths, and, as there are no thousandths, we take 1 hundredth from the three hundredths. 1 *hundredth* = 10 *thousandths* = 100 *ten-thousandths*. 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 *thousandths* + 6 *ten-thousandths*. Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths makes it but 2 hundredths now). Since we can not do this, we take 1 tenth from 6 tenths. 1 tenth (= 10 hundredths) + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this can not be done, we take 1 unit from the 9 units. 1 *unit* = 10 *tenths;* 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (one unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

(*b*) 81.963 − 1.700 = 80.263 Ans.

(*c*) 18.00 − .18 = 17.82 Ans.

(*d*) 1.000 − .001 = .999 Ans.

(*e*) $872.1 - (.8721 + .008) = ?$ In this problem we are to subtract $(.8721 + .008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

$$\begin{array}{r} .8721 \\ .008 \\ \hline .8801 \end{array}$$ *sum.*

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

$$\begin{array}{r} 872.1000 \\ .8801 \\ \hline 871.2199 \end{array}$$ Ans.

(*f*) $(5.028 + .0073) - (6.704 - 2.38) = ?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

$$\begin{array}{r} 5.0280 \\ .0073 \\ \hline 5.0353 \end{array}$$ *sum.*

$$\begin{array}{r} 6.704 \\ 2.380 \\ \hline 4.324 \end{array}$$ *difference.*

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324. The sign between the parentheses indicates that the quantities obtained by performing the above operations, are to be subtracted, namely, that 4.324 is to be subtracted from 5.0353. Performing this operation we obtain .7113 as the final result.

$$\begin{array}{r} 5.0353 \\ 4.324 \\ \hline .7113 \end{array}$$ Ans.

(67) In subtracting a decimal from a fraction, or subtracting a fraction from a decimal, either reduce the fraction to a decimal before subtracting, or reduce the decimal to a fraction and then subtract.

(*a*) $\frac{7}{8} - .807 = ?$ $\frac{7}{8}$ reduced to a decimal becomes

$$\frac{7}{8})\overline{7.000}$$
$$.875$$

Subtracting .807 from .875 the remainder is .068, as shown.

$$\begin{array}{r} .875 \\ .807 \\ \hline .068 \end{array}$$ Ans.

(*b*) $.875 - \frac{3}{8} = ?$ Reducing .875 to a fraction we have $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; hence, $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$. Ans.

Or, by reducing $\frac{3}{8}$ to a decimal, $\frac{3}{8}$)3.000(.375 and then subtracting, we obtain $.875 - .375 = .5 = \frac{5}{10} = \frac{1}{2}$, the same answer as above.

.875
.375
.500 Ans.

(*c*) $\left(\frac{5}{32} + .435\right) - \left(\frac{21}{100} - .07\right) = ?$ We first perform the operations as indicated by the signs between the numbers enclosed by the parentheses. Reduce $\frac{5}{32}$ to a decimal and we obtain $\frac{5}{32} = .15625$ (see example 57).

Adding .15625 and .435, .15625 + .435 = *sum* .59125

$\frac{21}{100} = .21$; subtracting, .21 − .07 = *difference* .14

We are now prepared to perform the operation indicated by the minus sign between the parentheses, which is,

.59125
.14
difference .45125 Ans.

(*d*) This problem means that 33 millionths and 17 thousandths are to be added. Also, that 53 hundredths and 274 thousandths are to be added, and the smaller of these sums is to be subtracted from the larger sum. Thus, $(.53 + .274) - (.000033 + .017) = ?$

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
.0	0	0	0	3	3	
.0	1	7				
.0	1	7	0	3	3	*sum.*

tenths.	hundredths.	thousandths.	
.5	3		
.2	7	4	
.8	0	4	*sum*

.804 *larger sum,*
.017033 *smaller sum,*
difference .786967 Ans.

(68) In addition of decimals the *decimal points must be placed directly under each other*, so that *tenths* will come *under tenths*, *hundredths under hundredths*, *thousandths under thousandths*, etc. The addition is then performed as in whole numbers, *the decimal point of the sum being placed directly under the decimal points above.*

.125
.7
.089
.4005
.9
.000027

2.214527 Ans.

(69)

927.416
8.274
372.6
62.07938

1370.36938 **Ans.**

(70)

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.
.0	1	7			
.2					
.0	0	0	0	4	7

.217047 = **Ans.**

(71) (*a*)

.107
.013

321
107

.001391 Ans.

There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3 or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

(*b*)

203
2.03

609
4060

412.09
.203

123627
824180

83.65427 **Ans.**

There are two decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0 or two decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2 or 5 decimal places in the second product.

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parenthesis, and then perform whatever may be required by the sign before the parenthesis.

Multiply together the numbers 2.7 and 31.85.

The parenthesis shows that .316 is to be taken from 3.16.

```
  3.160
   .316
 -------
  2.844
```

```
  31.85
    2.7
 -------
  22295
  6370
 -------
  85.995
```

The product obtained by the first operation is now multiplied by the remainder obtained by performing the operation indicated by the signs within the parenthesis.

```
   85.995
    2.844
 ---------
   343980
  343980
 687960
171990
-----------
244.569780   Ans.
```

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

```
   107.8
+    6.541
 ---------
   114.341
-   31.96
 ---------
    82.381
 x   1.742
 ---------
   164762
  329524
 576667
 82381
-----------
143.507702   Ans.
```

(72) (*a*) $\left(\frac{7}{16} - .13\right) \times \overline{.625 + \frac{5}{8}} = ?$

First perform the operation indicated by the parenthesis.

$\frac{7}{16} = \frac{7}{16}$)7.0000(.4375

64

60

48

120

112

80

80

We point off four decimal places since we annexed four ciphers.

.4375

.13

Subtracting, we obtain .3075

The vinculum has the same meaning as the parenthesis; hence, we perform the operation indicated by it. We point off three decimal places, since three ciphers were annexed to the 5.

$\frac{5}{8} = \frac{5}{8}$) 5.000

.625

Adding the terms included by the vinculum, we obtain

.625

.625

1.250

The final operation is to perform the work indicated by the sign between the parenthesis and the vinculum, thus,

.3075

1.25

15375

6150

3075

.384375 Ans.

(*b*) $\left(\frac{19}{32} \times .21\right) - \left(.02 \times \frac{3}{16}\right) = ?$

$.21 = \frac{21}{100}$. $\frac{19}{32} \times \frac{21}{100} = \frac{399}{3200}$. $.02 = \frac{2}{100}$. $\frac{2}{100} \times \frac{3}{16} = \frac{6}{1600} = \frac{3}{800}$.

$\frac{3}{800} = \frac{3}{800} \times \frac{4}{4} = \frac{12}{3200}$. $\frac{399}{3200} - \frac{12}{3200} = \frac{399 - 12}{3200} = \frac{387}{3200}$.

Reducing $\frac{387}{3200}$ to a decimal, we obtain

$\frac{387}{3200}$) 387.0000000 (.1209375 Ans.

3200

6700

6400

30000

28800

12000

9600

24000

22400

16000

16000

Point off seven decimal places, since seven ciphers were annexed to the dividend.

(c) $\left(\frac{13}{4} + .013 - 2.17\right) \times \overline{13\frac{1}{4} - 7\frac{5}{16}} = ?$

$\frac{13}{4} = \frac{13}{4}$) 13.00 (3.25

Point off two decimal places, since two ciphers were annexed to the dividend.

$\frac{5}{16}$ reduced to a decimal is .3125, since

$\frac{5}{16}$) 5.0000 (.3125

48

20

16

40

32

80

80

Point off four decimal places, since four ciphers were annexed to the dividend.

3.25

+ .013

3.263

− 2.17

1.093 .

Then, $7\frac{5}{16} = 7.3125$, and $13\frac{1}{4} = 13.25$, since $\frac{1}{4} = \frac{1}{4}$) 1.00 (.25

$$\begin{array}{r} 13.25 \\ -\ 7.3125 \\ \hline 5.9375 \end{array} \qquad \begin{array}{r} 5.9375 \\ \times\ 1.093 \\ \hline 178125 \\ 534375 \\ 593750 \\ \hline 6.4896875 \end{array}$$ Ans.

(73) (*a*) $.875 \div \frac{1}{2} = .875 \div .5 \left(\text{since } \frac{1}{2} = .5\right) = 1.75$. Ans.

Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875 = \frac{7}{8}$, since $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; then, $\frac{7}{8} \div \frac{1}{2} = \frac{7}{8} \times \frac{2}{1} = \frac{7}{4} = 1\frac{3}{4}$. Since $\frac{3}{4} = \frac{3}{4})\,3.00\,(.75$, $1\frac{3}{4} = 1.75$,

$\begin{array}{r} 28 \\ \hline 20 \\ 20 \\ \hline \end{array}$

the same answer as above.

(*b*) $\frac{7}{8} \div .5 = \frac{7}{8} \div \frac{1}{2} \left(\text{since } .5 = \frac{1}{2}\right) = \frac{7}{8} \times \frac{2}{1} = \frac{7}{4} = 1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8} = .875$; $.875 \div .5 = 1.75$. Since there are three decimal places in the dividend and one in the divisor, there are $3 - 1$, or 2 decimal places in the quotient.

(*c*) $\frac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125} = ?$ We shall solve this problem by first reducing the decimals to their equivalent common fractions.

$.375 = \frac{375}{1{,}000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$. $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$, or the value of the numerator of the fraction.

$.125 = \frac{125}{1{,}000} = \frac{25}{200} = \frac{1}{8}$. Reducing $\frac{1}{8}$ to sixteenths, we have $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$. Then, $\frac{5}{16} - \frac{2}{16} = \frac{3}{16}$, or the value of the de-

nominator of the fraction. The problem is now reduced to

$$\frac{\frac{3}{32}}{\frac{3}{16}} = ? \qquad \frac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{\cancel{3}}{\cancel{32}_{2}} \times \frac{\cancel{16}}{\cancel{3}} = \frac{1}{2} \text{ or } .5. \quad \text{Ans.}$$

(74) $\dfrac{1.25 \times 20 \times 3}{87 + (11 \times 8)} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

$$\begin{array}{r} 1.25 \\ \times \quad 20 \\ \hline 25.00 \\ \times \quad 3 \\ \hline 75 \end{array}$$

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\dfrac{87 + (11 \times 8)}{459 + 32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87 + 88 = 175$.

The numerator is combined as though it were written $87 + (11 \times 8)$, and its result is

$$\begin{array}{r} 11 \\ 8 \text{ times} \\ \hline 88 \\ +87 \\ \hline 175 \end{array}$$

The value of the denominator of this fraction is equal to $459 + 32 = 491$. The problem then becomes

$$\frac{\frac{75}{175}}{\frac{}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \frac{\overset{3}{\cancel{75}} \times 491}{\underset{7}{\cancel{175}}} = \frac{1,473}{7} = 210\frac{3}{7}. \quad \text{Ans.}$$

(75) 1 plus $.001 = 1.001$. $.01$ plus $.000001 = .010001$. And $1.001 - .010001 =$

$$\begin{array}{r} 1.001 \\ .010001 \\ \hline .990999 \end{array} \quad \text{Ans.}$$

ARITHMETIC.

(QUESTIONS 76–167.)

(76) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. Hence, 25% of 8,428 lb. = .25 × 8,428 lb. = 2,107 lb. Ans.

(77) Here \$100 is the base and 1% = .01 is the rate. Then, .01 × \$100 = \$1. Ans.

(78) $\frac{1}{2}$% means one-half of one per cent. Since 1% is .01, $\frac{1}{2}$% is .005, for, $2\overline{)}.010$ / .005. And .005 × \$35,000 = \$175. Ans.

(79) Here 50 is the base, 2 is the percentage, and it is required to find the rate. Applying rule, Art. **193,**

rate = percentage ÷ base;
rate = 2 ÷ 50 = .04 or 4%. Ans.

(80) By Art. **193,** rate = percentage ÷ base.*

As percentage = 10 and base = 10, we have rate = 10 ÷ 10 = 1 = 100%. Hence, 10 is 100% of 10. Ans.

(81) (*a*) Rate = percentage ÷ by base. Art. **193.**

As percentage = \$176.54 and base = \$2,522, we have

rate = 176.54 ÷ 2,522 = .07 = 7%. Ans.

$$2522\overline{)}176.54 \;/\; .07$$

* Remember that an expression of this form means that the first term is to be *divided by* the second term. Thus, as above, it means percentage *divided by* base.

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(*b*) Base = percentage ÷ rate. Art. **192.**

As percentage = 16.96 and rate = 8% = .08, we have

base = 16.96 ÷ .08 = 212. Ans.

$$.08\overline{)16.96}\quad 212$$

(*c*) Amount is the sum of the base and percentage; hence, the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base. Art. **193.**

Therefore, rate = 3.2025 ÷ 213.5 = .015 = 1½%. Ans.

$$213.5\overline{)3.2025}\ (.015 = 1\tfrac{1}{2}\%$$

2135

10675

10675

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base − the difference = the percentage. Base = 207 and difference = 201.825, hence percentage = 207 − 201.825 = 5.175.

Rate = percentage ÷ base. Art. **193.**

Therefore, rate = 5.175 ÷ 207 = .025 = $.02\frac{1}{2} = 2\frac{1}{2}\%$. Ans.

$$207\overline{)5.175}\ (.025$$

414

1035

1035

(82) In this problem $5,500 is the amount, since it equals what he paid for the farm + what he gained; 15% is the rate, and the cost (to be found) is the base. Applying rule, Art. **197,**

base = amount ÷ (1 + rate); hence,

base = $5,500 ÷ (1 + .15) = $4,782.61. Ans.

$$
\begin{array}{r}
1.15\overline{)5500.0000}(4782.61 \\
\underline{466} \\
900 \\
\underline{805} \\
950 \\
\underline{920} \\
300 \\
\underline{230} \\
700 \\
\underline{690} \\
100 \\
\underline{115}
\end{array}
$$

The example can also be solved as follows: 100% = cost; if he gained 15%, then 100 + 15 = 115% = $5,500, the selling price.

If 115% = $5,500, $1\% = \frac{1}{115}$ of $5,500 = $47.8261, and 100%, or the cost, = 100 × $47.8261 = $4,782.61. Ans.

(83) 24 % of $950 = .24 × 950 = $228

$12\frac{1}{2}$% of $950 = .125 × 950 = 118.75

17 % of $950 = .17 × 950 = 161.50

$53\frac{1}{2}$% of $950 = $508.25

The total amount of his yearly expenses, then, is $508.25, hence his savings are $950 − $508.25 = $441.75. Ans.

Or, as above, $24\% + 12\frac{1}{2}\% + 17\% = 53\frac{1}{2}\%$, the total percentage of expenditures; hence, $100\% - 53\frac{1}{2}\% = 46\frac{1}{2}\%$ = per cent. saved. And $950 × .465 = $441.75 = his yearly savings. Ans.

(84) The percentage is 961.38, and the rate is $37\frac{1}{2}$. By Art. **192**,

Base = percentage ÷ rate

= 961.38 ÷ .375 = 2,563.68, the number. Ans.

Another method of solving is the following:

If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38 and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

```
.375)961.38000(2563.68
     750
    2113
    1875
     2388
     2250
      1380
      1125
       2550
       2250
        3000
        3000
```

(85) Here \$1,125 is 30% of some number; hence, \$1,125 = the percentage, 30% = the rate, and the required number is the base. Applying rule, Art. **192,**

Base = percentage ÷ rate = \$1,125 ÷ .30 = \$3,750.

Since \$3,750 is $\frac{3}{4}$ of the property, one of the fourths is $\frac{1}{3}$ of \$3,750 = \$1,250, and $\frac{4}{4}$ or the entire property, is 4 × \$1,250 = \$5,000. Ans.

(86) Here \$4,810 is the difference and 35% the rate. By Art. **198,**

Base = difference ÷ (1 − rate)
= \$4,810 ÷ (1 − .35) = \$4,810 ÷ .65 = \$7,400. Ans.

```
.65)4810.00(7400
    455
    260
    260
      00
```

```
1.00
 .35
 .65
```

Solution can also be effected as follows: 100% = the sum diminished by 35%, then (1 − .35) = .65, which is \$4,810.

If $65\% = \$4{,}810$, $1\% = \frac{1}{65}$ of $4{,}810 = \$74$, and $100\% = 100 \times \$74 = \$7{,}400$. Ans.

(87) In this example the sales on Monday amounted to \$197.55, which was $12\frac{1}{2}\%$ of the sales for the entire week; i. e., we have given the percentage, \$197.55, and the rate, $12\frac{1}{2}\%$, and the required number (or the amount of sales for the week) equals the base. By Art. **192,**

Base = percentage ÷ rate = \$197.55 ÷ .125;

or,

$$
\begin{array}{r}
.125\overline{)197.5500}(1580.4 \text{ Ans.} \\
125 \\
\hline
725 \\
625 \\
\hline
1005 \\
1000 \\
\hline
500 \\
500 \\
\hline
\end{array}
$$

Therefore, base = \$1,580.40, which also equals the sales for the week.

(88) 16.5 miles = $12\frac{1}{2}\%$ of the entire length of the road. We wish to find the *entire* length.

16.5 miles is the percentage, $12\frac{1}{2}\%$ is the rate, and the entire length will be the base. By Art. **192,**

Base = percentage ÷ rate = $16.5 \div .12\frac{1}{2}$.

$$
\begin{array}{r}
.125\overline{)16.500}(132 \text{ miles. Ans.} \\
125 \\
\hline
400 \\
375 \\
\hline
250 \\
250 \\
\hline
\end{array}
$$

(89) Here we have given the difference, or \$35, and the rate, or 60%, to find the base. We use the rule in Art. **198,**

Base = difference ÷ (1 − rate)

= \$35 ÷ (1 − .60) = \$35 ÷ .40 = \$87.50. Ans.

```
.40)35.000(87.5
    320
    ---
     300
     280
     ---
      200
      200
      ---
```

Or, 100% = whole debt; 100% − 60% = 40% = \$35.

If 40% = \$35, then 1% = $\frac{1}{40}$ of \$35 = $\frac{35}{40}$, and 100% = $\frac{35}{40} \times 100$ = \$87.50. Ans.

(90) 28 rd. 4 yd. 2 ft. 10 in. to inches.

```
  5½
 ×
 ---
  154
+   4
 ---
  158 yards
×   3
 ---
  474
+   2
 ---
  476 feet
×  12
 ---
 5712
+  10
 ---
 5722 inches. Ans.
```

Since there are 5½ yards in one rod, in 28 rods there are 28 × 5½ or 154 yards; 154 yards plus 4 yards = 158 yards. There are 3 feet in one yard; therefore, in 158 yards there are 3 × 158 or 474 feet; 474 feet + 2 feet = 476 feet. There are 12 inches in one foot, and in 476 feet there are 12 × 476 or 5,712 inches; 5,712 inches + 10 inches = 5,722 inches. Ans.

(91)

```
12 ) 5722 inches.
 3 ) 476 + 10 inches.
5½ ) 158 + 2 feet.
       28 + 4 yards.
```

Ans. = 28 rd. 4 yd. 2 ft. 10 in.

EXPLANATION.—There are 12 inches in 1 foot; hence, in 5,722 inches there are as many feet as 12 is contained times in 5,722 inches, or 476 ft. and 10 inches remaining. Write these 10 inches as a remainder. There are 3 feet in 1 yard; hence, in 476 feet there are as many yards as 3 is contained times in 476 feet, or 158 yards and 2 feet remaining. There are $5\frac{1}{2}$ yards in one rod; hence, in 158 yards there are 28 rods and 4 yards remaining. Then, in 5,722 inches there are 28 rd. 4 yd. 2 ft. 10 in.

(92)

5 weeks 3.5 days.
× 7
35 days in 5 weeks.
+ 3.5
38.5 days.

Then, we find how many seconds there are in 38.5 days.

38.5 days
× 24 hours in one day.
1540
770
924.0 hours in 38.5 days.
× 60 minutes in one hour.
55440 minutes in 38.5 days.
× 60 seconds in one minute.
3326400 seconds in 38.5 days. Ans.

(93) Since there are 24 gr. in 1 pwt., in 13,750 gr. there are as many pennyweights as 24 is contained times in 13,750, or 572 pwt. and 22 gr. remaining. Since there are 20 pwt. in 1 oz., in 572 pwt. there are as many ounces as 20 is contained times in 572, or 28 oz. and 12 pwt. remaining.

Since there are 12 oz. in 1 lb. (Troy), in 28 oz. there are as many pounds as 12 is contained times in 28, or 2 lb. and 4 oz. remaining. We now have the pounds and ounces required by the problem; therefore, in 13,750 gr. there are 2 lb. 4 oz. 12 pwt. 22 gr.

24) 13750 gr.
20) 572 pwt. + 22 gr.
12) 28 oz. + 12 pwt.
2 lb. + 4 oz.

Ans. = 2 lb. 4 oz. 12 pwt. 22 gr.

(94) 100) 4763254 li.
80) 47632 + 54 li.
595 + 32 ch.

Ans. = 595 mi. 32 ch. 54 li.

EXPLANATION.—There are 100 li. in one chain; hence, in 4,763,254 li. there are as many chains as 100 is contained times in 4,763,254 li., or 47,632 ch. and 54 li. remaining. Write the 54 li. as a remainder. There are 80 ch. in one mile; hence, in 47,632 ch. there are as many miles as 80 is contained times in 47,632 ch., or 595 miles and 32 ch. remaining. Then, in 4,763,254 li. there are 595 mi. 32 ch. 54 li.

(95) 1728) 764325 cu. in.
27) 442 + 549 cu. in.
16 cu. yd. + 10 cu. ft.

Ans. = 16 cu. yd. 10 cu. ft. 549 cu. in.

EXPLANATION.—There are 1,728 cu. in. in one cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cu. ft. and 549 cu. in. remaining. Write the 549 cu. in. as a remainder. There are 27 cu. ft. in one cubic yard; hence, in 442 cu. ft. there are as many cubic yards as 27 is contained times in 442 cu. ft., or 16 cu. yd. and 10 cu. ft. remaining. Then, in 764,325 cu. in. there are 16 cu. yd. 10 cu. ft. 549 cu. in.

(96) We must arrange the different terms in columns, taking care to have like denominations in the same column.

	rd.	yd.	ft.	in.	
	2	2	2	3	
		4	1	9	
			2	7	
	3	$2\frac{1}{2}$	0	7	
or	3	2	2	1	Ans.

EXPLANATION.—We begin to add at the right-hand column. $7 + 9 + 3 = 19$ in.; as 12 in. make one foot, 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the next column.

1 (reserved) $+ 2 + 1 + 2 = 6$ ft. Since 3 ft. make 1 yard, 6 ft. = 2 yd. and 0 ft. remaining. Place the cipher in the column of feet and reserve the 2 yd. for the next column.

2 (reserved) $+ 4 + 2 = 8$ yd. Since $5\frac{1}{2}$ yd. = 1 rod, 8 yd. = 1 rd. and $2\frac{1}{2}$ yd. Place $2\frac{1}{2}$ yd. in the yards column, and reserve 1 rd. for the next column; 1 (reserved) $+ 2 = 3$ rd.

Ans. =	3 rd.	$2\frac{1}{2}$ yd.	0 ft.	7 in.
or,	3 rd.	2 yd.	1 ft.	13 in.
or,	3 rd.	2 yd.	2 ft.	1 in. Ans.

(97) We write the compound numbers so that the units of the same denomination shall stand in the same column. Beginning to add with the lowest denomination, we find that the sum of the gills is $1 + 2 + 3 = 6$. Since there are 4 gi. in 1 pint, in 6 gi. there are as many pints as 4 is contained times in 6, or 1 pt. and 2 gi. We place 2 gi. under the gills column and reserve the 1 pt. for the pints column; the sum of the pints is 1 (reserved) $+ 5 + 1 + 1 = 8$. Since there are 2 pt. in 1 quart, in 8 pt. there are as many quarts as 2 is contained times in 8, or 4 qt. and 0 pt. We place the cipher under the column of pints and reserve the 4 for the quarts column. The sum of the quarts is 4 (reserved) $+ 8 + 3 = 15$. Since there are 4 qt. in 1 gallon, in 15 qt. there are as many gallons as 4 is contained times in 15, or 3 gal. and 3 qt. remaining. We now place the 3 under the quarts column and reserve the 3 gal. for the gallons column. The sum of the gallons column is 3 (reserved) $+ 4 + 6 + 3 = 16$ gal. Since we can not reduce 16 gal. to any higher denomination, we have 16 gal. 3 qt. 0 pt. and 2 gi. for the answer.

gal.	qt.	pt.	gi.
3	3	1	3
6	0	1	2
4	0	0	1
	8	5	0

16 gal. 3 qt. 0 pt. 2 gi.

(98) Reduce the grains, pennyweights, and ounces to higher denominations.

24) 240 gr. 20) 125 pwt. 12) 50 oz.

10 pwt. 6 oz. 5 pwt. 4 lb. 2 oz.

Then, 3 lb. + 4 lb. 2 oz. + 6 oz. 5 pwt. + 10 pwt. =

lb.	oz.	pwt.
3		
4	2	
	6	5
		10
7 lb.	8 oz.	15 pwt. Ans.

(99) Since "seconds" is the lowest denomination in this problem, we find their sum first, which is 11 + 29 + 25 + 30 + 12, or 107 seconds. Since there are 60 seconds in 1 minute, in 107″ there are as many minutes as 60 is contained times in 107, or 1 minute and 47 seconds remaining. We place the 47 under the seconds column and reserve the 1 for the minutes column. The sum of the minutes is 1 (reserved) + 17 + 26 + 19 + 16, or 79. Since there are 60 minutes in 1 degree, in 79 minutes there are as many degrees as 60 is contained times in 79, or 1 degree and 19 minutes remaining. We place the 19 under the minutes column and reserve the 1 degree for the degrees column. The sum of the degrees is 1 (reserved) + 10 + 20 + 13 + 11, or 55 degrees. Since we can not reduce 55 degrees to any higher denominations, we have 55° 19′ 47″ for the answer.

deg.	min.	sec.
11	16	12
13	19	30
20	0	25
0	26	29
10	17	11
55°	19′	47″

(100) Since "inches" is the lowest denomination in this problem, we find their sum first, which is 11 + 8 + 6, or 25 inches. Since there are 12 inches in 1 foot, in 25 inches there are as many feet as 12 is contained times in 25, or 2 feet and 1 inch remaining. Place the 1 inch under the inches column, and reserve the 2 feet to add to the column

of feet. The sum of the feet is 2 feet (reserved) + 2 + 1 = 5 feet. Since there are 3 feet in 1 yard, in 5 feet there are as many yards as 3 is contained times in 5 feet, or 1 yard and 2 feet remaining. Place the 2 feet under the column of feet, and reserve the 1 yard to add to the column of yards. The sum of the yards is 1 yard (reserved) + 4 + 5 = 10 yards. Since there are $5\frac{1}{2}$ yards in 1 rod, in 10 yards there are as many rods as $5\frac{1}{2}$ is contained times in 10, or 1 rod and $4\frac{1}{2}$ yards remaining. Place the $4\frac{1}{2}$ yards under the column of yards, and reserve the 1 rod for the column of rods. The sum of the rods is 1 (reserved) + 304 + 215 + 130 = 650 rods. Place 650 rods under the column of rods. Therefore, the sum is 650 rd. $4\frac{1}{2}$ yd. 2 ft. 1 in. Or, since $\frac{1}{2}$ a yard = 1 ft. 6 in., and since there are 320 rods in 1 mile, the sum may be expressed as 2 mi. 10 rd. 5 yd. 0 ft. 7 in. Ans.

rd.	yd.	ft.	in.
130	5	1	6
215	0	2	8
304	4	0	11
650	$4\frac{1}{2}$	2	1

mi.
or, 2 10 5 0 7 Ans.

(101) Since "square links" is the lowest denomination in this problem, we find their sum first, which is 21 + 23 + 16 + 18 + 23 + 21, or 122 square links. Place 122 square links under the column of square links. The sum of the square rods is 2 + 3 + 2 + 2 + 2 + 3, or 14 square rods. Place 14 square rods under the column of square rods. The sum of the square chains is 323 square chains. Since there are 10 square chains in 1 acre, in 323 square chains there are as many acres as 10 is

A.	sq. ch.	sq. rd.	sq. li.
21	67	3	21
28	78	2	23
47	6	2	18
56	59	2	16
25	38	3	23
46	75	2	21
255	3	14	122

contained times in 323 square chains, or 32 acres and 3 square chains remaining. Place 3 square chains under the column of square chains, and reserve the 32 acres to add to the column of acres. The sum of the acres is 32 acres (reserved) + 46 + 25 + 56 + 47 + 28 + 21, or 255 acres. Place 255 acres under the column of acres. Therefore, the sum is 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li. Ans.

(102) Before we can subtract 300 ft. from 20 rd. 2 yd. 2 ft. and 9 in., we must reduce the 300 ft. to higher denominations.

Since there are 3 feet in 1 yard, in 300 feet there are as many yards as 3 is contained times in 300, or 100 yards. There are $5\frac{1}{2}$ yards in 1 rod, hence in 100 yards there are as many rods as $5\frac{1}{2}$ or $\frac{11}{2}$ is contained times in $100 = 18\frac{2}{11}$ rods.

$$100 \div \frac{11}{2} = 100 \times \frac{2}{11} = \frac{100 \times 2}{11} = \frac{200}{11}$$

$$\begin{array}{r} 11 \overline{)\,200\,(\,18\tfrac{2}{11}\text{ rd.}} \\ \underline{11} \\ 90 \\ \underline{88} \\ 2 \end{array}$$

Since there are $5\frac{1}{2}$ or $\frac{11}{2}$ yards in 1 rod, in $\frac{2}{11}$ rods there are $\frac{2}{11} \times \frac{11}{2}$, or one yard, so we find that 300 feet equals 18 rods and 1 yard. The problem now is as follows: From 20 rd. 2 yd. 2 ft. and 9 in. take 18 rd. and 1 yd.

We place the smaller number under the larger one, so that units of the same denomination fall in the same column. Beginning with the lowest denomination, we see that 0 inches from 9 inches leaves 9 inches. Going to the next higher denomination, we see that 0 feet from 2 feet leaves 2 feet. Subtracting 1 yard from 2

rd.	yd.	ft.	in.
20	2	2	9
18	1	0	0
2	1	2	9

yards, we have 1 yard remaining, and 18 rods from 20 rods leaves 2 rods. Therefore, the difference is 2 rd. 1 yd. 2 ft. 9 in. Ans.

(103)

A.	sq. rd.	sq. yd.
114	80	25
75	70	30
39	9	$25\frac{1}{4}$ Ans.

EXPLANATION.—Place the subtrahend under the minuend so that like denominations are under each other. Then begin at the right with the lowest denomination. We can not subtract 30 from 25, so we take one square rod ($= 30\frac{1}{4}$ square yards) from 80 square rods, leaving 79 square rods; adding $30\frac{1}{4}$ square yards to 25 square yards, we have $55\frac{1}{4}$ square yards; subtracting 30 from $55\frac{1}{4}$ square yards leaves $25\frac{1}{4}$ square yards; we now subtract 70 square rods from 79 square rods, which leaves 9 square rods; next, we subtract 75 acres from 114 acres, which leaves 39 acres, which we place under the column of acres.

(104) If 10 gal. 2 qt. and 1 pt. of molasses are sold from a hogshead at one time, and 26 gal. 3 qt. are sold at another time, then the total amount of molasses sold equals 10 gal. 2 qt. 1 pt. plus 26 gal. 3 qt.

Since the pint is the lowest denomination, we add the pints first, which equal $0 + 1$, or 1 pint. We can not reduce 1 pint to any higher denomination, so we place it under the pint column. The number of quarts is $3 + 2$, or 5. Since there are 4 quarts in 1 gallon, in 5 quarts there are as many gallons as 4 is contained times in 5, or 1 gallon and 1 quart remaining. We place the 1 quart under the quart column, and reserve the 1 gallon to add to the column of

gal.	qt.	pt.
10	2	1
26	3	0
37 gal.	1 qt.	1 pt.

gallons. The number of gallons equals 1 (reserved) + 26 + 10, or 37 gallons.

If 37 gal. 1 qt. and 1 pt. are sold from a hogshead of molasses (63 gal.), there remains the difference between 63 gal. and 37 gal. 1 qt. 1 pt., or 25 gal. 2 qt. and 1 pt.

63 gal. is the same as 62 gal. 3 qt. 2 pt., since 1 gal. equals 4 qt. and 1 qt. = 2 pt.

Beginning with the lowest denomination, 1 pt. from the 2 pt. 1 pint from 2 pints leaves 1 pint. One quart from 3 quarts leaves 2 quarts, and 37 gallons from 62 gallons leaves 25 gallons. Therefore, there are 25 gal. 2 qt. and 1 pt. of molasses remaining in the hogshead. Ans.

gal.	qt.	pt.
62	3	2
37	1	1
25	2	1

(105) If a person were born June 19, 1850, in order to find how old he would be on Aug. 3, 1892, subtract the earlier date from the later date.

On August 3, 7 mo. and 3 da. have elapsed from the beginning of the year, and on June 19, 5 mo. and 19 da.

Beginning with the lowest denomination, we find that 19 days can not be taken from 3 days, so we take 1 month from 7 months. The 1 month which we took equals 30 days, for in all cases 30 days are allowed to a month. Adding 30 days to the 3 days, we have 33 days; subtracting 19 days from 33 days, we have 14 days remaining. Since we borrowed 1 month from the months column, we have 7 — 1, or 6 months remaining; subtracting 5 months from 6 months, we have 1 month remaining. 1850 from 1892 leaves 42 years. Therefore, he would be 42 years 1 month and 14 days old. Ans.

yr.	mo.	da.
1892	7	3
1850	5	19
42	1	14

(106) If a note given Aug. 5, 1890, were paid June 3, 1892, in order to find the length of time it was due, subtract the earlier date from the later date.

Beginning with the lowest denomination, we find that 5 can not be subtracted from 3, so we take a unit from the next

yr.	mo.	da.
1892	5	3
1890	7	5
1	9	28

higher denomination, which is months. The 1 month which we take equals 30 days. Adding the 30 days to the 3 days, we have 33 days. 5 days from 33 days leaves 28 days. Since we took 1 month from the months column, only 4 months remain. 7 months cannot be taken from 4 months, so we take 1 year from the years column, which equals 12 months. 12 months + 4 months = 16 months. 7 months from 16 months = 9 months. Since we took 1 year from the years column, we have 1892 − 1, or 1891 remaining. 1890 from 1891 leaves 1 year. Hence, the note ran 1 year 9 months and 28 days. Ans.

(107) Write the number of the year, month, day, hour, and minute of the earlier date under the year, month, day, hour, and minute of the later date, and subtract.

.22 minutes before 8 o'clock is the same as 38 minutes after 7 o'clock. 7 o'clock P. M. is 19 hours from the beginning of the day, as there are 12 hours in the morning and 7 in the afternoon. December is 11 months from the beginning of the year.

10 o'clock A. M. is 10 hours from the beginning of the day. July is 6 months from the beginning of the year. The minuend would be the later date, or 1,888 years, 11 months, 11 days, 19 hours, and 38 minutes.

The subtrahend would be the earlier date, or 1,883 years, 6 months, 3 days, 10 hours, and 16 minutes.

Subtracting, we have

yr.	mo.	da.	hr.	min.
1888	11	11	19	38
1883	6	3	10	16
5	5	8	9	22

or, 5 yr. 5 mo. 8 da. 9 hr. and 22 min. Ans.

16 minutes subtracted from 38 minutes leaves 22 minutes; 10 hours from 19 hours leaves 9 hours; 3 days from 11 days leaves 8 days; 6 months subtracted from 11 months leaves 5 months; 1,883 from 1,888 leaves 5 years.

(108) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

17 ft.	3 in.
	51
879 ft.	9 in.

and begin at the right to multiply. $51 \times 3 = 153$ in. As there are 12 inches in 1 foot, in 153 in. there are as many feet as 12 is contained times in 153, or 12 feet and 9 inches remaining. Place the 9 inches under the inches, and reserve the 12 feet. 51×17 ft. $= 867$ ft. 867 ft. $+ 12$ ft. (reserved) $= 879$ ft.

879 feet can be reduced to higher denominations by dividing by 3 feet to find the number of yards, and by $5\frac{1}{2}$ yards to find the number of rods.

3) 8 7 9 ft. 9 in.
5.5) 2 9 3 yd.
5 3 rd. $1\frac{1}{2}$ yd.

Then, answer = 53 rd. $1\frac{1}{2}$ yd. 0 ft. 9 in.; or 53 rd. 1 yd. 2 ft. 3 in.

(109)

	qt.	pt.	gi.	
	3	**1**	3	
			4.7	
	1 8.2 qt.	0	.1	
or,	1 8 qt.	0 pt.	1.7 gi.	
or, 4 gal.	2 qt.	0 pt.	1.7 gi.	Ans.

Place the multiplier under the lowest denomination of the multiplicand, and proceed to multiply. 4.7×3 gi. $= 14.1$ gi. As 4 gi. $= 1$ pt., there are as many pints in 14.1 gi. as 4 is contained times in $14.1 = 3.5$ pt. and .1 gi. over. Place .1 under gills and carry the 3.5 pt. forward. 4.7×1 pt. $= 4.7$ pt.; $4.7 + 3.5$ pt. $= 8.2$ pt. As 2 pt. $= 1$ qt., there are as many quarts in 8.2 pt. as 2 is contained times in $8.2 = 4.1$ qt. and no pints over. Place a cipher under the pints, and carry the 4.1 qt. to the next product. 4.7×3 qt. $= 14.1$; $14.1 + 4.1 = 18.2$ qt. The answer now is 18.2 qt. 0 pt. .1

gi. Reducing the fractional part of a quart, we have 18 qt. 0 pt. 1.7 gi. (.2 qt. = .2 × 8 = 1.6 gi.; 1.6 + .1 gi. = 1.7 gi.). Then, we can reduce 18 qt. to gallons (18 ÷ 4 = 4 gal. and 2 qt.) = 4 gal. 2 qt. 1.7 gi. Ans.

The answer may be obtained in another and much easier way by reducing all to gills, multiplying by 4.7, and then changing back to quarts and pints. Thus,

3 qt.
× 2 pt.
6 pt.
+ 1 pt.
7 pt.
× 4 gi.
28 gi.
+ 3 gi.
31 gi.

3 qt. 1 pt. 3 gi. = 31 gi.
31 gi. × 4.7 = 145.7 gi.

4)145.7 gi.
2)36 pt. + 1.7 gi.
18 qt. + 0 pt.

Ans. = 18 qt. 1.7 gi.;
or, 4 gal. 2 qt. 1.7 gi.

(110) (3 lb. 10 oz. 13 pwt. 12 gr.) × 1.5 = ?

3 lb. 10 oz. 13 pwt. 12 gr.
×12
36 oz.
+10
46 oz.
× 20
920 pwt.
+ 13
933 pwt.
× 24
22392 gr.
+ 12
22404 gr.

22.404 gr. × 1.5 = 33,606 gr.

24)33606 gr.
20)1400 pwt. + 6 gr
12)70 oz. + 0 pwt.
5 lb. + 10 oz.

Since there are 24 gr. in 1 pwt., in 33,606 gr. there are as many pwt. as 24 is contained times in 33,606, or 1,400 pwt. and 6 gr. remaining. This gives us the number of grains in the answer. We now reduce 1,400 pwt. to higher denominations. Since there are 20 pwt. in 1 oz., in 1,400 pwt. there are as many ounces as 20 is contained times in 1,400, or 70 oz. and 0 pwt. remaining; therefore, there are 0 pwt. in the answer. We reduce 70 oz. to higher denominations. Since there are 12 oz. in 1 lb., in 70 oz. there are as many pounds as 12 is contained times in 70, or 5 lb. and 10 oz. remaining. We can not reduce 5 lb. to any higher denominations. Therefore, our answer is 5 lb. 10 oz. 6 gr.

Another but more complicated way of working this problem is as follows:

lb.	oz.	pwt.	gr.	
3	10	13	12	
			1.5	
4.5	15	19.5	18	
or, 4	21	19	30	
or, 5	10	0	6	Ans.

To get rid of the decimal in the pounds, reduce .5 of a pound to ounces. Since 1 lb. = 12 oz., .5 of a pound equals .5 lb. × 12 = 6 oz. 6 oz. + 15 oz. = 21 oz. We now have 4 lb. 21 oz. 19.5 pwt. and 18 gr., but we still have a decimal in the column of pwt., so we reduce .5 pwt. to grains to get rid of it. Since 1 pwt. = 24 gr., .5 pwt. = .5 pwt. × 24 = 12 gr. 12 gr. + 18 gr. = 30 gr. We now have 4 lb. 21 oz. 19 pwt. and 30 gr. Since there are 24 gr. in 1 pwt., in 30 gr. there is 1 pwt. and 6 gr. remaining. Place 6 gr. under the column of grains and add 1 pwt. to the pwt. column. Adding 1 pwt., we have 19 + 1 = 20 pwt. Since there are 20 pwt. in 1 oz., we have 1 oz. and 0 pwt. remaining. Write the 0 pwt. under the pwt. column, and reserve the 1 oz. to the oz. column. 21 oz. + 1 oz. = 22 oz. Since there are 12 oz. in 1 lb., in 22 oz. there is 1 lb. and 10 oz. remaining. Write the 10 oz. under the ounce column, and reserve the 1 lb. to add to the lb. column. 4 lb. + 1 lb. (reserved) = 5 lb. Hence, the answer equals 5 lb. 10 oz. 6 gr.

(111) If each barrel of apples contains 2 bu. 3 pk. and 6 qt., then 9 bbl. will contain 9 × (2 bu. 3 pk. 6 qt.).

We write the multiplier under the lowest denomination of the multiplicand, which is quarts in this problem. 9 times 6 qt. equals 54 qt. There are 8 qt. in 1 pk., and in 54 qt. there are as many pecks as 8 is contained times in 54, or 6 pk. and 6 qt. We write the 6 qt. under the column of quarts, and reserve the 6 pk. to add to the product of the pecks. 9 times 3 pk. equals 27 pk.; 27 pk. plus the 6 pk. reserved equals 33 pk. Since there are 4 pk. in 1 bu., in 33 pk. there are as many bushels as 4 is contained times in 33, or 8 bu. and 1 pk. remaining. We write the 1 pk. under the column of pecks, and reserve the 8 bu. for the product of the bushels. 9 times 2 bu. plus the 8 bu. reserved equals 26 bu. Therefore, we find that 9 bbl. contain 26 bu. 1 pk. 6 qt. of apples. Ans.

bu.	pk.	qt.
2	3	6
		9
18	27	54
or, 26	1	6

(112) (7 T. 15 cwt. 10.5 lb.) × 1.7 = ? When the multiplier is a decimal, instead of multiplying the denominate numbers as in the case when the multiplier is a whole number, it is much easier to reduce the denominate numbers to the lowest denomination given; then, multiply that result by the decimal, and, lastly, reduce the product to higher denominations. Although the correct answer can be obtained by working examples involving decimals in the manner as in the last example, it is much more complicated than this method.

7 T. 15 cwt. 10.5 lb.

× 20

140 cwt.

15

155 cwt.

× 100

15500 lb.

10.5

15510.5 lb.

15,510.5 lb. × 1.7 = 26,367.85 lb.

There are 100 lb. in 1 cwt., and in 26,367.85 lb. there are as many cwt. as 100 is contained times in 26,367.85, which equals 263 cwt. and 67.85 lb. remaining. Since we have the number of pounds for our answer, we reduce 263 cwt. to higher denominations

100) 26367.85 lb.
20) 263 cwt. + 67.85 lb.
13 T. + 3 cwt.

There are 20 cwt. in 1 ton, and in 263 cwt. there are as many tons as 20 is contained times in 263, or 13 tons and 3 cwt. remaining. Since we cannot reduce 13 tons any higher, our answer is 13 T. 3 cwt. 67.85 lb. Or, since .85 lb. = .85 lb. × 16 = 13.6 oz., the answer may be written 13 T. 3 cwt. 67 lb. 13.6 oz.

(113) 7) 358 A. 57 sq. rd. 6 sq. yd. 2 sq. ft.
51 A. 31 sq. rd. 0 sq. yd. 8 sq. ft. Ans.

We begin with the highest denomination, and divide each term in succession by 7.

7 is contained in 358 A. 51 times and 1 A. remaining. We write the 51 A. under the 358 A. and reduce the remaining 1 A. to square rods = 160 sq. rd.; 160 sq. rd. + the 57 sq. rd. in the dividend = 217 sq. rd. 7 is contained in 217 sq. rd. 31 times and 0 sq. rd. remaining. 7 is not contained in 6 sq. yd., so we write 0 under the sq. yd. and reduce 6 sq. yd. to square feet. 9 sq. ft. × 6 = 54 sq. ft. 54 sq. ft. + 2 sq. ft. in the dividend = 56 sq. ft. 7 is contained in 56 sq. ft. 8 times. We write 8 under the 2 sq. ft. in the dividend.

(114) 12) 282 bu. 3 pk. 1 qt. 1 pt.
23 bu. 2 pk. 2 qt. ½ pt. Ans.

12 is contained in 282 bu. 23 times and 6 bu. remaining. We write 23 bu. under the 282 bu. in the dividend, and reduce the remaining 6 bu. to pecks = 24 pk. + the 3 pk. in the dividend = 27 pk. 12 is contained in 27 pk. 2 times and 3 pk. remaining. We write 2 pk. under the 3 pk. in the dividend, and reduce the remaining 3 pk. to quarts. 3 pk. = 24 qt.; 24 qt. + the 1 qt. in the dividend = 25 qt. 12 is contained in 25 qt. 2 times and 1 qt. remaining. We write

2 qt. under the 1 qt. in the dividend, and reduce 1 qt. to pints = 2 pt. + the 1 pt. in the dividend = 3 pt. $3 \div 12 = \frac{3}{12}$ or $\frac{1}{4}$ pt.

(115) We must first reduce 23 miles to feet before we can divide by 30 feet. 1 mi. contains 5,280 ft.; hence, 23 mi. contain $5{,}280 \times 23 = 121{,}440$ ft.

121,440 ft. ÷ 30 ft. = 4,048 rails for 1 side of the track.

The number of rails for 2 sides of the track = $2 \times 4{,}048$, or 8,096 rails. Ans.

(116) In this case where both dividend and divisor are compound, reduce each to the lowest denomination mentioned in either and then divide as in simple numbers.

1 bu. 1 pk. 7 qt.	356 bu. 3 pk. 5 qt.
× 4	× 4
4 pk.	1424 pk.
+ 1	+ 3
5 pk.	1427 pk.
× 8	× 8
40 qt.	11416 qt.
+ 7	+ 5
47 qt.	11421 qt.

47) 11421 (243
94
202
188
141
141

11,421 qt. ÷ 47 qt. = 243 boxes. Ans.

(117) We must first reduce 16 square miles to acres.

In 1 sq. mi. there are 640 A., and in 16 sq. mi. there are 16×640 A. = 10,240 A.

62) 10240 A.

165 A. 25 sq.rd. 24 sq.yd. 3 sq.ft. 80+sq. in. Ans.

62 is contained in 10,240 A. 165 times and 10 A. remaining. We write 165 A. under the 10,240 A. in the dividend and reduce 10 A. to sq. rd. In 1 A. there are 160 sq. rd., and in 10 A. there are $10 \times 160 = 1{,}600$ sq. rd. 62 is contained in 1,600 sq. rd. 25 times and 50 sq. rd. remaining. We write 25 sq. rd. in the quotient and reduce 50 sq. rd. to sq. yd. In 1 sq. rd. there are $30\frac{1}{4}$ sq. yd., and in 50 sq. rd. there are 50 times $30\frac{1}{4}$ sq. yd. $= 1{,}512\frac{1}{2}$ sq. yd. 62 is contained in $1{,}512\frac{1}{2}$ sq. yd. 24 times and $24\frac{1}{2}$ sq. yd. remaining. In 1 sq. yd. there are 9 sq. ft., and in $24\frac{1}{2}$ sq. yd. there are $24\frac{1}{2} \times 9 = 220\frac{1}{2}$ sq. ft. 62 is contained in $220\frac{1}{2}$ sq. ft. 3 times and $34\frac{1}{2}$ sq. ft. remaining. We write 3 sq. ft. in the quotient and reduce $34\frac{1}{2}$ sq. ft. to sq. in. In 1 sq. ft. there are 144 sq. in., and in $34\frac{1}{2}$ sq. ft. there are $34\frac{1}{2} \times 144 = 4{,}968$ sq. in. 62 is contained in 4,968 sq. in. 80 times and 8 sq. in. remaining.

We write 80 sq. in. in the quotient.

It should be borne in mind that it is only for the purpose of illustrating the method that this problem is carried out to square inches. It is not customary to reduce any lower than square rods in calculating the area of a farm.

(118) To square a number, we must multiply the number by itself once, that is, use the number twice as a factor. Thus, the second power of 108 is $108 \times 108 = 11{,}664$. Ans.

$$\begin{array}{r} 108 \\ 108 \\ \hline 864 \\ 108 \\ \hline 11664 \end{array}$$

(119)

```
      181.25
      181.25
      ------
      90625
     36250
    18125
  145000
  18125
  ------
  32851.56 25
      181.25
  ---------
  1642578125
  657031250
 32851562 5
2628125000
3285156 25
-----------
5954345.703125
```

The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is $181.25 \times 181.25 \times 181.25 = 5{,}954{,}345.703125$. Ans.

Since there are 2 decimal places in the multiplier, and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand, and 2 in the multiplier, there are $4 + 2 = 6$ decimal places in the final product.

(120)

```
      27.61
      27.61
      -----
      2761
     16566
    19327
    5522
    -------
    762.3121
      27.61
    -------
    7623121
   45738726
  5336184 7
  1524624 2
  ---------
  21047.437081
      27.61
  -----------
  210474370 81
 12628462248 6
147332059567
42094874162
-------------
581119.737806 41
```

The fourth power of 27.61 is the number obtained by using 27.61 as a factor four times. Thus, the fourth power of 27.61 is $27.61 \times 27.61 \times 27.61 \times 27.61 = 581{,}119.73780641$. Ans.

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are $4 + 2 = 6$ decimal places in the second product.

Since there are 6 decimal places in the multiplicand and 2 in the multiplier, there are $6 + 2 = 8$ decimal places in the final product.

(121) (*a*) $106^2 = 106 \times 106 = 11{,}236$. **Ans.**

106
106
636
1060
11236

(*b*) $\left(182\frac{1}{8}\right)^2 = 182\frac{1}{8} \times 182\frac{1}{8} = 33{,}169.515625$. Ans.

$\frac{1}{8} = 8\overline{)1.000}$
.125

182.125
182.125
910625
364250
182125
364250
1457000
182125
33169.515625

Since there are 3 decimal places in the multiplier and 3 in the multiplicand, there are $3 + 3 = 6$ decimal places in the product.

(*c*) $.005^2 = .005 \times .005 = .000025$. Ans.

.005
.005
.000025 Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the product.

(*d*) $.0063^2 = .0063 \times .0063 = .00003969$. Ans.

.0063
.0063
189
378
.00003969 Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the product.

(*e*) $10.06^2 = 10.06 \times 10.06 = 101.2036$. Ans.

10.06
10.06
6036
100600
101.2036

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, there are $2 + 2 = 4$ decimal places in the product.

(122) (*a*) $753^3 = 753 \times 753 \times 753 = 426{,}957{,}777$. Ans.

753
753
2259
3765
5271
567009
753
1701027
2835045
3969063
426957777

(*b*) $987.4^3 = 987.4 \times 987.4 \times 987.4 = 962{,}674{,}279.624$. Ans.

987.4
987.4
39496
69118
78992
88866
974958.76

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, there are $1 + 1 = 2$ decimal places in the first product.

987.4
389983504
682471132
779967008
877462884
962674279.624

Since there are 2 decimal places in the multiplicand and one in the multiplier, there are $2 + 1 = 3$ decimal places in the final product.

(*c*) $.005^3 = .005 \times .005 \times .005 = .000000125$. Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the first product; but, as there are only 2 figures in the product, we prefix four ciphers to make the six decimal places.

.005
.005
.000025
.005
000000125

Since there are six decimal places in the multiplicand and 3 in the multiplier, there are $6 + 3 = 9$ decimal places in the final product. In this case we prefix six ciphers to form the nine decimal places.

(d) $.4044^3 = .4044 \times .4044 \times .4044 = .066135317184$. Ans.

```
    .4044
    .4044
   16176
  16176
 161760
.16353936
    .4044
  65415744
 65415744
654157440
.066135317184
```

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the first product.

Since there are 8 decimal places in the second multiplicand and 4 in the multiplier, there are $8 + 4 = 12$ decimal places in the final product; but, as there are only 11 figures in the product, we prefix 1 cipher to make 12 decimal places.

(123) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. Ans.

(124) $3^4 = 3 \times 3 \times 3 \times 3 = 81$. Ans.

(125) (a) $67.85^2 = 67.85 \times 67.85 = 4{,}603.6225$. Ans.

```
  67.85
  67.85
  33925
 54280
47495
40710
4603.6225  Ans.
```

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the product.

(b) $967{,}845^2 = 967{,}845 \times 967{,}845 = 936{,}723{,}944{,}025$. Ans.

```
    967845
    967845
   4839225
  3871380
 7742760
6774915
5807070
8710605
936723944025
```

(*c*) A fraction may be raised to any power by raising both numerator and denominator to the required term.

Thus, $\left(\frac{3}{8}\right)^2 = \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3}{8 \times 8} = \frac{9}{64}$. Ans.

(*d*) $\left(\frac{1}{4}\right)^2 = \frac{1}{4} \times \frac{1}{4} = \frac{1 \times 1}{4 \times 4} = \frac{1}{16}$. Ans.

(126) (*a*) $5^{10} = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 =$ 9,765,625. Ans.

(*b*) $9^5 = 9 \times 9 \times 9 \times 9 \times 9 = 59{,}049$. Ans.

5	9
5	9
25	81
5	9
125	729
5	9
625	6561
5	9
3125	59049
5	
15625	
5	
78125	
5	
390625	
5	
1953125	
5	
9765625	

(127) (*a*) $1.2^4 = 1.2 \times 1.2 \times 1.2 \times 1.2 = 2.0736$. Ans.

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2 + 1 = 3$ decimal places in the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3 + 1 = 4$ decimal places in the final product.

```
    1.2
    1.2
    ---
    24
  12
  ----
   1.44
    1.2
   ----
    288
  144
  -----
  1.728
    1.2
  -----
   3456
 1728
 ------
2.0736
```

(*b*) $11^6 = 11 \times 11 \times 11 \times 11 \times 11 \times 11 = 1{,}771{,}561$. Ans

```
     11
     11
    ---
    121
     11
    ----
   1331
     11
   -----
  14641
     11
  ------
 161051
     11
 -------
1771561
```

(*c*) $1^7 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$. Ans.

(*d*) $.01^4 = .01 \times .01 \times .01 \times .01 = .00000001$. Ans.

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, we must point off $2 + 2 = 4$ decimal places in the first product; but, as there is only 1 figure in the product, we prefix 3 ciphers to make the 4 necessary decimal places.

```
  .01
  .01
 .0001
```

Since there are 4 decimal places in the second multiplicand and 2 in the multiplier, we must point off $4 + 2 = 6$ decimal places in the second product. It is necessary to prefix 5 ciphers to make 6 decimal places.

```
 .0001
  .01
.000001
```

Since there are 6 decimal places in the third multiplicand and 2 in the multiplier, we must point off $6 + 2 = 8$ decimal places in the product. It is necessary to prefix 7 ciphers to make 8 decimal places in the final product.

```
.000001
  .01
.00000001
```

(*e*) $.1^5 = .1 \times .1 \times .1 \times .1 \times .1 = .00001$. Ans.

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1 + 1 = 2$ decimal places in the first product. It is necessary to prefix 1 cipher to the product.

```
  .1
  .1
 .01
```

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2 + 1 = 3$ decimal places in the second product. It is necessary to prefix 2 ciphers to the second product.

```
 .01
  .1
.001
```

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3 + 1 = 4$ decimal places in the third product. It is necessary to prefix 3 ciphers to this product.

```
.001
  .1
.0001
```

Since there are 4 decimal places in the fourth multiplicand and 1 in the multiplier, we must point off $4 + 1$ or 5 decimal places in the final product. It is necessary to prefix 4 ciphers to this product.

```
.0001
  .1
.00001
```

(128) (*a*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$. Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we must point off $4 + 4 = 8$ decimal places in the product; but, as there are only 5 figures in the product, we prefix three ciphers to form the eight necessary decimal places in the first product.

```
   .0133
   .0133
  ------
    399
   399
  133
 ---------
 .00017689
```

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we must point off $8 + 4 = 12$ decimal places in the product; but, as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

```
     .0133
     53067
    53067
   17689
 ------------
 .000002352637
```

(*b*) $301.011^3 = 301.011 \times 301.011 \times 301.011 =$ 27,273,890.942264331. Ans.

```
      301.011
      301.011
     ---------
      301011
     301011
    3010110
   9030330
 ------------
 90607.622121
```

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we must point off $3 + 3 = 6$ decimal places in the first product.

```
      301.011
     ---------
 90607622121
90607622121
906076221210
27182286663630
--------------
27273890.942264331
```

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we must point off $6 + 3 = 9$ decimal places in the final product.

(*c*) $\left(\frac{1}{8}\right)^3 = \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1 \times 1 \times 1}{8 \times 8 \times 8} = \frac{1}{512}$. Ans.

(*d*) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer, and multiply the denominators together for the denominator of the answer.

$$\left(3\frac{3}{4}\right)^3 = \frac{15}{4} \times \frac{15}{4} \times \frac{15}{4} = \frac{15 \times 15 \times 15}{4 \times 4 \times 4} = \frac{3{,}375}{64} = 52.734+.$$

Ans.

$$3\frac{3}{4} = \frac{3 \times 4 + 3}{4} = \frac{12 + 3}{4} = \frac{15}{4}.$$

```
  15          64)3375.000(52.734+
  15             320
  ---            ---
  75             175
 15              128
 ---             ---
 225              470
  15              448
 ----            ----
 1125             220
 225              192
 ----             ---
 3375              280
                   256
                   ---
                    24
```

Since *three* ciphers were annexed to the dividend, *three* decimal places must be pointed off in the quotient. It is easy to see that the next figure will be a 3; hence, write the sign +, as shown.

(129) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(130) (*a*)

1
1
20
8
28
8
360
6
366
6
3720
7
3727
7
3734

$\sqrt{3'48'67'84.40'10} = 1867.29+$ Ans.

1
248
224
2467
2196
27184
26089

3734)1095.000(.293 or .29+
7468
34820
33606
12140

Explanation.—Applying the short method described in Art. **272,** we extract the root by the regular method to four figures, since there are six figures in the answer, and $6 \div 2 + 1 = 4$. The last remainder is 1095, and the last trial divisor (with the cipher omitted) is 3734. Dividing 1095 by 3734, as shown, the quotient is .293 +, or .29 + using two figures. Annexing to the root, gives 1,867.29 +. **Ans.**

$$\sqrt{9'00'00'99.40'09'00} = 3000.0165 + \text{Ans.}$$

	(*a*)	3		
		3	(*b*)	9
	(*d*)	60	(*c*)	0000994009
		0		600001
		600		39400800
		0		36000156
		6000		3400644
		0		
		60000		
		0		
		600000		
		1		
		600001		
		1		
		6000020		
		6		
		6000026		

EXPLANATION.—Beginning at the decimal point we point off the whole number into periods of *two* figures each, proceeding from *right* to *left*; also, point off the decimal into periods of *two* figures each, proceeding from *left* to *right*. The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first figure in the root, or 3. The result is 9. Write 9 under the first period, 9, as at (*b*), subtract, and there is no remainder. Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the *trial divisor*, as shown at (*d*). Divide the dividend (*c*) by the trial divisor, and obtain 0 as the next figure in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained, and divide it by the trial divisor. 600 is contained in 0000, 0 times, so we place another cipher

in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 600, and annex a cipher, thereby making the next trial divisor 6,000. Bring down the next period, 99. The trial divisor 6,000 is contained in 000099, 0 times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor 6,000, and annex a cipher, thereby making the next trial divisor 60,000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor 60,000. 60,000 is contained in 00009940, 0 times, so we place another cipher in the root, as shown, and also add it to the trial divisor 60,000, and annex one cipher, thereby making the next trial divisor 600,000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor 600,000. 600,000 is contained in 0000994009 once, so we place 1 as the next figure in the root, and also add it to the trial divisor 600,000, thereby making the complete divisor 600,001. Multiply the complete divisor, 600,001, by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394,008, to which we annex the next period, 00, to form the next new dividend, or 39,400,800. Add the sixth figure of the root, or 1, to the divisor 600,001, and annex a cipher, thus obtaining 6,000,020 as the next trial divisor. Dividing 39,400,800 by 6,000,020, we find 6 to be the next figure of the root. Adding this last figure, 6, to the trial divisor, we obtain 6,000,026 for our next complete divisor, which, multiplied by the last figure of the root, or 6, gives 36,000,156, which write under 39,400,800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square, so we place + after the root. Since the next figure is 5, the answer is 3,000.017 —.

In this problem there are *seven* periods—four in the whole number and three in the decimal—hence, there will be *seven* figures in the root, *four* figures constituting the whole number, and three figures the decimal of the root. Hence, $\sqrt{9,000,099.4009} = 3,000.017 -$

(*c*)

3	$\sqrt{.00'12'25} = .035$. Ans.
3	00
60	12
5	9
65	325
	325

Pointing off periods, we find that the first period is composed of ciphers; hence, the first figure of the root will be a cipher. No further explanation is necessary, since this problem is solved in a manner exactly similar to the problem solved in Art. **264.** Since there are *three* decimal periods in the power, there will be three decimal figures in the root.

(131) (*a*)

1	$\sqrt{1'07'95.21} = 103.9$ Ans.
1	1
20	0795
0	609
200	18621
3	18621
203	
3	
2060	
9	
2069	

(*b*)

2	$\sqrt{7'30'08.04} = 270.2$ Ans.
2	4
40	330
7	329
47	10804
7	10804
5400	
2	
5402	

(*c*)

9	$\sqrt{90.00'00'00} = 9.487 -$
9	81 Ans.
180	900
4	736
184	16400
4	15104
1880	1896) 1296.00 (.68 + or .7 −
8	11376
1888	15840
8	15168
1896	

Having found the first three figures, we find the fourth by division, as shown.

(*d*) $\sqrt{.09} = .3$. Ans.

(132) (*a*)

6	36	$\sqrt[3]{.327'680'000} = .6894 +$
6	72	216 Ans.
12	10800	111680
6	1504	98432
180	12304	13248000
8	1568	12650769
188	1387200	1424163) 597231.00 (.41 + or .4 +
8	18441	5696652
196	1405641	2756580
8	18522	1424163
2040	1424163	
9		
2049		
9		
2058		
9		
2067		

Here we find the first three figures in the regular way, and the fourth figure by the short method. See Art. **284.**

EXPLANATION.—(1) When extracting the *cube* root we divide the power into periods of three figures each. Always begin at the decimal point, and proceed to the *left* in pointing off the whole number, and to the *right* in pointing off the decimal. In this power $\sqrt[3]{.32768}$, a cipher must be annexed to 68 to complete the second decimal period. Cipher periods may now be annexed until the root has as many figures as desired.

(2) We find by trial that the largest number whose cube is contained in the first period, 327, is 6. Write 6 as the first figure of the root, also at the extreme left at the head of column (1). Multiply the 6 in column (1) by the first figure of the root, 6, and write the product 36 at the head of column (2). Multiply the number in column (2) by the first figure of the root, 6, and write the product 216 under the figures in the first period. Subtract and bring down the next period 680; annex it to the remainder 111, thereby obtaining 111,680 for a new dividend. Add the first figure of the root, 6, to the number in column (1), obtaining 12, which we call the *first correction;* multiply the first correction 12 by the first figure of the root, and we obtain 72 as the product, which, added to 36 of column (2), gives 108. Annexing two ciphers to 108, we have 10,800 for the trial divisor. Dividing the dividend by the trial divisor, we see that it is contained about 8 times, so we write 8 as the second figure of the root. Add the first figure of the root to the first correction, and we obtain 18 as the *second correction.* To this annex *one* cipher, and add the second figure of the root, and we obtain 188. This, multiplied by the second figure of the root, 8, equals 1,504, which, added to the trial divisor 10,800, forms the *complete divisor* 12,304. Multiplying the complete divisor 12,304 by 8, the second figure of the root, the result is 98,432. Write 98,432 under the dividend 111,680; subtract, and there is a remainder of 13,248. To this remainder annex the next period 000, thereby obtaining 13,248,000 for the next new dividend.

(3) Adding the second figure of the root, 8, to the number in column (1), 188, we have 196 for the *first new*

correction. This, multiplied by the second figure of the root, 8, gives 1,568. Adding this product to the last complete divisor, and annexing two ciphers, gives 1,387,200 for the next trial divisor. Adding the second figure of the root, 8, to the first new correction, 196, we obtain 204 for the *new second correction.* Dividing the dividend by the trial divisor 1,387,200, we see that it is contained about 9 times. Write 9 as the third figure of the root. Annex *one* cipher to the *new second correction*, and to this add the third figure of the root, 9, thereby obtaining 2,049. This, multiplied by 9, the third figure of the root, equals 18,441, which, added to the trial divisor, 1,387,200, forms the complete divisor 1,405,641. Multiplying the complete divisor by the third figure of the root, 9, and subtracting, we have a remainder of 597,231. We then find the fourth figure by division, as shown.

(*b*)

4	16	$\sqrt[3]{74'088} = 42$ Ans.
4	32	64
8	4800	10088
4	244	10088
120	5044	
2		
122		

(*c*)

4	16	$\sqrt[3]{92'416} = 45.212 -$ Ans.
4	32	64
8	4800	28416
4	625	27125
120	5425	1291000
5	650	1220408
125	607500	612912)70592.000(.115
5	2704	612912
130	610204	930080
5	2708	612912
1350	612912	3171680
2		3064560
1352		107120
2		
1354		

(*d*)

7	49	$\sqrt[3]{.373248} = .72$ Ans.
7	98	343
14	14700	30248
7	424	30248
210	15124	
2		
212		

(133)

1	1	$\sqrt[4]{2.000000000} = 1.259921+$ Ans
1	2	1
2	300	1000
1	64	728
30	364	272000
2	68	225125
32	43200	46875000
2	1825	42491979
34	45025	4755243)4383021.000(9217 or .922–
2	1850	42797187
360	4687500	10330230
5	33831	9510486
365	4721331	8197440
5	33912	4755243
370	4755243	34421970
5		
3750		
9		
3759		
9		
3768		

This example shows what a great saving of figures is effected by using the short method. The figures obtained by the division are 9217, thus making the last figures of the answer 922, according to Art. **272.** This is not correct in this case; the true answer to eight decimal places being 1.25992104 +; hence, the first three figures

found by division should be used in this case. The reason for the apparent failure of the method in this case to give the seventh figure of the root correctly is because the fifth figure (the first obtained by division) is 9. Whenever the first figure obtained by division is 8 or 9, it is better to carry the root process one place further, before applying Art. **272,** if it is desired to obtain absolutely correct results.

(134) (*a*)

1	1	$\sqrt[3]{1'758.416'743} = 12.07$ Ans.
1	2	1
2	300	758
1	64	728
30	364	30416743
2	68	30416743
32	4320000	
2	25249	
34	4345249	
2		
3600		
7		
3607		

(*b*)

1	1	$\sqrt[3]{1'191'016} = 106$ Ans.
1	2	1
2	30000	191016
1	1836	191016
300	31836	
6		
306		

(*c*) $\sqrt[3]{\frac{4}{32}} = \sqrt[3]{\frac{1}{8}} = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \frac{1}{2}$. Ans.

(*d*) $\sqrt[3]{\frac{27}{512}} = \frac{\sqrt[3]{27}}{\sqrt[3]{512}} = \frac{3}{8}$. Ans.

(135)

1	1	$\sqrt[3]{3.000'000'000} = 1.442250 -$ Ans.
1	2	1
2	300	2000
1	136	1744
30	436	256000
4	152	241984
34	58800	14016000
4	1696	12458888
38	60496	6238092) 1557112.000 (.2496 or .250 -
4	1712	12476184
420	6220800	30949360
4	8644	24952368
424	6229444	59969920
4	8648	56142828
428	6238092	3827092
4		
4320		
2		
4322		
2		
4324		

(136) (*a*)

1	$\sqrt{1'23.21} = 11.1$ Ans.
1	1
20	23
1	21
21	221
1	221
220	
1	
221	

(*b*)

1	$\sqrt{1'14.92'10} = 10.72 +$ Ans.
1	1
200	1492
7	1449
207	4310
7	4284
2140	26
2	
2142	

(*c*)

7	$\sqrt{50'26'81} = 709$ Ans.
7	49
140	12681
0	12681
1400	
9	
1409	

(*d*)

2	$\sqrt{.00'04'12'09} = .0203$ Ans.
2	00
400	04
3	4
403	1209
	1209

(137) (*a*)

1	1	$\sqrt[3]{.006'500'000} = .18663 -$ **Ans**
1	2	1
2	300	5500
1	304	4832
30	604	668000
8	368	602856
38	97200	103788) 65144.00 (.627 or .63 —
8	3276	622728
46	100476	287120
8	3312	207576
540	103788	79544
6		
546		
6		
552		

(*b*)

2	4	$\sqrt[3]{.021'000'000} = .2759 -$ Ans.
2	8	8
4	1200	13000
2	469	11683
60	1669	1317000
7	518	1113875
67	218700	226875) 203125.0 (.89 or .9 —
7	4075	1815000
74	222775	216250
7	4100	
810	226875	
5		
815		
5		
820		

(*c*)

2	4	$\sqrt[3]{8'036'054'027} = 2{,}003$ Ans.
2	8	8
4	12000000	036054027
2	18009	36054027
6000	12018009	
3		
6003		

(*d*)

1	1	$\sqrt[3]{.000'004'096} = .016$ Ans.
1	2	000
2	300	004
1	216	1
30	516	3096
6		3096
36		

(*e*)

2	4	$\sqrt[3]{17.000'000} = 2.5713-$ Ans.
2	8	8
4	1200	9000
2	325	7625
60	1525	1375000
5	350	1349593
65	187500	198147)25407.00(.128 or .13-
5	5299	198147
70	192799	559230
5	5348	396294
750	198147	162936
7		
757		
7		
764		

(138) (*a*) In this example the index is 4, and equals 2×2. The root indicated is the fourth root, hence the square root must be extracted twice. Thus, $\sqrt[4]{\ } = \sqrt{\ }$ of the $\sqrt{\ }$ and $\sqrt[4]{6561} = \sqrt{\sqrt{6561}} = \sqrt{81} = 9$. Ans.

8	$\sqrt{65'61} = 81$	$\sqrt{81} = 9$ Ans.
8	64	
160	161	
1	161	
161		

(*b*) In this example the index is 6, and 6 equals 3×2 or 2×3. The root indicated is the sixth root; hence, extract both the square and cube root, it making no particular difference as to which root is extracted first. Thus,

$\sqrt[6]{\ } = \sqrt[3]{\ }$ of the $\sqrt{\ }$, or $\sqrt{\ }$ of the $\sqrt[3]{\ }$.

Hence, $\sqrt[6]{117,649} = \sqrt[3]{\sqrt{117,649}} = \sqrt[3]{343} = 7$. Ans.

3	$\sqrt{11'76'49} = 343$	$\sqrt[3]{343} = 7$ Ans.
3	9	
60	276	
4	256	
64	2049	
1	2049	
680		
3		
683		

(*c*) $\sqrt[6]{.000064} = \sqrt[3]{\sqrt{.000064}} = .2$. Ans.

$\sqrt{.000064} = .008$. $\sqrt[3]{.008} = .2$. Hence, $\sqrt[6]{.000064} = .2$. Ans.

(*d*) $\sqrt[3]{\frac{3}{8}} = ?$ $\frac{3}{8} = .375$, since 8)3.000
.375

7	49	$\sqrt[3]{.375'000'000} = .72112+$ Ans.
7	98	343
14	14700	32000
7	424	30248
210	15124	1752000
2	428	1557361
212	1555200	1559523)194639.00(.124 or .12+
2	2161	1559523
214	1557361	3868670
2	2162	3119046
2160	1559523	749624
1		
2161		
1		
2162		

Hence, $\sqrt[3]{\frac{3}{8}} = .72112+$. Ans.

(139) (*a*) $\sqrt{\frac{1225}{5476}} = \frac{\sqrt{1225}}{\sqrt{5476}}$.

3	$\sqrt{12'25} = 35$
3	9
60	325
5	325
65	

Hence, $\sqrt{\frac{1225}{5476}} = \frac{35}{74}$. Ans.

7	$\sqrt{54'76} = 74$
7	49
140	576
4	576
144	

(*b*)

5	$\sqrt{.33'64} = .58$	
5	25	Ans.
100	864	
8	864	
108		

(*c*)

3	$\sqrt{.10'00'00'00} = .31623-$	
3	9	Ans.
60	100	
1	61	
61	3900	
1	3756	
620	632)144.00(.227 or .23-	
6	1264	
626	1760	
6	1264	
632	496	

(*d*) $25.0\frac{3}{4} = 25.075.$

5	$\sqrt{25.07'50'00'00'00} = 5.00749 +$	
5	25	Ans.
10000	075000	
7	70049	
10007	495100	
7	400576	
100140	9452400	
4	9013401	
100144	438999	
4		
1001480		
9		
1001489		

(*e*) $.000\frac{4}{9} = .0004444444 +.$

2	$\sqrt{.00'04'44'44'44} = .02108+$	
2	00	Ans.
40	04	
1	4	
41	44	
1	41	
4200	34444	
8	33664	
4208	780	

(140) (*a*) $\sqrt[4]{2} = \sqrt{\sqrt{2}}$.

1	$\sqrt{2.0\,0'0\,0'0\,0'0\,0} = 1.41421356+$	
1	1	
20	100	
4	96	
24	400	
4	281	
280	11900	
1	11296	
281	60400	
1	56564	
2820	28284) 3836.0000 (.13562 or .1356+	
4	28284	
2824	100760	
4	84852	
28280	159080	
2	141420	
28282	176600	
2	169704	
28284	6896	

1	$\sqrt{1.41'42'13'56} = 1.1892+$ Ans.	
1	1	
20	41	
1	21	
21	2042	
1	1824	
220	21813	
8	21321	
228	49256	It is required in this problem to extract the fourth root of 2 to four decimal places; hence, we must extract the square root twice, since $\sqrt[4]{\ } = \sqrt{\ }$ of the $\sqrt{\ }$.
8	47564	
2360	1692	
9		
2369		
9		
23780		In the first operation we carry the root to 8 decimal places, in order to carry the root in the second operation to 4 decimal places.
2		
23782		

(*l*) $\sqrt[6]{6} = \sqrt[3]{\sqrt{6}}$.

2	$\sqrt{6.00'00'00'00'00'00} = 2.4494897428+$	
2	4	
40	200	
4	176	
44	2400	
4	1936	
480	46400	
4	44001	
484	239900	
4	195936	
4880	4396400	
9	3919104	
4889	489896)477296.00000	(.974280 or .97428+
9	4409064	
48980	3638960	
4	3429272	
48984	2096880	
4	1959584	
489880	1372960	
8	979792	
489888	3931680	
8	3919168	
489896	12512	

It is required in this problem to find the sixth root of 6; hence it is necessary to extract both the square and cube roots in succession, since the index, 6, equals 2×3 or 3×2. It makes no particular difference as to which root we extract first, but it will be more convenient to extract the square root first. The result has been carried to 10 decimal places; since the answer requires but 5 decimal places, the remaining decimals will not affect the cube root in the fifth decimal place, as the student can see for himself if he will continue the operation.

1	1	$\sqrt[5]{2.449'489'742'800} = 1.34801 -$
1	2	1 Ans.
2	300	1449
1	99	1197
30	399	252489
3	108	209104
33	50700	43385742
3	1576	43352192
36	52276	5451312) 33550.000 (.006 or .01 –
3	1592	32707872
390	5386800	842128
4	32224	
394	5419024	
4	32288	
398	5451312	
4		
4020		
8		
4028		
8		
4036		

(141) (*a*)

1		$\sqrt{3.14'16} = 1.7725 -$ Ans.
1		1
20		214
7		189
27		2516
7		2429
340		354) 87.00 (.245 + or 25 –
7		708
347		1620
7		1416
354		204

(*b*)

```
  8
  8
---
160
  8
---
168
  8
---
1760
   6
---
1766
   6
---
1772
```

$\sqrt{.78'54'00} = .8862 +$ Ans

```
  64
----
1454
1344
-----
 11000
 10596
-------
1772 ) 404.0 (.22 or .2 -
       3544
       ----
        496
```

(142) (*a*)

```
1
1
-
2
1
--
30
 4
--
34
 4
--
38
 4
---
420
  6
---
426
  6
---
432
  6
----
4380
   4
----
4384
   4
----
4388
```

```
   1
   2
 ---
 300
 136
 ---
 436
 152
 ----
 58800
  2556
 -----
 61356
  2592
 ------
 6394800
   17536
 -------
 6412336
   17552
 -------
 6429888
```

$\sqrt[3]{3.141'600'000} = 1.4646 -$ Ans.

```
  1
-----
 2141
 1744
------
  397600
  368136
  ------
   29464000
   25649344
   --------
6429888 ) 3814656.0 ( .59 or .6-
          32149440
          --------
           5997120
```

(*b*)

8	64	$\sqrt[3]{.523'600'000}$	$=.80599+$ or $.8060-$
8	128	512	Ans.
16	1920000	11600000	
8	12025	9660125	
2400	1932025	1944075)1939875.00 (.99	
5	12050	17496675	
2405	1944075	1902075	
5			
2410			

(143) $11.7 : 13 :: 20 : x.$ The product of the means equals the product of the extremes.

$11.7\,x = 13 \times 20$

$11.7\,x = 260$

$$x = \frac{260}{11.7}$$

260.000 (22.22+ Ans.

234

260

234

260

234

260

234

26

(144) (*a*) $20 + 7 : 10 + 8 :: 3 : x.$

$27 : 18 :: 3 : x$

$27\,x = 18 \times 3$

$27\,x = 54$

$$x = \frac{54}{27} = 2. \quad \text{Ans.}$$

(*b*) $12^3 : 100^2 :: 4 : x.$

$144 : 10{,}000 :: 4 : x$

$144\,x = 10{,}000 \times 4$

$144\,x = 40{,}000$

$$x = \frac{40{,}000}{144}$$

144) 40000.0 (277.7 + Ans.

288

1120

1008

1120

1008

1120

1008

112

(145) (*a*) $\frac{4}{x} = \frac{7}{21}$, is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$$7x = 4 \times 21$$
$$7x = 84$$
$$x = \frac{84}{7} \text{ or } 12. \quad \text{Ans.}$$

(*b*) In like manner,

$\frac{x}{24} = \frac{8}{16}$ is equivalent to $x : 24 :: 8 : 16$

$$16x = 24 \times 8$$
$$16x = 192$$
$$x = \frac{192}{16} = 12. \quad \text{Ans.}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x : 100$.

$$10x = 2 \times 100$$
$$10x = 200$$
$$x = \frac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to

$15 : 45 :: 60 : x$.

$15x = 45 \times 60$

$15x = 2{,}700$

$x = \frac{2{,}700}{15} = 180.$ Ans.

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to

$10 : 150 :: x : 600$.

$150x = 10 \times 600$

$150x = 6{,}000$

$x = \frac{6{,}000}{150} = 40.$ Ans.

(146) $x : 5 :: 27 : 12.5$.

$$12.5\overline{)135.0}\,(10\tfrac{4}{5} \text{ Ans.}$$

$$\frac{125}{100}$$

$$\frac{125}{4}$$

(147) $45 : 60 :: x : 24$

$$60x = 45 \times 24$$

$$60x = 1{,}080$$

$$x = \frac{1{,}080}{60} = 18. \text{ Ans.}$$

(148) $x : 35 :: 4 : 7$.

$$7x = 35 \times 4$$

$$7x = 140$$

$$x = \frac{140}{7} = 20. \text{ Ans.}$$

(149) $9 : x :: 6 : 24$.

$$6x = 9 \times 24$$

$$6x = 216$$

$$x = \frac{216}{6} = 36. \text{ Ans}$$

(150)

$\sqrt[3]{1{,}000} : \sqrt[3]{1{,}331} :: 27 : x$.

$10 : 11 :: 27 : x$.

$10x = 297$.

$$x = \frac{297}{10} = 29.7.$$

Ans.

$\sqrt[3]{1{,}000} = 10$.

$\sqrt[3]{1{,}331} = 11$.

$1'331\,(11$

1

331

331

$$\begin{array}{r} 1 \\ 1 \\ \hline 2 \\ 1 \\ \hline 30 \\ 1 \\ \hline 31 \end{array} \qquad \begin{array}{r} 1 \\ 2 \\ \hline 300 \\ 31 \\ \hline 331 \end{array}$$

(151) $64 : 81 = 21^3 : x^3$.

Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64} : \sqrt{81} = \sqrt{21^3} : \sqrt{x^3}$ is the same as

$$8 : 9 = 21 : x$$

$$8x = 189$$

$$x = 23.625. \text{ Ans.}$$

(152) $7 + 8 : 7 = 30 : x$ is equivalent to

$$15 : 7 = 30 : x.$$

$$15x = 7 \times 30$$

$$15x = 210$$

$$x = \frac{210}{15} = 14. \text{ Ans.}$$

(153) 2 ft. 5 in. = 29 in.; 2 ft. 7 in. = 31 in. Stating as a direct proportion, $29 : 31 = 2{,}480 : x$. Now, it is easy to see that x will be greater than 2,480. But x should be less than 2,480, since, when a man lengthens his steps, the number of steps required for the same distance is less; hence, the proportion is an inverse one, and

$$29 : 31 = x : 2{,}480,$$
$$\text{or,} \quad 31x = 71{,}920;$$
$$\text{whence,} \quad x = 71{,}920 \div 31 = 2{,}320 \text{ steps.} \quad \textbf{Ans.}$$

(154) This is evidently a direct proportion. 1 hr. 36 min. = 96 min.; 15 hr. = 900 min. Hence,

$$96 : 900 = 12 : x,$$
$$\text{or,} \quad 96x = 10{,}800;$$
$$\text{whence,} \quad x = 10{,}800 \div 96 = 112.5 \text{ mi.} \quad \textbf{Ans.}$$

(155) This is also a direct proportion; hence,

$$27.63 : 29.4 = .76 : x,$$
$$\text{or,} \quad 27.63x = 29.4 \times .76 = 22.344;$$
$$\text{whence,} \quad x = 22.344 \div 27.63 = .808 + \text{lb.} \quad \textbf{Ans.}$$

(156) 2 gal. 3 qt. 1 pt. = 23 pt.; 5 gal. 3 qt. = 46 pt. Hence,

$$23 : 46 = 5 : x,$$
$$\text{or,} \quad 23x = 46 \times 5 = 230;$$
$$\text{whence,} \quad x = 230 \div 23 = 10 \text{ days.} \quad \textbf{Ans.}$$

(157) Stating as a direct proportion, and squaring the distances, as directed by the statement of the example, $6^2 : 12^2 = 24 : x$. Inverting the second couplet, since this is an inverse proportion,

$$6^2 : 12^2 = x : 24.$$

Dividing both terms of the first couplet (see Art. **310**) by 6

$$1^2 : 2^2 = x : 24; \text{ or } 1 : 4 = x : 24;$$
$$\text{whence, } 4x = 24, \text{ or } x = 6 \text{ degrees.} \quad \textbf{Ans.}$$

(158) Taking the dimensions as the causes,

$$\begin{array}{|cc} 12 & 15 \\ 4 & 5 \\ 2 & 2 \\ 3 & 6 \end{array} = 12 \quad \Big| \; x, \text{ whence, } 2x = 75, \text{ or, } x = \$37.50. \text{ Ans.}$$

(159) 2 hr. = 120 min.; 14 hr. 28 min. = 868 min.

Hence, $120 : 868 = 100 : x$,

or, $120x = 86{,}800$;

whence, $x = 723\frac{1}{3}$ gal. Ans.

(160) Taking the dimensions as the causes,

$$\begin{array}{|ccc} 14 & 2 & \\ 28 & 20 & \\ 2 & & = 798 \\ 12 & 17 & 57 \\ 10 & 6 & \end{array} \quad \Big| \; x, \text{ whence, } 2x = 17 \times 57 = 969, \text{ or, } x = 484\frac{1}{2} \text{ bbl. Ans}$$

(161) 8 hr. 40 min. = 520 min. Hence,

$$444 : 1{,}060 = 520 : x,$$

$$\text{or, } x = \frac{1{,}060 \times \overset{130}{\cancel{520}}}{\underset{111}{\cancel{444}}} = \frac{137{,}800}{111} = 1{,}241.44+ \text{ min.} = 20 \text{ hr. } 41.44+ \text{ min. Ans.}$$

(162) 1 min. = 60 sec. Hence,

$$5\tfrac{1}{2} : 60 = 6{,}160 : x,$$

$$\text{or, } x = \frac{60 \times 6{,}160}{5.5} = 67{,}200 \text{ ft. Ans.}$$

(163) Writing the statement as a direct proportion, $8 : 10 = 5 : x$, it is easy to see that x will be greater than 5; but, it should be smaller, since by working longer hours, fewer men will be required to do the same work. Hence, the proportion is inverse. Inverting the second couplet,

$$8 : 10 = x : 5,$$

$$\text{or, } x = \frac{\overset{4}{\cancel{8}} \times 5}{\underset{2}{\cancel{10}}} = 4 \text{ men. Ans.}$$

(164) Taking the times as the causes,

$$\begin{array}{|cc|c|} 20 & 25 & 14 \\ & 5 & 70 \\ & = 540 & 630 \\ 10 & x \quad 27 & \\ 2 & 3 & \end{array}$$

; whence, $3x = 2 \times 14 = 28$, or $x = 9\frac{1}{3}$ hr. Ans.

(165) Taking the horsepowers as the effects, we have for the known causes in example 4, Art. **349**, 14^2, 500, and 48, and for the known effect 112 horsepower. Hence,

$$\begin{array}{|cc|} 14^2 & 30^2 \\ 500 & 660 = 112 \\ 48 & 42 \end{array} \quad x, \text{ or } \quad \begin{array}{|cc|} & 9 & \\ 14 & 900 & \\ 196 & 22 & \\ 5 & 110 & \\ 500 & 660 = 112 & x; \\ 6 & 3 & 8 \\ 48 & 42 & \end{array}$$

whence, $x = 9 \times 22 \times 3 = 594$ horsepower. Ans.

(166) First find the volume of the cylinder in cubic inches, as in the example, Art. **345.** The volume, multiplied by the weight of one cubic inch (.261 lb.), will evidently be the weight of the cylinder. Hence,

$$\begin{array}{|cc|} 10^2 & 12^2 \\ 20 & 60 \end{array} = 1{,}570.8 \quad x, \text{ or } \quad \begin{array}{|cc|} & 144 \\ 100 & 3 \\ 20 & 60 \end{array} = 1.570.8 \quad x;$$

whence, $x = \dfrac{144 \times 3 \times 1{,}570.8}{100} = 6{,}785.856$ cu. in. Therefore, weight of cylinder $= 6{,}785.856 \times .261 = 1{,}771.11$ lb. Ans.

(167) Referring to the example in Art. **348,**

$$\begin{array}{|cc|} 15 & 40 \\ & \\ 20^2 & 18^2 = 187 \\ 10 & 12 \end{array} \quad x, \text{ or } \quad \begin{array}{|cc|} & 4 \\ 5 & 40 \\ 15 & \\ 100 & \\ 400 & 324 = 187 \\ 10 & 4 \\ & 12 \end{array} \quad x;$$

whence, $x = \dfrac{324 \times 4 \times 187}{500} = 484.7$ lb. **Ans.**

ALGEBRA.

(QUESTIONS 168–217.)

(168) $-\dfrac{c-(a-b)}{c+(a+b)}=\dfrac{(a-b)-c}{c+(a+b)}$. Ans. (Art. **482.**)

(169) (*a*) Factoring each expression (Art. **457**), we have $9x^4+12x^2y^2+4y^4=(3x^2+2y^2)(3x^2+2y^2)=(3x^2+2y^2)^2$.

(*b*) $49a^4-154a^2b^2+121b^4=(7a^2-11b^2)(7a^2-11b^2)=(7a^2-11b^2)^2$. Ans.

(*c*) $64x^2y^2+64xy+16=16(2xy+1)^2$. Ans.

(170) (*a*) Arrange the dividend according to the decreasing powers of x and divide. Thus,

$3x-1\,)\,9x^3+3x^2+x-1\,(\,3x^2+2x+1$ **Ans.**

$9x^3-3x^2$

$6x^2+x$

$6x^2-2x$ (Art. **444.**)

$3x-1$

$3x-1$

(*b*) $a-b\,)\,a^3-2ab^2+b^3\,(\,a^2+ab-b^2$ **Ans.**

a^3-a^2b

a^2b-2ab^2

a^2b-ab^2 (Art. **444.**)

$-ab^2+b^3$

$-ab^2+b^3$

(*c*) Arranging the terms of the dividend according to the decreasing powers of x, we have

$7x-3\,)\,7x^3-24x^2+58x-21\,(\,x^2-3x+7$ **Ans.**

$7x^3-3x^2$

$-21x^2+58x$

$-21x^2+9x$

$49x-21$

$49x-21$

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(171) See Arts. **352** and **353.**

(172) (*a*) In the expression $4x^3y - 12x^2y^2 + 8xy^3$, it is evident that each term contains the common factor $4xy$. Dividing the expression by $4xy$, we obtain $x^2 - 3xy + 2y^2$ for a quotient. The two factors, therefore, are $4xy$ and $x^2 - 3xy + 2y^2$. Hence, by Art. **452,**

$$4x^3y - 12x^2y^2 + 8xy^3 = 4xy(x^2 - 3xy + 2y^2). \quad \text{Ans.}$$

(*b*) The expression $(x^4 - y^4)$ when factored, equals $(x^2 + y^2)(x^2 - y^2)$. (Art. **463.**) But, according to Art. **463,** $x^2 - y^2$ may be further resolved into the factors $(x + y)(x - y)$.

Hence, $(x^4 - y^4) = (x^2 + y^2)(x + y)(x - y)$. Ans.

(*c*) $8x^3 - 27y^3$. See Art. **466.** The cube root of the first term is $2x$, and of the second term is $3y$, the sign of the second term being $-$. Hence, the first factor of $8x^3 - 27y^3$ is $2x - 3y$. The second factor we find to be $4x^2 + 6xy + 9y^2$, by division. Hence, the factors are $2x - 3y$ and $4x^2 + 6xy + 9y^2$.

(173) Arranging the terms according to the decreasing powers of m.

$$\begin{array}{l}
3m^3 + 10m^2n + 10mn^2 + 3n^3 \\
3m^4n - 5m^3n^2 + 5m^2n^3 - mn^4 \\
\hline
9m^7n + 30m^6n^2 + 30m^5n^3 + 9m^4n^4 \\
\quad - 15m^6n^2 - 50m^5n^3 - 50m^4n^4 - 15m^3n^5 \\
\qquad + 15m^5n^3 + 50m^4n^4 + 50m^3n^5 + 15m^2n^6 \\
\qquad\quad - 3m^4n^4 - 10m^3n^5 - 10m^2n^6 - 3mn^7 \\
\hline
9m^7n + 15m^6n^2 - 5m^5n^3 + 6m^4n^4 + 25m^3n^5 + 5m^2n^6 - 3mn^7
\end{array}$$

Ans.

(174) $(2a^2bc^3)^4 = 16a^8b^4c^{12}$. Ans.

$(-3a^2b^3c)^5 = -243a^{10}b^{10}c^5$. Ans.

$(-7m^3nx^2y^4)^2 = 49m^6n^2x^4y^8$. Ans.

(175) (*a*) $4a^2 - b^2$ factored $= (2a + b)(2a - b)$. Ans.

(*b*) $16x^{10} - 1$ factored $= (4x^5 + 1)(4x^5 - 1)$. (Art. **463.**) Ans.

(*c*) $16x^6 - 8x^4y^2 + x^2y^4$ when factored =

$(4x^3 - xy^2)(4x^3 - xy^2)$. (Art. **457,** Rule.)

But, $(4x^3 - xy^2) = x(2x + y)(2x - y)$. (Arts. **452** and **463.**)

Hence, $16x^6 - 8x^4y^2 + x^2y^4 = x^2(2x + y)(2x + y)(2x - y)(2x - y)$. Ans.

(176)

$$\begin{array}{r|l} 4a^6 - 12a^5x + 5a^4x^2 + 6a^3x^3 + a^2x^4 & 2a^3 - 3a^2x - ax^2 \\ \hline 4a^6 & \text{Ans.} \\ \hline -12a^5x + 5a^4x^2 & \\ -12a^5x + 9a^4x^2 & \\ \hline -4a^4x^2 + 6a^3x^3 + a^2x^4 & \\ -4a^4x^2 + 6a^3x^3 + a^2x^4 & \end{array}$$

(177) (*a*) $6a^4b^4 + a^3b^3 - 7a^2b^3 + 2abc + 3.$

(*b*) $3 + 2abc + a^3b^3 - 7a^2b^3 + 6a^4b^4.$

(*c*) $1 + ax + a^2 + 2a^3$. Written like this, the a in the second term is understood as having 1 for an exponent; hence, if we represent the first term by a^0, in value it will be equal to 1, since $a^0 = 1$. (Art. **439.**) Therefore, 1 should be written as the first term when arranged according to the increasing powers of a.

(178) $\sqrt[4]{16a^{12}b^4c^8} = \pm 2a^3bc^2$. Ans. (Art. **521.**)

$\sqrt[5]{-32a^{15}} = -2a^3$. Ans.

$\sqrt[3]{-1,728a^6d^{12}x^3y^9} = -12a^2d^4xy^3$. Ans.

(179) (*a*) $(a - 2x + 4y) - (3z + 2b - c)$. Ans. (Art. **408.**)

(*b*) $-3b - 4c + d - (2f - 3e)$ becomes $-[3b + 4c - d + (2f - 3e)]$ when placed in brackets preceded by a minus sign. Ans. (Art. **408.**)

(*c*) The subtraction of one expression or quantity from another, when none of the terms are alike, can be represented only by combining the subtrahend with the minuend by means of the sign —.

In this case, where we are to subtract $2b-(3c+2d)-a$ from x, the result will be indicated by $x-[2b-(3c+2d)-a.]$ Ans. (Art. **408.**)

(180) (*a*)
$$\begin{array}{l} 2x^3+2x^2+2x-2 \\ x-1 \\ \hline 2x^4+2x^3+2x^2-2x \\ \quad -2x^3-2x^2-2x+2 \\ \hline 2x^4 \qquad\qquad -4x+2 \end{array}$$ Ans.

(*b*)
$$\begin{array}{l} x^2-4ax+c \\ 2x+a \\ \hline 2x^3-8ax^2+2cx \\ \quad ax^2 \qquad -4a^2x+ac \\ \hline 2x^3-7ax^2+2cx-4a^2x+ac \end{array}$$ Ans.

(*c*)
$$\begin{array}{l} -a^3+3a^2b-2b^3 \\ 5a^2+9ab \\ \hline -5a^5+15a^4b-10a^2b^3 \\ \quad -9a^4b \qquad +27a^3b^2-18ab^4 \\ \hline -5a^5+6a^4b-10a^2b^3+27a^3b^2-18ab^4 \end{array}$$

Arranging the terms according to the decreasing powers of a, we have $-5a^5+6a^4b+27a^3b^2-10a^2b^3-18ab^4$. Ans.

(181) (*a*)
$$\begin{array}{r} 4xyz \\ -3xyz \\ -5xyz \\ 6xyz \\ -9xyz \\ 3xyz \\ \hline -4xyz \end{array}$$ Ans.

The sum of the coefficients of the positive terms we find to be $+13$, since $(+3)+(+6)+(+4)=(+13)$.

When no sign is given before a quantity the $+$ sign must always be understood. The sum of the coefficients of the negative terms we find to be -17 since $(-9)+(-5)+(-3)=(-17)$. Subtracting the *lesser* sum from the *greater*, and prefixing the sign of the greater sum $(-)$ (Art. **390,** rule II), we have $(+13)+(-17)=-4$. Since the terms are all alike, we have only to annex the common symbols xyz to -4, thereby obtaining $-4xyz$ for the result or sum.

(*b*)
$$\begin{array}{r} 3a^2 + 2ab + 4b^2 \\ 5a^2 - 8ab + b^2 \\ -a^2 + 5ab - b^2 \\ 18a^2 - 20ab - 19b^2 \\ 14a^2 - 3ab + 20b^2 \\ \hline 39a^2 - 24ab + 5b^2 \end{array}$$ Ans.

When adding polynomials, always place like terms under each other. (Art. **393.**)

The coefficient of a^2 in the result will be 39, since $(+14)+(+18)+(-1)+(+5)+(+3)=39$. When the coefficient of a term is not written, 1 is always understood to be its coefficient. (Art. **359.**) The coefficient of ab will be -24, since $(-3)+(-20)+(+5)+(-8)+(+2)=-24$. The coefficient of b^2 will be $(+20)+(-19)+(-1)+(+1)+(+4)=+5$. Hence, the result or sum is $39a^2-24ab+5b^2$.

(*c*)
$$\begin{array}{r} 4mn + 3ab - 4c \\ +2mn - 4ab \qquad + 3x + 3m^2 - 4p \\ \hline 6mn - ab - 4c + 3x + 3m^2 - 4p \end{array}$$ Ans.

(182) The reciprocal of 3.1416 is $\frac{1}{3.1416} = .3183+$. **Ans.**

Reciprocal of $.7854 = \frac{1}{.7854} = 1.273+$. Ans.

Of $\frac{1}{64.32} = \frac{1}{\frac{1}{64.32}} = 1 \times \frac{64.32}{1} = 64.32$. Ans.
(See Art. **481.**)

(183) (*a*) $\frac{x}{x-y} + \frac{x-y}{y-x}$. If the denominator of the second fraction were written $x-y$, instead of $y-x$, then $x-y$ would be the common denominator.

By Art. **482,** the signs of the denominator and the sign before the fraction $\frac{x-y}{y-x}$ may be changed, giving $-\frac{x-y}{x-y}$. We now have

$$\frac{x}{x-y} - \frac{x-y}{x-y} = \frac{x-x+y}{x-y} = \frac{y}{x-y}.$$ Ans.

(*b*) $\frac{x^2}{x^2-1} + \frac{x}{x+1} - \frac{x}{1-x}$. If we write the denominator of the third fraction $x-1$ instead of $1-x$, x^2-1 will then be the common denominator.

By Art. **482,** the signs of the denominator and the sign before the fraction may be changed, thereby giving $\frac{x}{x-1}$. We now have

$$\frac{x^2}{x^2-1}+\frac{x}{x+1}+\frac{x}{x-1}=\frac{x^2+x(x-1)+x(x+1)}{x^2-1}=$$

$$\frac{x^2+x^2-x+x^2+x}{x^2-1}=\frac{3x^2}{x^2-1}. \quad \text{Ans.}$$

(*c*) $\frac{3a-4b}{7}-\frac{2a-b+c}{3}+\frac{13a-4c}{12}$, when reduced to a common denominator

$$=\frac{12(3a-4b)-28(2a-b+c)+7(13a-4c)}{84}.$$

Expanding the terms and removing the parentheses, we have

$$\frac{36a-48b-56a+28b-28c+91a-28c}{84}.$$

Combining like terms in the numerator, we have as the result,

$$\frac{71a-20b-56c}{84}. \quad \text{Ans.}$$

(184) (*a*) $45x^7y^{10}-90x^5y^7-360x^4y^6=45x^4y^7(x^3y^3-2x-8y)$. Ans. (Art. **452.**)

(*b*) $a^2b^2+2abcd+c^2d^2=(ab+cd)^2$. Ans. (Art. **457.**)

(*c*) $(a+b)^2-(c-d)^2=(a+b+c-d)\,(a+b-c+d)$. Ans. (Art. **463.**)

(185) (*a*) If a man builds 20 rods of stone wall, and we consider this work as positive, or $+$, the work which he does in tearing it down may be considered as negative, or $-$. If he tore down 10 rods, we could say that he *built* -10 rods.

(*b*) See Arts. **388** and **398.**

(186) (*a*) $\frac{2ax+x^2}{a^3-x^3}\div\frac{x}{a-x}=\frac{x(2a+x)}{a^3-x^3}\times\frac{a-x}{x}$.

(Art. **502.**)

Canceling common factors, the result equals $\frac{2a+x}{a^2+ax+x^2}$. Ans.

$$
\begin{array}{r|l}
a-x & a^3-x^3\,(\,a^2+ax+x^2 \\
\cline{2-2}
 & a^3-a^2x \\
\cline{2-2}
 & \quad a^2x-x^3 \\
 & \quad a^2x-ax^2 \\
\cline{2-2}
 & \qquad ax^2-x^3 \\
 & \qquad ax^2-x^3 \\
\cline{2-2}
\end{array}
$$

(*b*) Inverting the divisor and factoring, we have

$$\frac{3n(2m^2n-1)}{(2m^2n-1)\,(2m^2n-1)} \times \frac{(2m^2n+1)\,(2m^2n-1)}{3n}.$$

Canceling common factors, we have $2m^2n+1$. Ans.

(*c*) $9+\frac{5y^2}{x^2-y^2} \div \left(3+\frac{5y}{x-y}\right)$ simplified $= \frac{9x^2-4y^2}{x^2-y^2} \div \frac{3x+2y}{x-y}$.

Inverting the divisor, we have $\frac{9x^2-4y^2}{x^2-y^2} \times \frac{x-y}{3x+2y}$.

Canceling common factors, the result equals $\frac{3x-2y}{x+y}$. Ans.

(187) According to Art. **456,** the trinomials $1-2x^2+x^4$ and $4x^2+4x+1$ are perfect squares. (See Art. **458.**) The remaining trinomials are not perfect squares, since they do not comply with the foregoing principles.

(188) (*a*) By Art. **481,** the reciprocal of $\frac{24}{49} = 1 \div \frac{24}{49}$ $= 1 \times \frac{49}{24} = \frac{49}{24}$. Ans.

(*b*) Since, by Art. **481,** a number may be found from its reciprocal by dividing 1 by the reciprocal, the number $= 1 \div 700 = .0014\frac{2}{7}$. Ans.

(189) Applying the method of Art. **474,**

$x+y$	$12xy\,(x^2-y)$,	$2x^2(x^2+2xy+y^2)$,	$3y^2(x-y)^2$, $6(x^2+xy)$
$x-y$	$12xy\,(x-y)$,	$2x^2(x+y)$,	$3y^2(x-y)^2$, $6x$
$3xy$	$12xy$	$2x^2(x+y)$,	$3y^2(x-y)$, $6x$
2	4,	$2x\,(x+y)$,	$y(x-y)$, 2
	2,	$x\,(x+y)$,	$y(x-y)$, 1

Whence, L. C. M. $= (x+y)\ (x-y)\ 3\ x\ y \times 2 \times 2 \times x\,(x+y) \times y\ (x-y) = 12x^2y^2\ (x+y)^2\ (x-y)^2$.

(190) (*a*) $2 + 4a - 5a^2 - 6a^3$

$7a^3$

$14a^3 + 28a^4 - 35a^5 - 42a^6$ Ans. (Art. **423.**)

(*b*) $4x^2 - 4y^2 + 6z^2$

$3x^2y$

$12x^4y - 12x^2y^3 + 18x^2yz^2$ Ans.

(*c*) $3b + 5c - 2d$

$6a$

$18ab + 30ac - 12ad$ Ans.

(191) (*a*) See Arts. **359** and **361.**

(*b*) See Arts. **419** and **440.**

(*c*) See Art. **416.**

(192) (*a*) On removing the vinculum, we have

$2a - [3b + \{4c - 4a - (2a + 2b)\} + \{3a - b - c\}]$.

(Art. **405.**)

Removing the parenthesis,

$2a - [3b + \{4c - 4a - 2a - 2b\} + \{3a - b - c\}]$.

Removing the braces,

$2a - [3b + 4c - 4a - 2a - 2b + 3a - b - c]$.

(Art. **406.**)

Removing the brackets,

$2a - 3b - 4c + 4a + 2a + 2b - 3a + b + c$.

Combining like terms, the result is $5a - 3c$. Ans.

(b) Removing the parenthesis, we have

$$7a - [3a - \{2a - 5a + 4a\}].$$

Removing the brace,

$$7a - [3a - 2a + 5a - 4a].$$

Removing the brackets,

$$7a - 3a + 2a - 5a + 4a.$$

Combining terms, the result is $5a$. Ans.

(c) Removing the parentheses, we have

$$a - [2b + \{3c - 3a - a - b\} + \{2a - b - c\}].$$

Removing the braces,

$$a - [2b + 3c - 3a - a - b + 2a - b - c].$$

Removing the brackets,

$$a - 2b - 3c + 3a + a + b - 2a + b + c.$$

Combining like terms, the result is $3a - 2c$. Ans.

(193) (a) $(x^3 + 8) = (x + 2)(x^2 - 2x + 4)$. Ans.

(b) $x^3 - 27y^3 = (x - 3y)(x^2 + 3xy + 9y^2)$. Ans.

(c) $xm - nm + xy - ny = m(x - n) + y(x - n)$,

or $(x - n)(m + y)$. Ans.

(Arts. **466** and **468.**)

(194) Arrange the terms according to the decreasing powers of x. (Art. **523.**)

$4x^4 + 8ax^3 + 4a^2x^2 + 16b^2x^2 + 16ab^2x + 16b^4$ $(2x^2 + 2ax + 4b^2$. Ans.

$(2x^2)^2 = 4x^4$.

$4x^2 + 2ax$	$8ax^3 + 4a^2x^2$
	$8ax^3 + 4a^2x^2$
$4x^2 + 4ax + 4b^2$	$16b^2x^2 + 16ab^2x + 16b^4$
	$16b^2x^2 + 16ab^2x + 16b^4$

(195) $\dfrac{c(a+b)+cd}{(a+b)c} = \dfrac{ac+bc+cd}{ac+bc}$. Canceling c, which is common to each term, we have $\dfrac{a+b+d}{a+b} = 1 + \dfrac{d}{a+b}$. Ans.

(196) (*a*) $x+y+z-(x-y)-(y+z)-(-y)$ becomes $x+y+z-x+y-y-z+y$ on the removal of the parentheses. (Art. **405.**) Combining like terms,

$$x-x+y+y-y+y+z-z=2y. \quad \text{Ans.}$$

(*b*) $(2x-y+4z)+(-x-y-4z)-(3x-2y-z)$ becomes $2x-y+4z-x-y-4z-3x+2y+z$, on the removal of the parentheses. (Arts. **405** and **406.**) Combining like terms,

$$2x-x-3x-y-y+2y+4z-4z+z=z-2x. \quad \text{Ans.}$$

(*c*) $a-[2a+(3a-4a)]-5a-\{6a-[(7a+8a)-9a]\}$.

In this expression we find aggregation marks of different shapes, thus, [, (,and {. In such cases look for the corresponding part (whatever may intervene), and all that is included between the two parts of each aggregation mark must be treated as directed by the sign before it (Arts. **405** and **406**), no attention being given to any of the other aggregation marks. It is always best to begin with the *innermost pair*, and remove each pair of aggregation marks in order. First removing the parentheses, we have

$$a-[2a+3a-4a]-5a-\{6a-[7a+8a-9a]\}.$$

Removing the brackets, we have

$$a-2a-3a+4a-5a-\{6a-7a-8a+9a\}.$$

Removing the brace, we have

$$a-2a-3a+4a-5a-6a+7a+8a-9a.$$

Combining like terms, the result is $-5a$. Ans.

(197) (*a*) *A* square x square, plus $2a$ cube b fifth, minus the quantity a plus b.

(*b*) The cube root of x, plus y into the quantity a minus n square to the $\frac{2}{3}$ power.

(*c*) The quantity m plus n, into the quantity m minus n squared into the quantity m minus the quotient of n divided by 2.

(198)

$$a^2-a^3-2a-1)2a^6-4a^5-5a^4+3a^3+10a^2+7a+2(2a^3-2a^2-3a-2$$

$$2a^6-2a^5-4a^4-2a^3 \qquad \text{Ans.}$$

$$-2a^5-a^4+5a^3+10a^2$$

$$-2a^5+2a^4+4a^3+2a^2$$

$$-3a^4+a^3+8a^2+7a$$

$$-3a^4+3a^3+6a^2+3a$$

$$-2a^3+2a^2+4a+2$$

$$-2a^3+2a^2+4a+2$$

(199) (*a*) Factoring according to Art. **452,** we have $x^2y^2(x^6-64)$. Factoring (x^6-64), according to Art. **463,** we have

$$(x^3+8)(x^3-8).$$

Art. **466,** rule. $x^3+8=(x+2)(x^2-2x+4)$.

Art. **466,** rule. $x^3-8=(x-2)(x^2+2x+4)$.

Therefore, $x^8y^2-64x^2y^2=x^2y^2(x+2)(x^2-2x+4)(x-2)(x^2+2x+4)$, or $x^2y^2(x+2)(x-2)(x^2+2x+4)(x^2-2x+4)$. Ans.

(*b*) $a^2-b^2-c^2+1-2a+2bc$. Arrange as follows (Art. **408**):

$(a^2-2a+1)-(b^2-2bc+c^2)=(a-1)^2-(b-c)^2$. (Art. **455.**)

By Art. **463,** we have

$$(a-1+b-c)(a-1-[b-c]),$$

or $(a-1+b-c)(a-1-b+c)$. Ans.

(*c*) $1-16a^2+8ac-c^2$. Placing the last three terms in parentheses (Art. **408**), $1-(16a^2-8ac+c^2)$.

$16a^2-8ac+c^2=(4a-c)^2$. (Art. **455.**)

$1-(16a^2-8ac+c^2)=1-(4a-c)^2$.

$1-(4a-c)^2=[1+(4a-c)][1-(4a-c)]$. (Art. **463.**)

Removing parentheses, and writing parentheses in place of the brackets,

$$1-(4a-c)^2=(1+4a-c)(1-4a+c). \quad \text{Ans.}$$

(200) See Art. **482.**

(201) The subtraction of one expression from another, if none of the terms are similar, may be represented only by connecting the subtrahend with the minuend by means of the sign —. Thus, it is required to subtract $5a^3b - 7a^2b^2 + 5ab^3$ from $a^4 - b^4$, the result will be represented by $a^4 - b^4 - (5a^3b - 7a^2b^2 + 5ab^3)$, which, on removing the parentheses (Art. **405**), becomes $a^4 - b^4 - 5a^3b + 7a^2b^2 - 5ab^3$. From this result, subtract $3a^4 - 4a^3b + 6a^2b^2 + 5ab^3 - 3b^4$.

$$\begin{array}{l} a^4 - b^4 - 5a^3b + 7a^2b^2 - 5ab^3 \quad \textit{minuend.} \\ \underline{-3a^4 + 3b^4 + 4a^3b - 6a^2b^2 - 5ab^3 \quad \textit{subtrahend,} \text{ with signs}} \\ -2a^4 + 2b^4 - a^3b + a^2b^2 - 10ab^3 \quad \text{changed. (Art. 401.)} \end{array}$$

Or, $-2a^4 - a^3b + a^2b^2 - 10ab^3 + 2b^4$. Answer arranged according to the decreasing powers of a.

(202) (*a*) $3a - 2b + 3c$ / $2a - 8b - c$ becomes $3a - 2b + 3c$ / $-2a + 8b + c$ / $a + 6b + 4c$

when the signs of the subtrahend are changed. Now, adding each term (with its sign changed) in the subtrahend to its corresponding term in the minuend, we have $(-2a) + (3a) = a$; $(+8b) + (-2b) = +6b$; $(+c) + (3c) = +4c$. Hence, $a + 6b + 4c$ equals the difference. Ans.

(*b*) $2x^3 - 3x^2y + 2xy^3$ / $x^3 + y^3 - xy^2$ becomes

$2x^3 - 3x^2y + 2xy^3$ / $-x^3 - y^3 + xy^2$ / $x^3 - 3x^2y + 2xy^3 - y^3 + xy^2$

when the signs of the subtrahend are changed. Adding each term in the subtrahend (with its sign changed) to its corresponding term in the minuend, we have $x^3 - 3x^2y + 2xy^3 - y^3 + xy^2$, which, arranged according to the decreasing powers of x, equals $x^3 - 3x^2y + xy^2 + 2xy^3 - y^3$. Ans.

(*c*) $14a + 4b - 6c - 3d$ / $11a - 2b + 4c - 4d$

On changing the sign of each term in the subtrahend, the problem becomes

$$\begin{array}{r} 14a + 4b - \ 6c - 3d \\ -11a + 2b - \ 4c + 4d \\ \hline 3a + 6b - 10c + \ d \end{array}$$

Adding each term of the subtrahend (with the sign changed) to its corresponding term in the minuend, the difference, or result, is $3a + 6b - 10c + d$. Ans.

(203) The numerical values of the following, when $a = 16$, $b = 10$, and $x = 5$, are:

(*a*) $(ab^2x + 2abx)\ 4a = (16 \times 10^2 \times 5 + 2 \times 16 \times 10 \times 5) \times 4 \times 16$. It must be remembered that when no sign is expressed between symbols or quantities, the sign of multiplication is understood.

$(16 \times 100 \times 5 + 2 \times 16 \times 10 \times 5) \times 64 = (8{,}000 + 1{,}600) \times 64 = 9{,}600 \times 64 = 614{,}400$. Ans.

(*b*) $2\sqrt{4a} - \dfrac{2bx}{a-b} + \dfrac{b-x}{x} = 2\sqrt{64} - \dfrac{2 \times 10 \times 5}{16 - 10} + \dfrac{10 - 5}{5} = 16 - \dfrac{100}{6} + 1 = \dfrac{96 - 100 + 6}{6} = \dfrac{2}{6} = \dfrac{1}{3}$. Ans.

(*c*) $(b - \sqrt{a})\ (x^3 - b^2)\ (a^2 - b^2) = (10 - \sqrt{16})\ (5^3 - 10^2)\ (16^2 - 10^2) = (10 - 4)\ (125 - 100)\ (256 - 100) = 6 \times 25 \times 156 = 23{,}400$. Ans.

(204) (*a*) Dividing both numerator and denominator by $15mxy^2$, $\dfrac{15mxy^2}{75mx^2y^3} = \dfrac{1}{5xy}$. Ans.

(*b*) $\dfrac{x^2 - 1}{4x(x+1)} = \dfrac{(x+1)(x-1)}{4x(x+1)}$ when the numerator is factored.

Canceling $(x + 1)$ from both the numerator and denominator (Art. **484**), the result is $\dfrac{x-1}{4x}$. Ans.

(*c*) $\dfrac{(a^3 + b^3)(a^3 + ab + b^2)}{(a^3 - b^3)(a^3 - ab + b^2)}$ when factored becomes

$\dfrac{(a+b)(a^2 - ab + b^2)(a^3 + ab + b^2)}{(a-b)(a^2 + ab + b^2)(a^3 - ab + b^3)}$. (Art. **466.**)

Canceling the factors common to both the numerator and denominator, we have

$$\frac{(a+b)(a^2-ab+b^2)(a^2+ab+b^2)}{(a-b)(a^2+ab+b^2)(a^2-ab+b^2)}=\frac{a+b}{a-b}.$$ Ans.

$$\begin{array}{r|l} a+b & a^3+b^3 \quad (a^2-ab+b^2 \\ \hline & a^3+a^2b \\ \hline & -a^2b+b^3 \\ & -a^2b-ab^2 \\ \hline & ab^2+b^3 \\ & ab^2+b^3 \\ \hline \end{array} \qquad \begin{array}{r|l} a-b & a^3-b^3 \quad (a^2+ab+b^2 \\ \hline & a^3-a^2b \\ \hline & a^2b-b^3 \\ & a^2b-ab^2 \\ \hline & ab^2-b^3 \\ & ab^2-b^3 \\ \hline \end{array}$$

(205) (*a*) $$\frac{\frac{1}{1-x}-\frac{1}{1+x}}{\frac{1}{1-x}+\frac{1}{1+x}}=\frac{\frac{1+x-1+x}{1-x^2}}{\frac{1+x+1-x}{1-x^2}}=\frac{2x}{1-x^2}\div$$

$$\frac{2}{1-x^2}=\frac{2x}{1-x^2}\times\frac{1-x^2}{2}=x.$$ Ans. (See Art. **509.**)

(*b*) $$\frac{\frac{a^3}{b^3}+\frac{1}{a}}{\frac{a}{b^2}-\frac{a-b}{ab}}=\frac{\frac{a^3+b^3}{ab^3}}{\frac{a^2b-b^2(a-b)}{ab^3}}=\frac{\frac{a^3+b^3}{ab^3}}{\frac{a^2b-ab^2+b^3}{ab^3}}=$$

$$\frac{a^3+b^3}{ab^3}\div\frac{a^2b-ab^2+b^3}{ab^3}=\frac{a^3+b^3}{ab^3}\times\frac{ab^3}{b(a^2-ab+b^2)}=\frac{a+b}{b}.$$

Ans.

(*c*) $$\frac{1}{x+\frac{1}{1+\frac{x+1}{3-x}}}=\frac{1}{x+\frac{3-x}{4}}=\frac{4}{3x+3}.$$ Ans.

(Art. **509.**)

(206) $\frac{3+2x}{2-x}-\frac{2-3x}{2+x}+\frac{16x-x^2}{x^2-4}$. If the denominator of the third fraction were written $4-x^2$, instead of x^2-4, the common denominator would then be $4-x^2$.

By Art. **482,** $\frac{16x-x^2}{x^2-4}$ becomes $-\frac{16x-x^2}{-x^2+4}=-\frac{16x-x^2}{4-x^2}$.

Hence, $\frac{3+2x}{2-x}-\frac{2-3x}{2+x}-\frac{16x-x^2}{4-x^2}$, when reduced to a common denominator, becomes

$$\frac{(3+2x)(2+x)-(2-3x)(2-x)-(16x-x^2)}{4-x^2}=$$
$$\frac{(6+7x+2x^2)-(4-8x+3x^2)-(16x-x^2)}{4-x^2}.$$

Removing the parentheses (Art. **405**), we have

$$\frac{6+7x+2x^2-4+8x-3x^2-16x+x^2}{4-x^2}.$$

Combining like terms in the numerator, we have

$$\frac{2-x}{4-x^2}.$$

Factoring the denominator by Art. **463,** we have

$$\frac{2-x}{(2+x)(2-x)}.$$

Canceling the common factor $(2-x)$, the result equals

$$\frac{1}{2+x}, \text{ or } \frac{1}{x+2}. \quad \text{Ans.} \quad \text{(Art. 373.)}$$

(207) (*a*)

$$1+2x-\frac{4x-4}{5x}=\frac{5x+10x^2-4x+4}{5x}=\frac{10x^2+x+4}{5x}. \quad \text{Ans.}$$

(Art. **504.**)

(*b*) $\dfrac{3x^2+2x+1}{x+4}=3x-10+\dfrac{41}{x+4}$. Ans. (Art. **505.**)

$$\begin{array}{r|l} x+4 & 3x^2+2x+1 \\ & 3x^2+12x \\ \hline & -10x+1 \\ & -10x-40 \\ \hline & 41 \end{array} \quad \left(3x-10+\frac{41}{x+4}\right.$$

(*c*) Reducing, the problem becomes

$$\frac{x^2+4x-5}{x^2}\times\frac{x-7}{x^2-8x+7}.$$

Factoring, we have

$$\frac{(x+5)\,(x-1)}{x^2}\times\frac{x-7}{(x-1)\,(x-7)}.$$

Canceling common factors, the result equals $\dfrac{x+5}{x^2}$. Ans.

(208) (*a*) Writing the work as follows, and canceling common factors in both numerator and denominator (Arts. **496** and **497**), we have

$$\frac{9m^2n^2}{8p^3q^3} \times \frac{5p^2q}{2xy} \times \frac{24x^2y^2}{90mn} =$$

$$\frac{9 \times 5 \times 24 \times m^2n^2p^2qx^2y^2}{8 \times 2 \times 90 \times m \times n \times p^3 \times q^3 \times x \times y} = \frac{3mnxy}{4pq^2}. \quad \text{Ans.}$$

(*b*) Factoring the numerators and denominators of the fraction (Art. **498**), and writing the factors of the numerators over the factors of the denominators, we have

$$\frac{(a-x)(a^2+ax+x^2)(a+x)(a+x)}{(a+x)(a^2-ax+x^2)(a-x)(a-x)} =$$

$$\frac{(a+x)(a^2+ax+x^2)}{(a-x)(a^2-ax+x^2)}. \quad \text{Ans.}$$

(*c*) This problem may be written as follows, according to Art. **480:**

$$\frac{3ax+4}{1} \times \frac{a^2}{a(3ax+4)(3ax+4)}.$$

Canceling a and $(3ax+4)$, we have $\frac{a}{3ax+4}$. Ans.

(209) (*a*) $-7my \,)\, 35m^3y + 28m^2y^2 - 14my^3$

$$-5m^2 - 4my + 2y^2$$

Ans. (Art. **442.**)

(*b*) $a^4 \,)\, 4a^4 - 3a^5b - a^6b^2$

$$4 - 3ab - a^2b^2 \quad \text{Ans.}$$

(*c*) $4x^2 \,)\, 4x^3 - 8x^5 + 12x^7 - 16x^9$

$$x - 2x^3 + 3x^5 - 4x^7 \quad \text{Ans.}$$

(210) (*a*) $16a^2b^3$; $a^4 + 4ab$; $4a^2 - 16a^3b + 5a^6 + 7ax$.

(*b*) Since the terms are not alike, we can only indicate the sum, connecting the terms by their proper signs. (Art. **389.**)

(*c*) Multiplication: $4ac^2d$ means $4 \times a \times c^2 \times d$. (Art. **358.**)

(211) $\frac{a^2+c^2+ac}{a^2+b^2-c^2-2ab} \times \frac{a^2+c^2-b^2-2ac}{a^4c-ac^4}$.

Arranging the terms, we have

$$\frac{a^2+ac+c^2}{a^3-2ab+b^2-c^2} \times \frac{a^2-2ac+c^2-b^2}{a^4c-ac^4},$$

which, being placed in parentheses, become

$$\frac{a^2+ac+c^2}{(a^2-2ab+b^2)-c^2} \times \frac{(a^2-2ac+c^2)-b^2}{a^4c-ac^4}.$$

By Art. **456,** we know that $a^2-2ab+b^2$, also $a^2-2ac+c^2$, are perfect squares, and may be written $(a-b)^2$ and $(a-c)^2$.

Factoring a^4c-ac^4 by Case I, Art. **452,** we have

$$\frac{a^2+ac+c^2}{(a-b)^2-c^2} \times \frac{(a-c)^2-b^2}{ac(a^3-c^3)} =$$

$$\frac{a^2+ac+c^2}{(a-b-c)(a-b+c)} \times \frac{(a-c-b)(a-c+b)}{ac(a-c)(a^2+ac+c^2)}.$$

(Arts. **463** and **466.**)

Canceling common factors and multiplying, we have

$$\frac{a-c+b}{(a-b+c)ac(a-c)}, \text{ or } \frac{a+b-c}{ac(a-b+c)(a-c)}. \quad \text{Ans.}$$

(212) The square root of the fraction a plus b plus c divided by n, plus the square root of a, plus the fraction b plus c divided by n, plus the square root of a plus b, plus the fraction c divided by n, plus the quantity a plus b, into c, plus a plus bc.

(213) (*a*) $\frac{4x+5}{3}-\frac{3x-7}{5x}+\frac{9}{12x^2}$.

We will first reduce the fractions to a common denominator. The L. C. M. of the denominator is $60x^2$, since this is the smallest quantity that each denominator will divide without a remainder. Dividing $60x^2$ by 3, the first denominator, the quotient is $20x^2$; dividing $60x^2$ by $5x$, the second denominator, the quotient is $12x$; dividing $60x^2$ by $12x^2$, the third denominator, the quotient is 5. Multiplying the corresponding numerators by these respective quotients, we obtain $20x^2(4x+5)$ for the first new numerator; $12x(3x-7)$ for the second new numerator, and $5 \times 9 = 45$ for the third new numerator. Placing these new numerators over the common denominator and expanding the terms, we have

$$\frac{20x^2(4x+5)-12x(3x-7)+45}{60x^2}=\frac{80x^3+100x^2-36x^2+84x+45}{60x^2}.$$

Collecting like terms, the result is

$$\frac{80x^3+64x^2+84x+45}{60x^2}. \quad \text{Ans.}$$

(*b*) In $\frac{1}{2a(a+x)}+\frac{1}{2a(a-x)}$, the L. C. M. of the denominators is $2a(a^2-x^2)$, since this is the smallest quantity that each denominator will divide without a remainder. Dividing $2a(a^2-x^2)$ by $2a(a+x)$, the first denominator, we will have $a-x$; dividing $2a(a^2-x^2)$ by $2a(a-x)$, the second denominator, we have $a+x$. Multiplying the corresponding numerators by these respective quotients, we have $(a-x)$ for the first new numerator, and $(a+x)$ for the second new numerator. Arranging the work as follows:

$1\times(a-x)=a-x=$ 1st numerator.
$1\times(a+x)=a+x=$ 2d numerator.
or $2a$ = the sum of the numerators.

Placing the $2a$ over the common denominator $2a(a^2-x^2)$, we find the value of the fraction to be

$$\frac{2a}{2a(a^2-x^2)}=\frac{1}{a^2-x^2}. \quad \text{Ans.}$$

(*c*) $\frac{x}{y}+\frac{y}{x+y}+\frac{x^2}{x^2+xy}=\frac{x}{y}+\frac{y}{x+y}+\frac{x}{x+y}$.

The common denominator $=y\,(x+y)$. Reducing the fractions to a common denominator, we have

$$\frac{x(x+y)+y^2+xy}{y(x+y)}=\frac{x^2+2xy+y^2}{y(x+y)}=\frac{x+y}{y}. \quad \text{Ans.}$$

(214) (*a*) Apply the method of Art. **474:**

$6ax$	$18ax^2,\ 72ay^2,\ 12xy$
$2y$	$3x,\ 12y^2,\ 2y$
3	$3x,\ 6y,\ 1$
	$x,\ 2y,\ 1$

Whence, $6ax\times 2y\times 3\times x\times 2y=72ax^2y^2$. Ans.

$$\begin{array}{rl|l} (b)\ 2(1+x) & 4(1+x),\ 4(1-x),\ 2(1-x^2) \\ \hline 2(1-x) & 2, \qquad 2(1-x), \quad 1-x \\ \hline & 1, \qquad 1, \qquad\quad 1 \end{array}$$

Hence, L. C. M. $= 2(1+x) \times 2(1-x) = 4(1-x^2)$. Ans.

$$\begin{array}{rl|l} (c)\ a-b & (a-b)\ (b-c),\ (b-c)\ (c-a),\ (c-a)\ (a-b) \\ \hline b-c & (b-c),\ (b-c)\ (c-a),\ (c-a) \\ \hline c-a & 1, \qquad\quad c-a, \quad c-a \\ \hline & 1, \qquad\quad 1, \qquad 1 \end{array}$$

Hence, L. C. M. $= (a-b)\ (b-c)\ (c-a)$. Ans.

(215) $3x^6 - 3 + a - ax^6 = (3-a)x^6 - 3 + a = (3-a)\ (x^6-1)$. Regarding $x^6 - 1$ as $(x^3)^2 - 1$, we have, by Art. **462,** $x^6 - 1 = (x^3)^2 - 1 = (x^3-1)\ (x^3+1)$. $x^3 - 1 = (x-1)\ (x^2+x+1)$; $x^3 + 1 = (x+1)\ (x^2-x+1)$. Art. **466.**

Hence, the factors are

$(x^2+x+1)\ (x^2-x+1)\ (x+1)\ (x-1)\ (3-a)$. Ans.

(216) Arranging the terms according to the decreasing powers of x, and extracting the square root, we have

$x^4 + x^3y + 4\frac{1}{4}x^2y^2 + 2xy^3 + 4y^4\ (x^2 + \frac{1}{2}xy + 2y^2$. Ans.

$$\begin{array}{rl|l} & x^4 & \\ \hline 2x^2 + \frac{1}{2}xy & x^3y + 4\frac{1}{4}x^2y^2 & \\ & x^3y + \ \frac{1}{4}x^2y^2 & \\ \hline 2x^2 + xy + 2y^2 & & 4x^2y^2 + 2xy^3 + 4y^4 \\ & & 4x^2y^2 + 2xy^3 + 4y^4 \end{array}$$

(217) The arithmetic ratio of $x^4 - 1$ to $x + 1$ is $x^4 - 1 - (x+1) = x^4 - x - 2$. Art. **381.**

The geometric ratio of $x^4 - 1$ to $x + 1$ is $\frac{x^4-1}{x+1} = x^3 - x^2 + x - 1 = (x^2+1)\ (x-1)$. Ans.

ALGEBRA.

(QUESTIONS 218-257.)

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(218) (*a*) According to Art. **528,** $x^{\frac{1}{2}}$ expressed radically is $\sqrt[4]{x^3}$;

$3x^{\frac{1}{2}}y^{-\frac{2}{3}}$ expressed radically is $3\sqrt{xy^{-3}}$;

$3x^{\frac{1}{2}}y^{\frac{2}{3}}z^{\frac{1}{3}} = 3\sqrt[6]{xy^{-5}z^2}$, since $z^{\frac{1}{3}} = z^{\frac{2}{6}}$. Ans.

(*b*) $a^{-1}b^{\frac{1}{2}} + \dfrac{c^{-2}}{a+b} + (m-n)^{-1} - \dfrac{a^2b^{-3}c}{c^{-3}} =$

$\dfrac{b^{\frac{1}{2}}}{a} + \dfrac{1}{c^2(a+b)} + \dfrac{1}{m-n} - \dfrac{a^2c^4}{b^3}$. Ans.

(*c*) $\sqrt[3]{x^6} = x^{\frac{6}{3}}$. Ans. $\sqrt[3]{x^{-1}} = x^{-\frac{1}{3}}$. Ans.

$(\sqrt[4]{b^5x^2})^3 = (b^{\frac{5}{4}}x^{\frac{1}{2}})^3 = b^{\frac{15}{4}}x^{\frac{3}{2}}$. Ans.

(219) $3\sqrt{21} = \sqrt{189}$. Ans. (Art. **542.**)

$a^2b\sqrt{b^3c} = \sqrt{a^4b^5c}$. Ans. $2x\sqrt[5]{x} = \sqrt[5]{32x^6}$. Ans.

(220) Let x = the length of the post.

Then, $\dfrac{x}{5}$ = the amount in the earth.

$\dfrac{3x}{7}$ = the amount in the water.

$$\frac{x}{5} + \frac{3x}{7} + 13 = x.$$

$$7x + 15x + 455 = 35x.$$

$$-13x = -455.$$

$$x = 35 \text{ feet.} \quad \text{Ans.}$$

(221) $t = \dfrac{W_1s_1t_1 + W_2s_2t_2}{W_1s_1 + W_2s_2}$.

In order to transform this formula so that t_2 may stand alone in the first member, we must first clear of fractions. Clearing of fractions, we have

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$$t\,W_1 s_1 + t\,W_2 s_2 = W_1 s_1 t_1 + W_2 s_2 t_2.$$

Transposing, we have

$$-W_2 s_2 t_2 = W_1 s_1 t_1 - t\,W_1 s_1 - t\,W_2 s_2.$$

Factoring (Arts. **452** and **408**), we have

$$-W_2 s_2 t_2 = W_1 s_1 t_1 - (W_1 s_1 + W_2 s_2)\,t;$$

whence, $t_2 = \dfrac{(W_1 s_1 + W_2 s_2)\,t - W_1 s_1 t_1}{W_2 s_2}$. Ans.

(222) Let x = number of miles he traveled per hour.

Then, $\dfrac{48}{x}$ = time it took him.

$\dfrac{48}{x+4}$ = time it would take him if he traveled 4 miles more per hour.

In the latter case the time would have been 6 hours *less;* whence, the equation

$$\frac{48}{x+4} = \frac{48}{x} - 6.$$

Clearing of fractions,

$$48x = 48x + 192 - 6x^2 - 24x.$$

Combining like terms and transposing,

$$6x^2 + 24x = 192.$$

Dividing by 6, $x^2 + 4x = 32.$

Completing the square, $x^2 + 4x + 4 = 36.$

Extracting square root, $x + 2 = \pm 6;$

whence, $x = -2 + 6 = 4$, or the number of miles he traveled per hour. Ans.

(223) (*a*) $S = \sqrt[3]{\dfrac{CPD^2}{f\left(2 + \dfrac{D^2}{d^2}\right)}} = \sqrt[3]{\dfrac{CPD^2}{2f + \dfrac{fD^2}{d^2}}}.$

Cubing both members to remove the radical,

$$S^3 = \frac{CPD^2}{2f + \dfrac{fD^2}{d^2}}.$$

Simplifying the result, $S^3 = \dfrac{CPD^2 d^2}{2fd^2 + fD^2}.$

Clearing of fractions,

$$2S^2fd^2 + S^2fD^2 = CPD^2d^2.$$

Transposing, $CPD^2d^2 = 2S^2fd^2 + S^2fD^2$;

whence, $$P = \frac{2S^2fd^2 + S^2fD^2}{CD^2d^2} = \frac{(2d^2 + D^2)fS^2}{CD^2d^2}.$$ Ans.

(*b*) Substituting the values of the letters in the given formula, we have

$$P = \frac{(2 \times 18^2 + 30^2) \times 864 \times 6^2}{10 \times 30^2 \times 18^2} = \frac{(648 + 900) \times 864 \times 216}{9{,}000 \times 324} =$$

$$\frac{288{,}893{,}952}{2{,}916{,}000} = 99.1, \text{ nearly.}$$ Ans.

(224) (*a*) $3x + 6 - 2x = 7x$. Transposing 6 to the second member, and $7x$ to the first member (Art. **561**),

$$3x - 2x - 7x = -6.$$

Combining like terms, $-6x = -6$;

whence, $x = 1$. Ans.

(*b*) $$5x - (3x - 7) = 4x - (6x - 35).$$

Removing the parentheses (Art. **405**),

$$5x - 3x + 7 = 4x - 6x + 35.$$

Transposing 7 to the second member, and $4x$ and $-6x$ to the first member, $5x - 3x - 4x + 6x = 35 - 7$. (Art. **561.**)

Combining like terms, $4x = 28$;

whence, $x = 28 \div 4 = 7$. Ans.

(*c*) $$(x + 5)^2 - (4 - x)^2 = 21x.$$

Performing the operations indicated, the equation becomes

$$x^2 + 10x + 25 - 16 + 8x - x^2 = 21x.$$

Transposing, $x^2 - x^2 + 10x + 8x - 21x = 16 - 25$.

Combining like terms, $-3x = -9$.

Dividing by -3, $x = 3$. Ans.

(225) (*a*) Simplifying by Art. **538,**

$$\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}.$$

$$2\sqrt{48} = 2\sqrt{16} \times \sqrt{3} = 8\sqrt{3}.$$

$$3\sqrt{108} = 3\sqrt{36} \times \sqrt{3} = 18\sqrt{3}.$$

Sum $= 29\sqrt{3}$. Ans. (Art. **544.**)

(*b*) $\sqrt[3]{128} = \sqrt[3]{64} \times \sqrt[3]{2} = 4\sqrt[3]{2}.$

$\sqrt[3]{686} = \sqrt[3]{343} \times \sqrt[3]{2} = 7\sqrt[3]{2}.$

$\sqrt[3]{16} = \sqrt[3]{8} \times \sqrt[3]{2} = 2\sqrt[3]{2}.$

Sum $= 13\sqrt[3]{2}.$ Ans. (Art. **544.**)

(*c*) $\sqrt{\frac{3}{8}} = \sqrt{\frac{3}{8} \times \frac{2}{2}} = \sqrt{\frac{6}{16}} = \frac{1}{4}\sqrt{6}.$ (Art. **540.**)

$\sqrt{\frac{1}{6}} = \sqrt{\frac{1}{6} \times \frac{6}{6}} = \sqrt{\frac{6}{36}} = \frac{1}{6}\sqrt{6}.$

$\sqrt{\frac{2}{27}} = \sqrt{\frac{2}{27} \times \frac{3}{3}} = \sqrt{\frac{6}{81}} = \frac{1}{9}\sqrt{6}.$

Sum $= \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{9}\right)\sqrt{6} = \frac{19}{36}\sqrt{6}.$ Ans.

(226) Let $x =$ the capacity.

Then, $x - 42 =$ amount held at first;

$7(x - 42) = x;$

$7x - 294 = x;$

$6x = 294;$

$x = 49$ gallons. Ans.

(227) (*a*) $2\sqrt{3x + 4} - x = 4.$

Transposing, Art. **579,** so that the radical stands alone in the first member, $2\sqrt{3x + 4} = x + 4.$

Squaring both members, since the index of the radical is understood to be 2, $4(3x + 4) = (x + 4)^2,$

or $12x + 16 = x^2 + 8x + 16.$

Transposing and uniting terms,

$-x^2 - 8x + 12x = 16 - 16.$

$-x^2 + 4x = 0.$

Dividing by $-x,$ $x - 4 = 0;$

whence, $x = 4.$ Ans.

(*b*) $\sqrt{3x - 2} = 2(x - 4).$

Squaring, $3x - 2 = 4(x - 4)^2,$

or $3x - 2 = 4x^2 - 32x + 64.$

Transposing, $-4x^2+32x+3x=64+2$.

Combining terms, $-4x^2+35x=66$.

Dividing by -4, $x^2-\frac{35x}{4}=-\frac{66}{4}$.

Completing the square,

$$x^2-\frac{35x}{4}+\left(\frac{35}{8}\right)^2=-\frac{66}{4}+\frac{1,225}{64}.$$

$$x^2-\frac{35x}{4}+\left(\frac{35}{8}\right)^2=-\frac{1,056}{64}+\frac{1,225}{64}=\frac{169}{64}$$

Extracting the square root,

$$x-\frac{35}{8}=\pm\frac{13}{8}.$$

Transposing, $x=\frac{35}{8}\pm\frac{13}{8}=6$, or $2\frac{3}{4}$. Ans.

(*c*) $\sqrt{x+16}=2+\sqrt{x}$ becomes $x+16=4+4\sqrt{x}+x$, when squared. Canceling x (Art. **562**), and transposing,

$$-4\sqrt{x}=4-16.$$

$$-4\sqrt{x}=-12.$$

$$\sqrt{x}=3;$$

whence, $x=3^2=9$. Ans.

(228) (*a*) $\sqrt{3x-5}=\frac{\sqrt{7x^2+36x}}{x}$.

Clearing of fractions,

$$x\sqrt{3x-5}=\sqrt{7x^2+36x}.$$

Removing radicals by squaring,

$$x^2(3x-5)=7x^2+36x.$$

$$3x^3-5x^2=7x^2+36x.$$

Dividing by x, $3x^2-5x=7x+36$.

Transposing and uniting,

$$3x^2-12x=36.$$

Dividing by 3, $x^2-4x=12$.

Completing the square,

$$x^2 - 4x + 4 = 16.$$

Extracting the square root,

$$x - 2 = \pm 4;$$

whence, $x = 6$, or -2. Ans.

(b) $x^2 - (b - a)c = ax - bx + cx.$

Transposing, $x^2 - ax + bx - cx = (b - a)c.$

Factoring, $x^2 - (a - b + c)x = bc - ac.$

Regarding $(a - b + c)$ as the coefficient of x, and completing the square,

$$x^2 - (a - b + c)x + \left(\frac{a - b + c}{2}\right)^2 = bc - ac + \left(\frac{a - b + c}{2}\right)^2.$$

$$x^2 - (a - b + c)x + \left(\frac{a - b + c}{2}\right)^2 =$$

$$\frac{a^2 - 2ab + b^2 - 2ac + 2bc + c^2.}{4}$$

$$x - \frac{a - b + c}{2} = \pm \frac{a - b - c}{2}.$$

$$x = \frac{2a - 2b}{2}, \text{ or } \frac{2c}{2}.$$

$$x = a - b, \text{ or } c. \quad \text{Ans.}$$

(c) $(x - 2)(x - 4) - 2(x - 1)(x - 3) = 0$, becomes

$x^2 - 6x + 8 - 2x^2 + 8x - 6 = 0$, when expanded.

Transposing and uniting terms,

$$-x^2 + 2x = -2.$$

Changing signs, $x^2 - 2x = 2.$

Completing the square, $x^2 - 2x + 1 = 3.$

Extracting the square root, $x - 1 = \pm \sqrt{3};$

whence, $x = 1 \pm \sqrt{3}.$ Ans.

(229) (a) $\sqrt{x - 4ab} = \dfrac{(a + b)(a - b)}{\sqrt{x}}.$

Expanding and clearing of fractions,

$$\sqrt{x^2 - 4abx} = a^2 - b^2.$$

Squaring both members,

$$x^2 - 4abx = a^4 - 2a^2b^2 + b^4.$$

Completing the square,

$$x^2 - 4abx + 4a^2b^2 = a^4 + 2a^2b^2 + b^4.$$
$$x - 2ab = \pm (a^2 + b^2).$$
$$x = (a^2 + 2ab + b^2),$$
$$\text{or} - (a^2 - 2ab + b^2).$$
$$x = (a + b)^2, \text{ or} - (a - b)^2. \quad \text{Ans.}$$

(*b*) $-\frac{1}{\sqrt{x+1}} + \frac{1}{\sqrt{x-1}} = \frac{1}{\sqrt{x^2-1}}$ becomes

$-\sqrt{x-1} + \sqrt{x+1} = 1$ when cleared of fractions.

Squaring,

$$x - 1 - 2\sqrt{x^2-1} + x + 1 = 1.$$
$$-2\sqrt{x^2-1} = 1 - 2x.$$

Squaring again, $4x^2 - 4 = 1 - 4x + 4x^2$.

Canceling $4x^2$ and transposing,

$$4x = 5.$$
$$x = \frac{5}{4} = 1\frac{1}{4} \quad \text{Ans.}$$

(230) $5x - 2y = 51.$ (1)

$19x - 3y = 180.$ (2)

We will first find the value of x by transposing $-2y$ to the second member of equation (1), whence $5x = 51 + 2y$, and

$$x = \frac{51 + 2y}{5}. \qquad (3)$$

This gives the value of x in terms of y. Substituting the value of x for the x in (2), (Art. **609.**)

$$\frac{19(51 + 2y)}{5} - 3y = 180.$$

Expanding, $\frac{969 + 38y}{5} - 3y = 180.$

Clearing of fractions, $969 + 38y - 15y = 900.$

Transposing and uniting, $23y = -69.$

$$y = -3. \quad \text{Ans.}$$

Substituting this value in equation (3), we have

$$x = \frac{51 - 6}{5} = 9. \quad \text{Ans.}$$

(231) (*a*) $2x^2 - 27x = 14.$

$$x^2 - \frac{27x}{2} = 7.$$

$$x^2 - \frac{27x}{2} + \left(\frac{27}{4}\right)^2 = 7 + \left(\frac{27}{4}\right)^2 = \frac{841}{16}.$$

$$x - \frac{27}{4} = \pm\frac{29}{4}.$$

$$x = \frac{56}{4} = 14,$$

$$\text{or} \quad x = -\frac{2}{4} = -\frac{1}{2}.$$

Hence, $x = 14$, or $-\frac{1}{2}$. Ans

(*b*) $x^2 - \frac{2x}{3} + \frac{1}{12} = 0.$

Transposing, $x^2 - \frac{2x}{3} = -\frac{1}{12}.$

Completing the square,

$$x^2 - \frac{2x}{3} + \left(\frac{1}{3}\right)^2 = -\frac{1}{12} + \frac{1}{9} = \frac{1}{36}.$$

Extracting the square root,

$$x - \frac{1}{3} = \pm\frac{1}{6}.$$

Transposing, $x = \frac{1}{3} + \frac{1}{6} = \frac{1}{2},$

$$\text{or} \quad x = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}.$$

Therefore, $x = \frac{1}{2}$ or $\frac{1}{6}$. Ans.

(*c*) $x^2 + ax = bx + ab.$

Transposing and factoring,

$$x^2 + (a - b)x = ab.$$

$$x^2 + (a - b)x + \left(\frac{a - b}{2}\right)^2 = ab + \left(\frac{a - b}{2}\right)^2 =$$

$$\frac{4ab + a^2 - 2ab + b^2}{4} = \frac{a^2 + 2ab + b^2}{4}.$$

Extracting square root,

$$x + \frac{a-b}{2} = \pm \frac{a+b}{2}.$$

$$x = -\frac{a-b}{2} + \frac{a+b}{2} = b,$$

or $$x = -\frac{a-b}{2} - \frac{a+b}{2} = -a.$$

Therefore, $x = b$, or $-a$. Ans.

(232) Let x = rate of current.
y = rate of rowing.

Down stream, the rowers are aided by the current, so $x + y = 12$.

Since it takes them twice as long to row a given distance *up* stream as it does down stream, they will go only $\frac{1}{2}$ as far in 1 hour, or $\frac{1}{2}$ of $12 = 6$ miles per hour up stream.

$$x + y = 12. \quad (1)$$
$$-x + y = 6. \quad (2)$$

Subtracting, $2x = 6$, and $x = 3$ miles per hour. Ans.

(233) (*a*) $\frac{10x+3}{3} - \frac{6x-7}{2} = 10(x-1)$.

Reducing the last member to a simpler form, the equation becomes

$$\frac{10x+3}{3} - \frac{6x-7}{2} = 10x - 10.$$

Clearing of fractions by multiplying each term of both members by 6, the L. C. M. of the denominators, and changing the sign of each term of the numerator of the second fraction, since it is preceded by the minus sign (Art. **567**), we have

$$20x + 6 - 18x + 21 = 60x - 60.$$

Transposing terms, $20x - 18x - 60x = -60 - 21 - 6$.

Combining like terms, $-58x = -87$.

Changing signs, $58x = 87$;

whence, $x = \frac{87}{58} = 1\frac{1}{2}$. Ans.

(*b*) $(a^2 + x)^2 = x^2 = 4a^2 + a^4.$

Performing the operation indicated in the first member, the equation becomes

$$a^4 + 2a^2x + x^2 = x^2 + 4a^2 + a^4.$$

Canceling x^2 (Art. **562**) and transposing,

$$2a^2x = 4a^2 + a^4 - a^4.$$

Combining like terms, $2a^2x = 4a^2.$

Dividing by $2a^2$, $x = 2.$ Ans.

(*c*) $$\frac{x-1}{x-2} - \frac{x+1}{x+2} = \frac{3}{x^2-4}.$$

Clearing of fractions, the equation becomes

$$(x-1)(x+2) - (x+1)(x-2) = 3.$$

Expanding, $x^2 + x - 2 - x^2 + x + 2 = 3.$

Uniting terms, $2x = 3.$

$$x = \frac{3}{2} = 1\frac{1}{2}.$$ Ans.

(234) $$11x + 3y = 100. \quad (1)$$
$$4x - 7y = 4. \quad (2)$$

Since the signs of the terms containing x in each equation are alike, x may be eliminated by subtraction. If the first equation be multiplied by 4, and the second by 11, the coefficients in each case will become equal. Hence,

Multiplying (1) by 4, $44x + 12y = 400.$ (3)

Multiplying (2) by 11, $44x - 77y = 44.$ (4)

Subtracting (4) from (3), $89y = 356.$

$y = 4.$ Ans.

Substituting this value for y in (2),

$4x - 28 = 4.$

$4x = 32.$

$x = 8.$ Ans.

(235) (*a*) $y^{\frac{1}{5}} = 243.$

Extracting fifth root of both terms,

$y^{\frac{1}{5}} = 3.$

Cubing both terms, $y = 3^5 = 27.$ Ans.

(*b*) $x^{10} + 31x^5 - 10 = 22,$

or $x^{10} + 31x^5 = 32.$

Completing the square,

$$x^{10} + 31x^5 + \left(\frac{31}{2}\right)^2 = 32 + \left(\frac{31}{2}\right)^2.$$

$$x^{10} + 31x^5 + \left(\frac{31}{2}\right)^2 = 32 + \frac{961}{4} = \frac{1,089}{4}$$

Extracting square root, $x^5 + \frac{31}{2} = \pm \frac{33}{2}.$

Transposing, $x^5 = -\frac{31}{2} + \frac{33}{2} = \frac{2}{2}$, or **1.**

$$x^5 = -\frac{31}{2} - \frac{33}{2} = -\frac{64}{2} = -32;$$

whence, $x = \sqrt[5]{1} = 1,$

or $x = \sqrt[5]{-32} = -2.$ Ans. (Art. **600.**)

(*c*) $x^3 - 4x^{\frac{3}{2}} = 96.$

Completing the square,

$$x^3 - 4x^{\frac{3}{2}} + 4 = 96 + 4 = 100.$$

Extracting square root, $x^{\frac{3}{2}} - 2 = \pm 10.$

Transposing and combining, $x^{\frac{3}{2}} = 12,$ or $-8.$

But, $x^{\frac{3}{2}} = \sqrt{x^3} = 12,$ or $-8.$

Removing the radical, $x^3 = 144,$ or $(-8)^2.$

$$\sqrt[3]{144} = \sqrt[3]{8} \times \sqrt[3]{18} = 2\sqrt[3]{18}.$$ (Art. **538.**)

$$\sqrt[3]{(-8)^2} = (-8)^{\frac{2}{3}}.$$

Hence, $x = 2\sqrt[3]{18},$ or $(-8)^{\frac{2}{3}}.$ Ans.

(236) (*a*) The value of a^0 is the same as 1. (Arts. **438** and **439.**)

(*b*) $\frac{a^0}{a^{-1}} = a.$ Ans. (Art. **530.**)

(*c*) $\sqrt[7]{(3x^2 + 5xy^3 + 6x^2y)^7} = 3x^2 + 5xy^3 + 6x^2y = 3 \times 2^2 + 5 \times 2 \times 4^3 + 6 \times 2^2 \times 4 = 12 + 640 + 96 = 748,$ when $x = 2,$ and $y = 4.$ Ans.

(237) (*a*) $\dfrac{6x+1}{15} - \dfrac{2x-4}{7x-16} = \dfrac{2x-1}{5}.$

Clearing of fractions,

$$(6x+1)(7x-16) - 15(2x-4) = 3(2x-1)(7x-16)$$

Removing parentheses and expanding,

$$42x^2 - 89x - 16 - 30x + 60 = 42x^2 - 117x + 48.$$

Canceling $42x^2$ (Art. **562**) and transposing,

$$117x - 89x - 30x = 16 - 60 + 48.$$

Combining terms, $$-2x = 4.$$

$$x = -2. \quad \text{Ans.}$$

(*b*) $$\frac{ax^2}{c-bx} + a + \frac{ax}{b} = 0.$$

$$abx^2 + abc - ab^2x + acx - abx^2 = 0.$$

Transposing and uniting, $acx - ab^2x = -abc.$

$$a(c-b^2)x = -abc.$$

$$x = -\frac{abc}{a(c-b^2)}.$$

Canceling the common factor a and changing two of the signs of the fraction (Art. **482**),

$$x = \frac{bc}{b^2-c}. \quad \text{Ans.}$$

(*c*) $$\frac{\sqrt{x}-3}{\sqrt{x}+7} = \frac{\sqrt{x}-4}{\sqrt{x}+1}.$$

Clearing of fractions,

$$(\sqrt{x}-3)(\sqrt{x}+1) = (\sqrt{x}+7)(\sqrt{x}-4)$$

$$x - 2\sqrt{x} - 3 = x + 3\sqrt{x} - 28.$$

Transposing and canceling x (Art. **562**),

$$-2\sqrt{x} - 3\sqrt{x} = 3 - 28.$$

$$-5\sqrt{x} = -25.$$

$$\sqrt{x} = 5.$$

$$x = 25. \quad \text{Ans.}$$

(238) (*a*) $\sqrt{\frac{3}{2}}$ by Art. **540** $= \sqrt{\frac{3}{2} \times \frac{2}{2}} = \sqrt{\frac{6}{4}} = \frac{1}{2}\sqrt{6}$.

Ans.

(*b*) $\frac{3}{11}\sqrt{\frac{4}{7}} = \frac{3}{11}\sqrt{\frac{4}{7} \times \frac{7}{7}} = \frac{3}{11} \times \frac{2}{7}\sqrt{7} = \frac{6}{77}\sqrt{7}$. Ans.

(*c*) $z\sqrt[3]{\frac{2x}{z}} = z\sqrt[3]{\frac{2x}{z} \times \frac{z^2}{z^2}} = \frac{z}{z}\sqrt[3]{2xz^2} = \sqrt[3]{2xz^2}$. Ans.

(239) (*a*) $\frac{9x + 20}{36} = \frac{4(x - 3)}{5x - 4} + \frac{x}{4} = ?$

When the denominators contain both simple and compound expressions, it is best to remove the simple expressions first, and then remove each compound expression in order. Then, after each multiplication, the result should be reduced to the simplest form.

Multiplying both sides by 36,

$$9x + 20 = \frac{144(x - 3)}{5x - 4} + 9x,$$

or $$\frac{144x - 432}{5x - 4} = 20.$$

Clearing of fractions,

$$144x - 432 = 100x - 80.$$

Transposing and combining,

$$44x = 352;$$

whence, $x = 8$. Ans.

(*b*) $ax - \frac{3a - bx}{2} = \frac{1}{2}$ becomes, when cleared of fractions,

$$2ax - 3a + bx = 1.$$

Transposing and uniting terms,

$$2ax + bx = 3a + 1.$$

Factoring, $$(2a + b)x = 3a + 1;$$

whence, $$x = \frac{3a + 1}{2a + b}.$$ Ans.

(*c*) $am - b - \frac{ax}{b} + \frac{x}{m} = 0$, when cleared of fractions =

$$abm^2 - b^2m - amx + bx = 0.$$

Transposing, $bx - amx = b^2m - abm^2.$

Factoring, $(b - am)x = bm(b - am);$

whence, $x = \frac{bm(b - am)}{(b - am)} = bm.$ Ans

(240) $x + y = 13.$ (1)

$xy = 36.$ (2)

Squaring (1) we have

$$x^2 + 2xy + y^2 = 169. \quad (3)$$

Multiplying (2) by 4, $4xy = 144.$ (4)

Subtracting (4) from (3),

$$x^2 - 2xy + y^2 = 25. \quad (5)$$

Extracting the square root of (5),

$$x - y = \pm 5. \quad (6)$$

Adding (6) and (1), $2x = 18$ or $8,$

$x = 9$ or $4.$ Ans.

Substituting the value of x in (1),

$9 + y = 13,$

or $4 + y = 13;$

whence, $y = 4,$ } Ans.

or $y = 9.$ }

(241) $x^3 - y^3 = 98.$ (1)

$x - y = 2.$ (2)

From (2), $x = 2 + y.$ (3)

Substituting the value of x in (1),

$$8 + 12y + 6y^2 + y^3 - y^3 = 98.$$

Combining and transposing,

$$6y^2 + 12y = 90.$$

$$y^2 + 2y = 15.$$

$$y^2 + 2y + 1 = 15 + 1 = 16.$$

$$y + 1 = \pm 4.$$

$y = 3,$ or $-5.$ Ans.

Substituting the value of y in (3), $x = 5,$ or $-3.$ Ans.

(242) Let $x =$ the whole quantity.

Then, $\frac{2x}{3} + 10 =$ the quantity of niter.

$\frac{x}{6} - 4\frac{1}{2} =$ the quantity of sulphur.

$\frac{1}{7}\left(\frac{2x}{3} + 10\right) - 2 =$ the quantity of charcoal.

Hence, $$x = \frac{2x}{3} + 10 + \frac{x}{6} - 4\frac{1}{2} + \frac{1}{7}\left(\frac{2x}{3} + 10\right) - 2.$$

Clearing of fractions and expanding terms,

$$42x = 28x + 420 + 7x - 189 + 4x + 60 - 84.$$

Transposing,

$$42x - 28x - 7x - 4x = 420 - 189 + 60 - 84.$$

$$3x = 207.$$

$$x = 69 \text{ lb.} \quad \text{Ans.}$$

(243) Let $x =$ number of revolutions of hind wheel.

Then, $51 + x =$ number of revolutions of fore wheel.

Since, in making these revolutions both wheels traveled the same distance, we have

$$16x = 14(51 + x).$$

$$16x = 714 + 14x.$$

$$2x = 714.$$

$$x = 357.$$

Since the hind wheel made 357 revolutions, and since the distance traveled for each revolution is equal to the circumference of the wheel, or 16 feet, the whole distance traveled $= 357 \times 16$ ft. $= 5{,}712$ feet. Ans.

(244) (*a*) Transposing,

$$5x^3 - 2x^3 = 24 + 9.$$

Uniting terms, $$3x^3 = 33.$$

$$x^3 = 11.$$

Extracting the square root of both members,

$$x = \pm\sqrt{11}. \quad \text{Ans.}$$

(*b*) $$\frac{3}{4x^2} - \frac{1}{6x^2} = \frac{7}{3}.$$

Clearing of fractions, $9 - 2 = 28x^2$.

Transposing terms, $28x^2 = 7$.

$$x^2 = \frac{1}{4}.$$

Extracting the square root of both members,

$$x = \pm\frac{1}{2}. \quad \text{Ans.}$$

(*c*) $$\frac{x^2}{5} - \frac{x^2 - 10}{15} = 7 - \frac{50 + x^2}{25}.$$

Clearing of fractions by multiplying each term of both members by 75, the L. C. M. of the denominators, and expanding,

$$15x^2 - 5x^2 + 50 = 525 - 150 - 3x^2.$$

Transposing and uniting terms,

$$13x^2 = 325.$$

Dividing by 13, $$x^2 = \frac{325}{13} = 25,$$

or $$x = \pm 5. \quad \text{Ans.}$$

(245) $$4x + 3y = 48. \qquad (1)$$

$$-3x + 5y = 22. \qquad (2)$$

From (1), $$y = \frac{48 - 4x}{3}. \qquad (3)$$

From (2), $$y = \frac{22 + 3x}{5}. \qquad (4)$$

Placing (3) and (4) equal to each other,

$$\frac{48 - 4x}{3} = \frac{22 + 3x}{5}.$$

Clearing of fractions,

$$240 - 20x = 66 + 9x.$$

Transposing and uniting terms,

$$-29x = -174,$$

or $$x = 6. \quad \text{Ans.}$$

Substituting this value in (4),

$$y = \frac{22 + 18}{5} = 8. \quad \text{Ans.}$$

(246) Let x = speed of one.

$x + 10$ = speed of other.

Then, $\frac{1,200}{x}$ = number of hours one train required.

$\frac{1,200}{x+10}$ = number of hours other train required.

$$\frac{1,200}{x} = \frac{1,200}{x+10} + 10.$$

$$1,200x + 12,000 = 1,200x + 10x^2 + 100x.$$

$$-10x^2 - 100x = -12,000.$$

$$10x^2 + 100x = 12,000.$$

$$x^2 + 10x = 1,200.$$

$$x^2 + 10x + 25 = 1,200 + 25 = 1,225.$$

$$x + 5 = \pm 35.$$

$x = 30$ miles per hour. }
$x + 10 = 40$ miles per hour. } Ans.

(247) $2x - \frac{y-3}{5} - 4 = 0$

$3y + \frac{x-2}{3} - 9 = 0$ } cleared of fractions, becomes

$$10x - y + 3 - 20 = 0. \quad (1)$$

$$9y + x - 2 - 27 = 0. \quad (2)$$

Transposing and uniting, $10x - y = 17$. (3)

$x + 9y = 29$. (4)

Multiplying (4) by 10 and subtracting (3) from the result,

$$10x + 90y = 290$$
$$10x - y = 17$$
$$91y = 273$$
$$y = 3. \quad \text{Ans.}$$

Substituting value of y in (4),

$$x + 27 = 29.$$
$$x = 2. \quad \text{Ans.}$$

(248) (*a*) $\sqrt[3]{2} \times \sqrt[4]{3} = 2^{\frac{1}{3}} \times 3^{\frac{1}{4}}$. (Art. **547.**)

$2^{\frac{4}{12}} \times 3^{\frac{3}{12}} = \sqrt[12]{2^4} \times \sqrt[12]{3^3} = \sqrt[12]{32 \times 27} = \sqrt[12]{864}$. Ans.

(*b*) $\sqrt[4]{2ax} \times \sqrt[3]{ax^2} = (2ax)^{\frac{1}{4}} \times (ax^2)^{\frac{1}{3}} = \sqrt[12]{8a^3x^3} \times \sqrt[12]{a^4x^8} = \sqrt[12]{8a^7x^{11}}$. Ans.

(*c*) $2\sqrt{xy} \times 3\sqrt[5]{x^3y} = 2 \times 3(xy)^{\frac{1}{2}} \times (x^3y)^{\frac{1}{5}} = 6\sqrt[10]{x^{11}y^7}$. Ans.

(249) Let $x =$ the part of the work which they all can do in 1 day when working together.

Then, since $\frac{1}{7\frac{1}{2}} = \frac{2}{15}$, $\frac{1}{5} + \frac{1}{6} + \frac{2}{15} = x$;

or, clearing of fractions and adding,

$$15 = 30x, \text{ and } x = \frac{1}{2}.$$

Since they can do $\frac{1}{2}$ the work in 1 day, they can do all of the work in 2 days. Ans.

(250) Let $x =$ value of first horse.
$y =$ value of second horse.

If the saddle be put on the first horse, its value will be $x + 10$. This value is double that of the second horse, or $2y$, whence the equation, $x + 10 = 2y$.

If the saddle be put on the second horse, its value is $y + 10$. This value is \$13 less than the first, or $x - 13$, whence the equation, $y + 10 = x - 13$.

$$x + 10 = 2y. \quad (1)$$
$$y + 10 = x - 13. \quad (2)$$

Transposing, $x - 2y = -10$. (3)
$-x + y = -23$. (4)

Adding (3) and (4), $-y = -33$.

$y =$ \$33, or value of second horse. Ans.

Substituting in (1), $x + 10 = 66$;

or $x =$ \$56, or value of first horse. Ans.

(251) Let $x =$ A's money.
$y =$ B's money.

If A should give B \$5, A would have $x - 5$, and B, $y + 5$. B would then have \$6 more than A, whence the equation,

$$y + 5 - (x - 5) = 6. \qquad (1)$$

But if A received \$5 from B, A would have $x + 5$, and B, $y - 5$, and 3 times his money, or 3 $(x + 5)$, would be \$20 more than 4 times B's, or 4 $(y - 5)$, whence the equation,

$$3(x + 5) - 4(y - 5) = 20. \qquad (2)$$

Expanding equations (1) and (2),

$$y + 5 - x + 5 = 6. \qquad (3)$$

$$3x + 15 - 4y + 20 = 20. \qquad (4)$$

Transposing and combining,

$$y - x = -4. \qquad (5)$$

$$-4y + 3x = -15. \qquad (6)$$

Multiplying (5) by 4, and adding to (6),

$$\begin{aligned} 4y - 4x &= -16. \\ -4y + 3x &= -15. \\ \hline -x &= -31. \\ x &= 31. \end{aligned}$$

Substituting value of x in (5),

$$y - 31 = -4.$$

$$y = 27.$$

Hence, $x = \$31$, A's money. $y = \$27$, B's money. } Ans.

(252) (*a*) $x^2 - 6x = 16.$

Completing the square (Art. **597**),

$$x^2 - 6x + 9 = 16 + 9.$$

Extracting the square root, $x - 3 = \pm 5$.

Transposing, $x = 8$, or -2. Ans.

(*b*) $x^2 - 7x = 8.$

$$x^2 - 7x + \left(\frac{7}{2}\right)^2 = 8 + \left(\frac{7}{2}\right)^2 = \frac{81}{4}.$$

$$x - \frac{7}{2} = \pm \frac{9}{2};$$

whence, $x = 8$, or -1. Ans.

(*c*) $9x^2 - 12x = 21.$

Dividing by 9, $x^2 - \frac{12x}{9} = \frac{21}{9},$

or $x^2 - \frac{4x}{3} = \frac{7}{3}.$

Completing the square,

$$x^2 - \frac{4x}{3} + \left(\frac{2}{3}\right)^2 = \frac{7}{3} + \frac{4}{9} = \frac{25}{9}.$$

Extracting square root, $x - \frac{2}{3} = \pm\frac{5}{3}.$

Transposing, $x = \frac{2}{3} + \frac{5}{3} = \frac{7}{3},$

or $x = \frac{2}{3} - \frac{5}{3} = -\frac{3}{3} = -1.$

Therefore, $x = \frac{7}{3}$, or -1. Ans.

(253) $(c^{-\frac{1}{2}})^{-\frac{1}{3}} = c^{\frac{1}{6}}$. Ans. (Art. **526,** III.)

$(m\sqrt{n^3})^{-\frac{1}{2}} = m^{-\frac{1}{2}}(n^{\frac{3}{2}})^{-\frac{1}{2}} = m^{-\frac{1}{2}}n^{-\frac{3}{4}} = \frac{1}{m^{\frac{1}{2}}n^{\frac{3}{4}}}$. Ans.

$(cd^{-2})^{\frac{1}{a}} = c^{\frac{1}{a}}d^{-\frac{2}{a}}$, or $\sqrt[a]{cd^{-2}}$, or $\sqrt[a]{\frac{c}{d^2}}$. Ans. (Art. **530.**)

(254)

Let x = number of quarts of 90-cent wine in the mixture.

y = number of quarts of 50-cent wine in the mixture

Then, $x + y = 60,$ (1)

and $90x + 50y = 4{,}500 = 75 \times 60.$ (2)

Multiplying (1) by 50,

$50x + 50y = 3{,}000.$ (3)

Subtracting (3) from (2),

$40x = 1{,}500;$

whence, $x = 37\frac{1}{2}$ qt. Ans.

Multiplying (1) by 90, $90x + 90y = 5{,}400$ (4)

Subtracting (2), $90x + 50y = 4{,}500$ (2)

$40y = 900;$

whence, $y = 22\frac{1}{2}$ qt. Ans.

(255) Let x = the numerator of the fraction.
y = the denominator of the fraction.

Then, $\frac{x}{y}$ = the fraction.

From the conditions, $$\frac{2x}{y+7} = \frac{2}{3}, \qquad (1)$$

and $$\frac{x+2}{2y} = \frac{3}{5}. \qquad (2)$$

Clearing (1) and (2) of fractions, and transposing,

$$6x = 2y + 14, \quad (3)$$

and $$5x = 6y - 10. \quad (4)$$

Solving for x, $$x = \frac{2y+14}{6} = \frac{y+7}{3}. \quad (5)$$

$$x = \frac{6y-10}{5}. \quad (6)$$

Equating (5) and (6), $$\frac{y+7}{3} = \frac{6y-10}{5}.$$

Clearing of fractions, $$5y + 35 = 18y - 30$$

whence, $$13y = 65,$$

or, $$y = 5.$$

Substituting this value of y in (3),

$$6x = 10 + 14 = 24;$$

whence, $x = 4$.

Therefore, the fraction is $\frac{4}{5}$. Ans.

(256) Let x = digit in tens place.
y = digit in units place.

Then $10x + y$ = the number.

From the conditions of the example,

$$10x + y = 4(x+y) = 4x + 4y;$$

whence, $$3y = 6x,$$

or $$y = 2x.$$

From the conditions of the example,

$$10x + y + 18 = 10y + x;$$

whence, $$9y - 9x = 18.$$

Substituting the value of y, found above,

$$18x - 9x = 18;$$

whence, $x = 2.$

$$y = 2x = 4.$$

Hence, the number $= 10x + y = 20 + 4 = 24.$ **Ans.**

(257) Let $x =$ greater number.

$y =$ less number.

Then, $x + 4 = 3\frac{1}{4}y,$ (1)

and $y + 8 = \frac{x}{2}.$ (2)

Clearing of fractions, $4x + 16 = 13y,$

and $2y + 16 = x;$

whence, $13y - 4x = 16.$ (3)

$2y - x = -16.$ (4)

Multiplying (4) by 4, and subtracting from (3)

$$5y = 80,$$

or $y = 16.$ Ans.

Substituting in (4), $32 - x = -16;$

whence, $x = 48.$ **Ans.**

LOGARITHMS.

(QUESTIONS 258–272.)

(258) First raise $\frac{200}{100}$ to the .29078 power. Since $\frac{200}{100} = 2$, $\left(\frac{200}{100}\right)^{.29078} = 2^{.29078}$, and $\log 2^{.29078} = .29078 \times \log 2 = .29078 \times .30103 = .08753$. Number corresponding $= 1.2233$. Then,

$$1 - \left(\frac{200}{100}\right)^{.29078} = 1 - 1.2233 = -.2233.$$

We now find the product required by adding the logarithms of 351.36, 100, 24, and .2233, paying no attention to the negative sign of .2233 until the product is found. (Art. **647.**)

$$\begin{aligned} \text{Log } 351.36 &= 2.54575 \\ \log \quad 100 &= 2 \\ \log \quad 24 &= 1.38021 \\ \log \quad .2233 &= \bar{1}.34889 \\ \textit{sum} &= 5.27485 = \end{aligned}$$

$$\log 351.36 \times 100 \times 24 \left(1 - \left(\frac{200}{100}\right)^{.29078}\right)$$

Number corresponding $= 188{,}300$.

The number is negative, since multiplying positive and negative signs gives negative; and the sign of .2233 is minus. Hence,

$$x = -188{,}300. \quad \text{Ans.}$$

(259) (*a*) Log 2,376 = 3.37585. Ans. (See Arts. **625** and **627.**)

(*b*) Log .6413 = $\bar{1}$.80706. Ans.

(*c*) Log .0002507 = $\bar{4}$.39915. Ans.

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(260) (*a*) Apply rule, Art. **652.**

Log 755.4 = 2.87818
log .00324 = $\bar{3}$.51055
difference = 5.36763 = logarithm of quotient.

The mantissa is not found in the table. The next less mantissa is 36754. The difference between this and the next greater mantissa is 773 − 754 = 19, and the P. P. is 763 − 754 = 9. Looking in the P. P. section for the column headed 19, we find opposite 9.5, 5, the fifth figure of the number. The fourth figure is 1, and the first three figures 233; hence, the figures of the number are 23315. Since the characteristic is 5, 755.4 ÷ .00324 = 233,150. Ans.

(*b*) Apply rule, Art. **652.**

Log .05555 = $\bar{2}$.74468
log .0008601 = $\bar{4}$.93455
difference = 1.81013 = logarithm of quotient.

The number whose logarithm is 1.81013 equals 64.584.

Hence, .05555 ÷ .0008601 = 64.584. Ans.

(*c*) Apply rule, Art. **652.**

Log 4.62 = .66464
log .6448 = $\bar{1}$.80943
difference = .85521 = logarithm of quotient.

Number whose logarithm = .85521 = 7.1648.

Hence, 4.62 ÷ .6448 = 7.1648. Ans.

(261) $x^{.74} = \dfrac{238 \times 1{,}000}{.0042^{.6602}}$.

Log 238 = 2.37658
log 1,000 = 3.
sum = 5.37658 = log (238 × 1,000)

Log .0042 = $\bar{3}$.62325
.6602
124650
373950
373950
.411469650 or .41147.

$$.6602$$
$$-3$$
$$-1.9806 = \text{characteristic.}$$

Adding, $.41147$
-1.9806
$\overline{2}.43087$ (See Art. **659.**)

Then, $\log\left(\frac{238 \times 1{,}000}{.0042^{.6602}}\right) = 5.37658 - \overline{2}.43087 = 6\ 94571 =$ 74 log x; whence, $\log x = \frac{6.94571}{.74} = 9.38609$. Number whose logarithm = 9.38609 is 2,432,700,000 = x. Ans.

(262) Log .00743 = $\overline{3}$.87099.
log .006 = $\overline{3}$.77815.

$\sqrt[5]{.00743} = \log .00743 \div 5$ (Art. **662**), and $\sqrt[6]{.006} = \log .006 \div .6$. Since these numbers are wholly decimal, we apply Art. **663.**

$$5\,)\,\overline{3}.87099$$
$$\overline{1}.57419 = \log \sqrt[5]{.00743}.$$

The characteristic $\overline{3}$ will not contain 5. We then add $\overline{2}$ to it, making $\overline{5}$. 5 is contained in $\overline{5}$, $\overline{1}$ times. Hence, the characteristic is $\overline{1}$. Adding the same number, 2, to the mantissa, we have 2.87099. $2.87099 \div 5 = .57419$. Hence, $\log \sqrt[5]{.00743} = \overline{1}.57419$.

.6) $\overline{3}.77815$ .6 is contained in $\overline{3}$, -5 times.
$\overline{5}.$ .6 is contained in .77815, 1.29691 times.
1.29691
sum = $\overline{4}.29691 = \log \sqrt[6]{.006}$.

Log $\sqrt[5]{.00743} = \overline{1}.57419$
log $\sqrt[6]{.006} = \overline{4}.29691$
difference = $\overline{3}.27728$ = log of quotient.

Number corresponding = 1,893.6.

Hence, $\sqrt[5]{.00743} \div \sqrt[6]{.006} = 1{,}893.6$. Ans.

(263) Apply rule, Art. **647.**

$$\begin{aligned}
\text{Log } 1{,}728 &= 3.23754\\
\log .00024 &= \overline{4}.38021\\
\log .7462 &= \overline{1}.87286\\
\log 302.1 &= 2.48015\\
\log 7.6094 &= .88135\\
\textit{sum} &= 2.85211 =
\end{aligned}$$

$\log (1{,}728 \times .00024 \times .7462 \times 302.1 \times 7.6094)$. Number whose logarithm is $2.85211 = 711.40$, the product. Ans.

(264)

$$\begin{aligned}
\text{Log } \sqrt{5.954} &= .77481 \div 2 = .38741\\
\log \sqrt[3]{61.19} &= 1.78668 \div 3 = .59556\\
\textit{sum} &= .98297
\end{aligned}$$

$$\log \sqrt[5]{298.54} = 2.47500 \div 5 = .49500.$$

Then, $\dfrac{\sqrt{5.954} \times \sqrt[3]{61.19}}{\sqrt[5]{298.54}} = \log (\sqrt{5.954} \times \sqrt[3]{61.19}) - \log \sqrt[5]{298.54} = .98297 - .49500 = .48797 =$ logarithm of the required result.

Number corresponding $= 3.0759$. Ans.

(265) $\sqrt[7]{.0532864} = \log .0532864 \div 7.$

Log $.0532864 = \overline{2}.72661.$

Adding $\overline{5}$ to characteristic $\overline{2} = \overline{7}.$

Adding 5 to mantissa $= 5.72661.$

$\overline{7} \div 7 = \overline{1}.$

$5.72661 \div 7 = .81809$, nearly.

Hence, $\log \sqrt[7]{.0532864} = \overline{1}.81809.$

Number corresponding to $\log \overline{1}.81809 = .65780.$ Ans.

(266) (*a*) $32^{4.8}$.

Log $32 = 1.50515.$

$$\begin{array}{r}
1.50515\\
4.8\\
\hline
120412\,0\\
60206\,0\\
\hline
7.22472\,0
\end{array}$$

7.22472 is the logarithm of the required power. (Art. **657.**)

Number whose logarithm $= 7.22472$ is 16,777,000.

Hence, $32^{4.8} = 16{,}777{,}000.$ Ans.

(*b*) $.76^{3.62}$.

Log .76 = $\bar{1}$.88081.

(See Arts. **658** and **659.**)

$$\begin{array}{r} \bar{1}+.88081 \\ \quad 3.62 \\ \hline 176162 \\ 528486 \\ 264243 \\ \hline 3.1885322 \\ -3.62 \\ \hline \bar{1}.56853 = \log .37028. \end{array}$$

Hence, $.76^{3.62} = .37028$. Ans.

(*c*) $.84^{.38}$.

Log .84 = $\bar{1}$.92428.

$$\begin{array}{r} \bar{1}+.92428 \\ \quad .38 \\ \hline 739424 \\ 277284 \\ \hline .3512264 \\ -.38 \\ \hline \bar{1}.97123 = \log .93590. \end{array}$$

Hence, $.84^{.38} = .93590$. Ans.

(267) Log $\sqrt[6]{\frac{1}{249}}$ − log $\sqrt[5]{\frac{23}{71}}$ = logarithm of answer.

Log $\sqrt[6]{\frac{1}{249}} = \frac{1}{6}(\log 1 - \log 249) = \frac{1}{6}(0 - 2.39620) = -.39937$ = (adding + 1 and − 1)$\bar{1}$.60063.

Log $\sqrt[5]{\frac{23}{71}} = \frac{1}{5}(\log 23 - \log 71) = \frac{1}{5}(1.36173 - 1.85126) =$ $\frac{1}{5}(-.48953) = -.097906$ = (adding + 1 and − 1) $\bar{1}$.902094, or $\bar{1}$.90209 when using 5-place logarithms.

Hence, $\bar{1}.60063 - \bar{1}.90209 = \bar{1}.69854 = \log .49950$. Therefore,

$$\sqrt[6]{\frac{1}{249}} \div \sqrt[5]{\frac{23}{71}} = .49950. \quad \text{Ans.}$$

(268) The mantissa is not found in the table. The next less mantissa is .81291; the difference between this and the

next greater mantissa is $298 - 291 = 7$, and the P. P. is $.81293 - .81291 = 2$. Looking in the P. P. section for the column headed 7, we find opposite 2.1, 3, the fifth figure of the number; the fourth figure is 0, and the first three figures, 650. Hence, the number whose logarithm is .81293 is 6.5003. Ans.

2.52460 = logarithm of 334.65. Ans. (See Art. **640.**)

$\bar{1}.27631$ = logarithm of .18893. Ans. We choose 3 for the fifth figure because, in the proportional parts column headed 23, 6.9 is nearer 8 than 9.2.

(269) The most expeditious way of solving this example is the following:

$$p\,v^{1.41} = p_1 v_1^{1.41}, \text{ or } v_1 = \sqrt[1.41]{\frac{p\,v^{1.41}}{p_1}} = v\sqrt[1.41]{\frac{p}{p_1}}.$$

Substituting values given, $v_1 = 1.495\sqrt[1.41]{\frac{134.7}{16.421}}$.

$$\text{Log } v_1 = \log 1.495 + \frac{\log 134.7 - \log 16.421}{1.41} = .17464 + \frac{2.12937 - 1.21540}{1.41} = .17464 + .64821 = .82285 = \log 6.6504;$$

whence, $v_1 = 6.6504$. Ans.

(270) $\text{Log}\sqrt[5]{\frac{7.1895 \times 4{,}764.2^2 \times 0.00326^5}{.000489 \times 457^3 \times .576^2}} = \frac{1}{5}[\log 7.1895 + 2 \log 4{,}764.2 + 5 \log .00326 - (\log .000489 + 3 \log 457 + 2 \log .576)] = \frac{\bar{5}.77878 - 4.18991}{5} = \bar{2}.31777 = \log .020786$. Ans.

$$\begin{aligned} \text{Log } 7.1895 &= .85670 \\ 2 \log 4{,}764.2 = 2 \times 3.67799 &= 7.35598 \\ 5 \log .00326 = 5 \times \bar{3}.51322 &= \bar{13}.56610 \\ \textit{sum} &= \bar{5}.77878 \end{aligned}$$

$$\begin{aligned} \text{Log } .000489 &= \bar{4}.68931 \\ 3 \log 457 = 3 \times 2.65992 &= 7.97976 \\ 2 \log .576 = 2 \times \bar{1}.76042 &= \bar{1}.52084 \\ \textit{sum} &= 4.18991. \end{aligned}$$

(271) Substituting the values given,

$$p = \frac{\overset{8,000}{\cancel{960,000}} \times \left(\frac{3}{16}\right)^{2.18}}{\cancel{120} \times 2.25} = \frac{8,000\left(\frac{3}{16}\right)^{2.18}}{2.25}$$

$\text{Log } p = \log 8,000 + 2.18 \log \frac{3}{16} - \log 2.25 = 3.90309 + 2.18$ (log 3 − log 16) − .35218 = 3.55091 + 2.18 × (.47712 − 1.20412) = 1.96605 = log 92.480. Ans.

(272) Solving for t, $t = \sqrt[2.18]{\frac{p\,l\,d}{960,000}}$.

Substituting values given,

$$t = \sqrt[2.18]{\frac{\overset{44}{\cancel{160} \times \cancel{132} \times \cancel{2}}}{\overset{960,000}{\cancel{960,000}}\ \overset{6,000}{\cancel{6,000}}\ \overset{3,000}{\cancel{3,000}}\ 1,000}} = \sqrt[2.18]{.044}.$$

$$\text{Log } t = \frac{\log .044}{2.18} = \frac{\bar{2}.64345}{2.18} = \frac{-2.18 + .82345}{2.18} =$$

$\bar{1}.37773 = \log .23863$. **Ans.**

GEOMETRY AND TRIGONOMETRY.

(1) When one straight line meets another straight line at a point between the ends, the sum of the two adjacent angles equals two right angles. Therefore, since one of the angles equals $\frac{4}{5}$ of a right angle, the other angle equals two right angles (or $\frac{10}{5}$) minus $\frac{4}{5}$. We have, then, $\frac{10}{5} - \frac{4}{5} = \frac{6}{5}$, or $1\frac{1}{5}$ right angles. Ans.

(2) It is an isosceles triangle, since the sides opposite the equal angles are equal.

(3) A regular decagon has 10 equal sides; therefore, the length of one side is $40 \div 10 = 4$ in. Ans.

(4) The sum of all the interior angles of any polygon equals two right angles, multiplied by the number of sides in the polygon less two. As a regular dodecagon has 12 equal sides, the sum of the interior angles equals two right angles $\times$ 10 ($= 12 - 2$), or 20 right angles. Since there are 12 equal angles, the size of any one of them equals $20 \div 12$, or $1\frac{2}{3}$ right angles. Ans.

(5) Equilateral triangle.

(6) Since the two angles A and C, Fig. I, are equal, the triangle is isosceles, and a line drawn from the vertex B will bisect the line $A\,C$, the length of which is 7 inches; therefore,

$$A\,D = D\,C = 7 \div 2 = 3\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

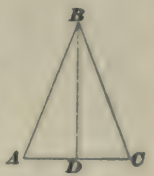

FIG. I.

(7) The length of the line $= \sqrt{12^2 - 9^2} + \sqrt{15^2 - 9^2}$, or 19.94 in. Ans.

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(8) One of the angles of an equiangular octagon is equal to $\frac{1}{8}$ of 12 right angles, or $1\frac{1}{2}$ right angles, since the sum of the interior angles of the equiangular octagon equals 12 right angles.

(9) See Art. **71.**

(10) In Fig. II, $AB = 4$ inches, and $OA = 6$ inches. We first find the length of OD. $OD = \sqrt{\overline{OA}^2 - \overline{DA}^2}$; but

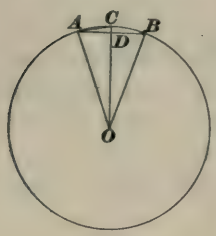

FIG. II.

$\overline{OA}^2 = 6^2$, or 36, and $\overline{DA}^2 = \left(\frac{4}{2}\right)^2$, or 4; therefore, $OD = \sqrt{36 - 4}$, or 5.657.

$DC = OC - OD$, or $DC = 6 - 5.657$, or .343 inch. In the right triangle ADC, we have AC, which is the chord of half the arc ACB, equals $\sqrt{2^2 + .343^2}$, or 2.03 in. Ans.

(11) Given, $OC = 5\frac{3}{4}$ inches and $OA = 17 \div 2 = 8\frac{1}{2}$ inches, to find AB (see Fig. III). AC, which is one-half the chord AB, equals $\sqrt{\overline{OA}^2 - \overline{OC}^2}$; therefore,

$AC = \sqrt{(8\frac{1}{2})^2 - (5\frac{3}{4})^2}$, or 6.26 inches.

Now, $AB = 2 \times AC$; therefore, $AB = 2 \times .6.26$, or 12.52 in. Ans.

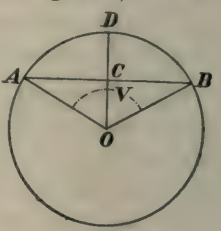

FIG. III.

(12) The arc intercepted equals $\frac{3}{4}$ of 4, or 3 quadrants. As the inscribed angle is measured by one-half the intercepted arc, we have $3 \div 2 = 1\frac{1}{2}$ quadrants as the size of the angle.

(13) Four right angles $\div \frac{2}{7} = 4 \times \frac{7}{2}$, or 14 equal sectors.

(14) Since 24 inches equals the perimeter, we have $24 \div 8 = 3$ inches, as the length of each side or chord.

Then, $\sqrt{\left(\frac{3}{2}\right)^2 + 3.62^2} = 3.92$ in., radius.

$3.92 \times 2 = 7.84$ in., diameter. Ans.

(15) In 19° 19′ 19″ there are 69,559 seconds, and in 360°, or a circle, there are 1,296,000 seconds. Therefore,

69,559 seconds equal $\frac{69,559}{1,296,000}$, or .053672 part of a circle. Ans.

(16) Referring to Fig. 75 of the text and using the values given in the example, we have $AB = 26$ feet 7 inches, or 26.583 feet; $AC = 40$ feet; and the included angle $A = 36° 20' 43''$. Then in the right triangle ADB, AB is known, and also the angle A.

Hence, by rule **1**, Art. **98**, $BD = 26.583 \times \sin 36° 20' 43''$ $= 26.583 \times .59265 = 15.754$ feet. By rule **3**, Art. **98**, $AD = 26.583 \times \cos 36° 20' 43'' = 26.583 \times .80546 = 21.411$. $AC - AD = 40 - 21.411 = 18.589$ feet $= DC$. In the right triangle CDB, the two sides BD and DC are known; hence, $\tan C = \frac{BD}{DC} = \frac{15.754}{18.589} = .84749$, and angle C $= 40° 16' 52''$. Ans.

Applying rule **2**, Art. **98**, $BC = \frac{BD}{\sin C} = \frac{15.754}{\sin 40° 16' 52''}$ $= \frac{15.754}{.64654} = 24.37$, or 24 ft. 4.4 in. Ans.

Angle $B = 180° - (36° 20' 43'' + 40° 16' 52'') = 180° -$ $76° 37' 36'' = 103° 22' 25''$. Ans.

(17) See Fig. 76 of the text. Solving the triangle ABC, we first find BD. By rule **1**, Art. **98**, $BD = AB \times \sin A$ $= 16\frac{5}{12} \times \sin 54° 54' 54'' = 16\frac{5}{12} \times .81830 = 13.434$ feet. $\sin BCD = \frac{BD}{BC} = \frac{13.434}{13.542} = .99202$; whence, angle BCD $= 82° 45' 30''$. Ans.

By rule **3**, $CD = BC \times \cos C = 13\frac{1}{4} \times \cos 82° 45' 30''$ $= 13\frac{1}{4} \times .12605 = 1.7069$ feet.

By rule **3**, $AD = AB \times \cos A = 16\frac{5}{12} \times \cos 54° 54' 54''$ $= 16\frac{5}{12} \times .57479 = 9.4361$ feet.

In the triangle ABC, the angle ACB is the supplement (see Art. **27**) of the angle BCD and equals $180°$ − angle BCD, or $ACB = 180° - 82° 45' 30'' = 97° 14' 30''$. Ans.

Angle $ABC = 180° -$ (angle $BAC +$ angle ACB) $= 180°$ $- (54° 54' 54'' + 97° 14' 30'') = 180° - 152° 9' 24'' = 27°$ $50' 36''$. Ans.

Side $AC = AD - CD$, or $AC = 9.4361 - 1.7069 = 7.7292$ = 7 ft. $8\frac{3}{4}$ in. Ans.

For the triangle ABC', angle $C' = BCD$ (isosceles triangle), or $C' = 82° 45' 30''$.

Angle $ABC' = 180° -$ (angle A + angle C') $= 180°$ $- (54° 54' 54'' + 82° 45' 30'') = 180° - 137° 40' 24''$ $= 42° 19' 36''$. Ans.

$AC' = AD + C'D = 9.4361 + 1.7069 = 11.143 = 11$ ft. $1\frac{3}{4}$ in. Ans.

(18) If one-third of a certain angle equals $14° 47' 10''$, then the angle must be $3 \times 14° 47' 10''$, or $44° 21' 30''$. $2\frac{1}{2} \times 44° 21' 30''$, or $110° 53' 45''$, equals one of the other two angles. The third angle equals $180° - (110° 53' 45'' + 44° 21' 30'')$, or $24° 44' 45''$.

(19) Referring to Fig. 70 of the text, let $BC = 437$ feet and $AC = 792$ feet, to find the hypotenuse AB and the angles A and B.

$AB = \sqrt{\overline{AC}^2 + \overline{BC}^2} = \sqrt{\overline{792}^2 + \overline{437}^2} = \sqrt{818{,}233} = 904$ ft. $6\frac{3}{4}$ in. Ans.

By rule **4**, Art. **98**, $\tan A = \dfrac{\text{side opposite}}{\text{side adjacent}}$, or $\tan A = \dfrac{437}{792}$ $= .55177$; therefore, $A = 28° 53' 19''$. Ans.

Angle $B = 90° - 28° 53' 19''$, or $61° 6' 41''$. Ans.

(20) See Fig. IV. Angle $B = 180° - (29° 21' + 76° 44' 18'')$ $= 180° - 106° 5' 18'' = 73° 54' 42''$.

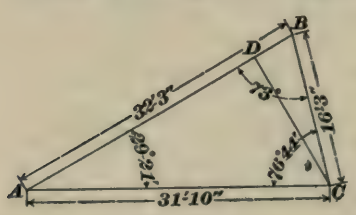

FIG. IV.

From C, draw CD perpendicular to AB.

$AD = AC \cos A = 31.833$ $\times \cos 29° 21' = 31.833 \times .87164$ $= 27.747$ feet. $CD = AC \sin A$ $= 31.833 \times \sin 29° 21' = 31.833$ $\times .49014 = 15.603$.

$$BC = \frac{CD}{\sin B} = \frac{15.603}{\sin 73° 54' 42''} = 16.24 \text{ feet} = 16 \text{ ft. } 3 \text{ in.}$$

$$DB = \frac{CD}{\tan B} = \frac{15.603}{\tan 73° 54' 42''} = 4.5 \text{ feet.}$$

$A\,B = A\,D + D\,B = 27.747 + 4.5 = 32.247 = 32$ feet, 3 inches.

Ans. $\begin{cases} B\,C = 16 \text{ ft. } 3 \text{ in.} \\ A\,B = 32 \text{ ft. } 3 \text{ in.} \\ B = 73^\circ\ 54'\ 42''. \end{cases}$

(21) By rule, Art. **135,**

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{89.42}{.7854}} = \sqrt{113.8528},$$ or 10.67 in. Ans.

Circumference equals 10.67×3.1416, or 33.52 in. Ans.

In a regular hexagon inscribed in a circle, each side is equal to the radius of the circle; therefore, $\frac{10.67}{2} = 5.335$ in. is the length of a side. Ans.

(22) Angle $m\,O\,B = \frac{1}{16}$ of 360°, or $22\frac{1}{2}^\circ$. $m\,O = \frac{1}{2}$ of $m\,n = \frac{1}{2}$ of 2, or 1 inch. See Fig. V.

Side $m\,B = O\,m \times \tan 22\frac{1}{2}^\circ$, or $m\,B = 1 \times .41421 = .41421$.

$A\,B = 2\,m\,B$; therefore, $A\,B = .82842$ inch.

Area of $A\,O\,B = \frac{1}{2} \times .82842 \times 1 = .41421$ square inch, which, multiplied by 8, the number of equal triangles, equals 3.31368 square inches.

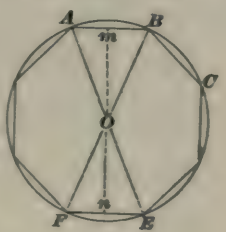

FIG. V.

Volume of bar $= 3.31368 \times 10 \times 12 = 397.6416$ cu. in.

Weight of bar equals $397.6416 \times .282 = 112.1349$, or 112 lb. 2 oz. Ans.

(23) $.5236 \times 16^3 = 2{,}144.66$ cubic inches equals the volume of a sphere 16 inches in diameter.

$.5236 \times 12^3 = 904.78$ cubic inches equals the volume of a sphere 12 inches in diameter.

The difference of the two volumes equals the volume of the spherical shell, and this multiplied by the weight per cubic inch equals the weight of the shell. Hence, we have $(2{,}144.66 - 904.78) \times .261 = 323.61$ lb. Ans.

(24) The circumference of the circle equals $\frac{5\frac{1}{3}\frac{3}{2} \times 360}{27}$, or 72.0833 inches. The diameter, therefore, is $\frac{72.0833}{3.1416}$, or 22.95 in., nearly. Ans.

(25) (*a*) $17\frac{1}{64}$ inches = 17.016 inches.

Area of circle = $.7854d^2 = .7854 \times 17.016^2 = 227.41$ sq. in. Ans.

(*b*) $16° 7' 21'' = 16.1225°$. By rule, Art. **132,**

$$l = \frac{\pi d n}{360} = \frac{3.1416 \times 17.016 \times 16.1225}{360} = 2.394 \text{ in.} \quad \text{Ans.}$$

(26) (*a*) By rule, Art. **144,** area = $12 \times 8 \times .7854 = 75.4$ sq. in. Ans.

(*b*) Applying the formula, Art. **143,** $a = 6$, $b = 4$, $D = \frac{6-4}{6+4} = \frac{2}{10} = \frac{1}{5}$.

Perimeter = $3.1416\,(6+4)\,\frac{64 - 3(\frac{1}{5})^4}{64 - 16(\frac{1}{5})^2} = 31.731$ in. Ans.

(27) Area of base = $.7854 \times 7^2 = 38.484$ square inches.

Slant height of cone = $\sqrt{11^2 + 3\frac{1}{2}^2}$, or 11.5434 inches.

Circumference of base = $7 \times 3.1416 = 21.9912$ inches.

Convex area of cone = $21.9912 \times \frac{11.5434}{2} = 126.927$ square inches.

Total area = $126.927 + 38.484 = 165.41$ sq. in. Ans.

(28) Volume of sphere equals $.5236 \times 10^3 = 523.6$ cubic inches.

Area of base of cone = $.7854 \times 10^2 = 78.54$ square inches.

$\frac{3 \times 523.6}{78.54} = 20$ in., altitude of cone. Ans.

(29) Volume of sphere = $.5236 \times 12^3 = 904.7808$ cubic inches.

Area of base of cylinder = $.7854 \times 12^2 = 113.0976$ square inches.

Height of cylinder = $\frac{904.7808}{113.0976} = 8$ in. Ans.

(30) (*a*) Area of the triangle equals $\frac{AC \times BD}{2}$, or $\frac{9\frac{1}{2} \times 12}{2} = 57$ sq. in. Ans.

(*b*) See Fig. VI. Angle $BAD = 79° 22'$; angle $ABD = 90° - 79° 22' = 10° 38'$. Side $AB = BD \div \sin 79° 22' = 12 \div .98283 = 12.209$ inches.

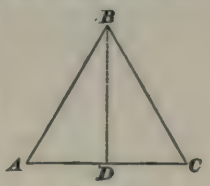

FIG. VI.

Side $AD = BD \times \tan 10° 38' = 12 \times .18775 = 2.253$ inches.

Side $DC = AC - AD = 9.5 - 2.253 = 7.247$ inches.

Side $BC = \sqrt{DB^2 + DC^2} = \sqrt{12^2 + 7.247^2} = \sqrt{196.519} = 14.018$ inches.

Perimeter of triangle equals $AB + BC + AC = 12.209 + 14.018 + 9.5 = 35.73$ in. Ans.

(31) The diagonal divides the trapezium into two triangles; the sum of the areas of these two triangles equals the area of the trapezium, which is, therefore,

$$\frac{11 \times 7}{2} + \frac{11 \times 4\frac{1}{4}}{2} = 61\tfrac{7}{8} \text{ sq. in.} \quad \text{Ans.}$$

(32) Referring to Fig. III, example 11, we have OA or $OB = 10 \div 2 = 5$ inches, and $AB = 6\frac{3}{4}$ inches.

$\sin COB = \dfrac{CB}{OB} = \dfrac{6\frac{3}{4} \div 2}{5} = .67500$; therefore, angle $COB = 42° 27' 14.3''$.

Angle $AOB = (42° 27' 14.3'') \times 2 = 84° 54' 28.6''$. Ans.

$OC = OB \times \cos COB = 5 \times .73782 = 3.6891$.

Area of sector $= 10^2 \times .7854 \times \dfrac{84° 54' 28.6''}{360°} = 18.524$ square inches.

Area of triangle $= \dfrac{6.75 \times 3.6891}{2} = 12.450$ square inches.

$18.524 - 12.450 = 6.074$ sq. in., the area of the segment. Ans.

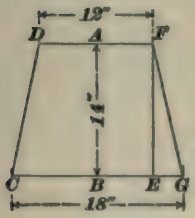

FIG. VII.

(33) See Fig. VII. Area of lower base $= 18^2 \times .7854 = 254.4696$ square inches.

Area of upper base $= 12^2 \times .7854 = 113.0976$ square inches.

$EG = BG - AF = 9 - 6$, or 3 inches.

Slant height $FG = \sqrt{EG^2 + EF^2} = \sqrt{3^2 + 14^2} = 14.32$ inches.

By formula, Art. **169,**

$$C = \frac{(p + p')s}{2} = \frac{37.6992 + 56.5488}{2} \times 14.32 = 674.8156 \text{ sq. in.}$$

Total area = 674.8157 + 254.4696 + 113.0976 = 1,042.38 sq. in. Ans.

By formula, Art. **170,** $V = (A + a + \sqrt{A + a})\frac{h}{3} = (113.0976 + 254.4696 + \sqrt{113.0976 \times 254.4696})\ 14\frac{2}{3} = 2{,}506.997$ cu. in. Ans.

(34) Area of surface of sphere 27 inches in diameter $= \pi d^2 = 3.1416 \times 27^2 = 2{,}290.2$ sq. in. Ans.

(35) Area of end $= 19^2 \times .7854 = 283.5294$ square inches. Volume $= 283.5294 \times 24 = 6{,}804.7056$ cu. in. $= 3.938$ cu. ft. Ans.

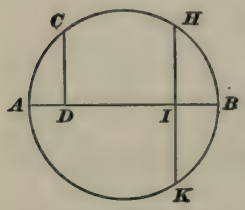

FIG. VIII.

(36) Given, $IB = 2$ inches and $HI = IK = 1\frac{4}{2} = 7$ inches to find the radius. See Fig. VIII.

$IB : HI = HI : AI$, or $2 : 7 = 7 : AI$.

Therefore, $AI = \frac{49}{2} = 24\frac{1}{2}$ inches.

$AB = AI + IB = 24\frac{1}{2} + 2 = 26\frac{1}{2}$ inches.

$$\text{Radius} = \frac{AB}{2} = \frac{26\frac{1}{2}}{2} = 13\frac{1}{4} \text{ in.} \quad \text{Ans.}$$

(37) (*a*) In Fig. IX, given $OB = \frac{16}{2}$ or 8 inches, and $OA = 13 \div 2 = 6\frac{1}{2}$ inches, to find the volume, area, and weight.

Radius of center circle equals $\frac{8 + 6.5}{2}$, or $7\frac{1}{4}$ inches. Length of center line $= 3.1416 \times 2 \times 7\frac{1}{4} = 45.5532$ inches. The radius of the inner circle is $6\frac{1}{2}$ inches, and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, the area of the imaginary cross-section is $(1\frac{1}{2})^2 \times .7854 = 1.76715$ sq. in. Ans.

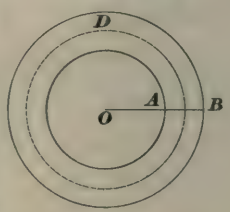

FIG. IX.

Volume of ring $= 1.76715 \times 45.5532 = 80.499$ cu. in. Ans.

(*b*) Weight of ring $= 80.4993 \times .261 = 21$ lb. Ans.

(38) The convex area $= 4 \times 5\frac{1}{4} \times 18 = 378$ sq. in. Ans.
Area of the bases $= 5\frac{1}{4} \times 5\frac{1}{4} \times 2 = 55.125$ square inches.
Total area $= 378 + 55.125 = 433.125$ sq. in. Ans.
Volume $= (5\frac{1}{4})^2 \times 18 = 496.125$ cu. in. Ans.

(39) In Fig. X, $OC = \dfrac{AC}{\tan 30^\circ}$. ($\frac{1}{6}$ of $360^\circ = 60^\circ$, and since $AOC = \frac{1}{2}$ of AOB, $AOC = 30^\circ$.)

$$OC = \frac{6}{.57735} = 10.392.$$

$$\text{Area of } AOB = \frac{12 \times 10.392}{2}$$

$= 62.352$ square feet.

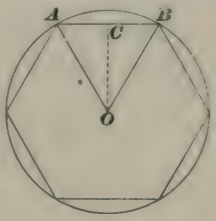

FIG. X.

Since there are 6 equal triangles in a hexagon, then the area of the base $= 6 \times 62.352$, or 374.112 square feet.

Perimeter $= 6 \times 12$, or 72 feet.

$$\text{Convex area} = \frac{72 \times 37}{2} = 1{,}332 \text{ sq. ft.} \quad \text{Ans.}$$

Total area $= 1{,}332 + 374.112 = 1{,}706.112$ sq. ft. Ans.

(40) Area of the base $= 374.112$ square feet, and altitude $= 37$ feet. Since the volume equals the area of the base multiplied by $\frac{1}{3}$ of the altitude, we have

$$\text{volume} = 374.112 \times \frac{37}{3} = 4{,}614 \text{ cu. ft.} \quad \text{Ans.}$$

(41) Given, $AB = 6\frac{3}{4}$ inches, and $OB = OA = 10 \div 2 = 5$ inches (see Fig. XI), to find the area of the sector.

Area of circle $= 10^2 \times .7854 = 78.54$ square inches.

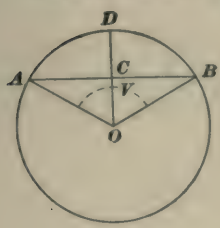

FIG. XI.

$$\text{Sin } AOC = \frac{AC}{OA} = \frac{6\frac{3}{4} \div 2}{5} = .68750;$$

therefore, $AOC = 43^\circ\ 25'\ 57''$.

$AOB = 2 \times AOC = 2 \times 43^\circ\ 25'\ 57''$
$= 86^\circ\ 51'\ 54'' = 86.865^\circ$.

By rule, Art. **137**,

$$\frac{86.865}{360} \times 78.54 = 18.95 \text{ sq. in.} \quad \text{Ans.}$$

(42) By rule, Art. **129,** $A = b\,h = 7 \times 10\frac{3}{4}$ (129 inches $= 10\frac{3}{4}$ feet) $= 75\frac{1}{4}$ sq. ft. Ans.

(43) See Art. **130.**

Area of trapezoid $= \left(\frac{a+b}{2}\right) h = \frac{15\frac{7}{12} + 21\frac{11}{12}}{2} \times 7\frac{2}{3} = 143.75$ sq. ft. Ans.

(44) (*a*) Side of square having an equivalent area $= \sqrt{143.75} = 11.99$ ft. Ans.

(*b*) Diameter of circle having an equivalent area $= \sqrt{\frac{143.75}{.7854}} = \sqrt{183.0277} = 13\frac{1}{2}$ ft. Ans.

(*c*) Perimeter of square $= 4 \times 11.99 = 47.96$ ft.
Circumference of circle $= 13\frac{1}{2} \times 3.1416 = 42.41$ ft.

Difference of perimeter $= 5.55$ ft. $= 5$ ft. 6.6 in. Ans.

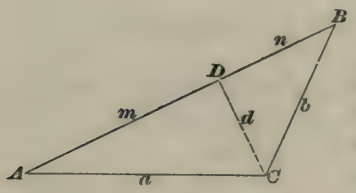

FIG XII.

(45) In the triangle $A\,B\,C$, Fig. XII, $A\,B = 24$ feet, $B\,C = 11.25$ feet, and $A\,C = 18$ feet. $m + n : a + b = a - b : m - n$, or $24 : 29.25 = 6.75 : m - n$.

$$m - n = \frac{29.25 \times 6.75}{24} = 8.226562.$$

Adding $m + n$ and $m - n$, we have

$$\begin{aligned} m + n &= 24 \\ m - n &= 8.226562 \\ \hline 2\,m &= 32.226562 \\ m &= 16.113281. \end{aligned}$$

Subtracting $m - n$ from $m + n$, we have

$$\begin{aligned} 2\,n &= 15.773438 \\ n &= 7.886719. \end{aligned}$$

In the triangle $A\,D\,C$, side $A\,C = 18$ feet, side $A\,D = 16.113281$; hence, according to rule **2,** Art. **98,** $\cos A = \frac{16.113281}{18} = .89518$, or angle $A = 26^\circ\ 28'\ 5''$. In the triangle $B\,D\,C$, side $B\,D = 7.886719$, and side $B\,C = 11.25$ feet. Hence, $\cos B = \frac{7.886719}{11.25} = .70104$, or angle $B = 45^\circ\ 29'\ 23''$. Angle $C = 180^\circ - (45^\circ\ 29'\ 23'' + 26^\circ\ 28'\ 5'') = 108^\circ\ 2'\ 32''$.

Ans. $\begin{cases} A = 26^\circ\ 28'\ 5''. \\ B = 45^\circ\ 29'\ 23''. \\ C = 108^\circ\ 2'\ 32''. \end{cases}$

NOTICE

The present set of answers on the subject of Geometry and Trigonometry are less in number than were contained in the former edition. As a consequence, there is a slight break in the page numbers between the last page of the answers on Geometry and Trigonometry and the Paper following.

ELEMENTARY MECHANICS.

(QUESTIONS 355–453.)

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(355) Use formulas **18** and **8.**

Time it would take the ball to fall to the ground $= t =$ $\sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5.5}{32.16}} = .58484$ sec.

The space passed through by a body having a velocity of 500 ft. per sec. in .58484 of a second $= S = Vt = 500 \times .58484 = 292.42$ ft. Ans.

(356) Use formula **7.**

$$\frac{\frac{8}{11} \times 3.1416 \times 160}{60} = 55.85 \text{ ft. per sec.} \quad \text{Ans.}$$

(357) $160 \div 60 \times 7 = \frac{8}{21}$ revolution in $\frac{1}{7}$ sec. $360° \times \frac{8}{21} = 137\frac{1}{7}° = 137° 8' 34\frac{2}{7}''$. Ans.

(358) (*a*) See Fig. 20. $36' = 3'$. $4 \div 3 = \frac{4}{3} =$ number of revolutions of pulley to one revolution of fly-wheel. $54 \times \frac{4}{3} = 72$ revolutions of pulley and drum per min. $100 \div \left(\frac{18}{12} \times 3.1416\right) = 21.22$ revolutions of drum to raise elevator 100 ft. $\frac{21.22}{72} \times 60 = 17.68$ sec. to travel 100 ft. Ans.

(*b*) $21.22 : x :: 30 : 60$, or $x = \frac{21.22 \times 60}{30} = 42.44$ rev.

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per min. of drum. The diameter of the pulley divided by

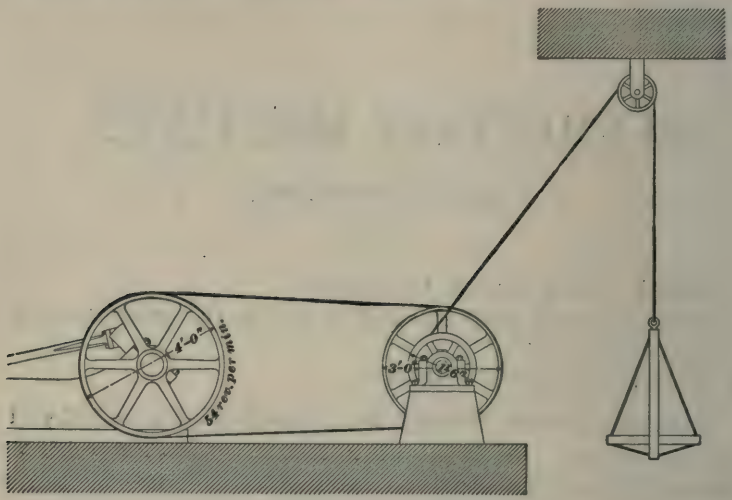

FIG. 20.

the diameter of the fly-wheel $= \frac{3}{4}$, which multiplied by $42.44 = 31.83$ revolutions per min. of fly-wheel. Ans.

(359) See Arts. **857** and **859.**

(360) See Art. **861.**

(361) See Arts. **843** and **871.**

(362) See Art. **871.**

(363) See Arts. **842, 886, 887,** etc.

The relative weight of a body is found by comparing it with a given standard by means of the balance. The absolute weight is found by noting the pull which the body will exert on a spring balance.

The absolute weight increases and decreases according to the laws of weight given in Art. **890;** the relative weight is always the same.

(364) See Art. **861.**

(365) See Art. **857.**

(366) See Art. **857.**

(367) If the mountain is at the same height above, and the valley at the same depth below sea-level respectively, it will weigh more at the bottom of the valley.

(368) $\frac{31,680}{5,280} = 6$ miles. Using formula **12,** $d^2 : R^2 :: W : w$, we have $w = \frac{R^2 W}{d^2} = \frac{3,960^2 \times 20,000}{3,966^2} = 19,939.53+$ lb. $= 19,939$ lb. $8\frac{1}{2}$ oz. Ans.

(369) Using formula **11,** $R : d :: W : w$, we have $w = \frac{d W}{R} = \frac{3,958 \times 20,000}{3,960} = 19,989.89$ lb. $= 19,989$ lb. $14\frac{1}{4}$ oz. Ans.

(370) See Art. **870.**

(371) See Art. **894.**

(372) The velocity which a body may have at the instant the time begins to be reckoned.

(373) Because the man after jumping tends to continue in motion with the same velocity as the train, and the sudden stoppage by the earth causes a shock, the severity of which varies with the velocity of the train.

(374) See Arts. **870** and **871.**

(375) See Art. **872.**

(376) That force which will produce the same final effect upon a body as all the other forces acting together is called the resultant.

(377) (*a*) If a 5-in. line $= 20$ lb., a 1-in. line $= 4$ lb. $1 \div 4 = \frac{1}{4}$ in. $= 1$ lb. Ans. (*b*) $6\frac{1}{4} \div 4 = 1.5625$ in. $= 6\frac{1}{4}$ lb. Ans.

(378) Those forces by which a given force may be

replaced, and which will produce the same effect upon a body.

(379) Southeast, in the direction of the diagonal of a square. See Fig. 21.

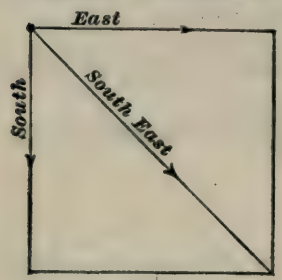

FIG. 21.

(380) $4'\ 6'' = 54''$. $54 \times 2 \times \frac{3}{4} \times .261 = 21.141$ lb. = weight of lever. Center of gravity of lever is in the middle, at a, Fig. 22, $27''$ from each end. Consider that the lever has no weight. The center of gravity of the two weights is at b, at a distance from c equal to $\frac{47 \times 54}{47 + 71} = 21.508'' = bc$. Formula **20**, Art. **911.**

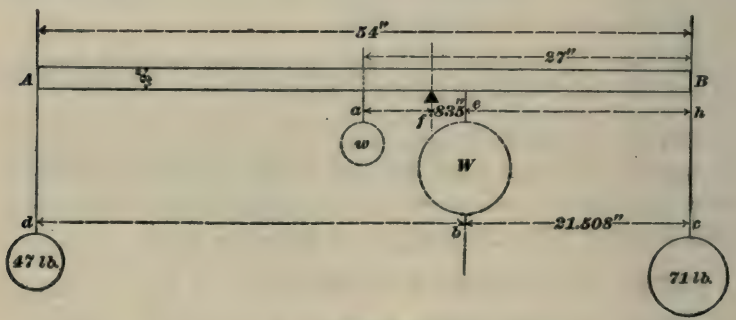

FIG. 22.

Consider both weights as concentrated at b, that is, imagine both weights removed and replaced by the dotted weight W, equal to $71 + 47 = 118$ lb. Consider the weight of the bar as concentrated at a, that is, as if replaced by a weight $w = 21.141$ lb. Then, the distance of the balancing point f, from e, or fe, $= \frac{21.141 \times 5.492}{21.141 + 118} = .834''$, since $ae = 27 - 21.508 = 5.492''$. Finally, $fe + eh = fh = .834 + 21.508 = 22.342'' =$ the short arm. Ans. $54 - 22.342 = 31.658'' =$ long arm. Ans.

(381) See Fig. 23.

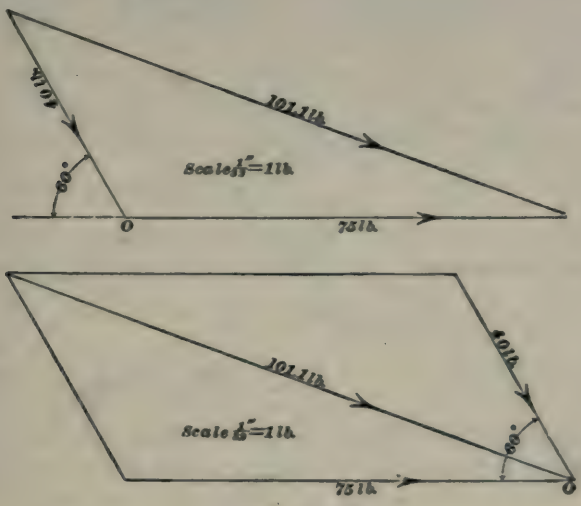

FIG. 23.

(382) See Fig. 24.

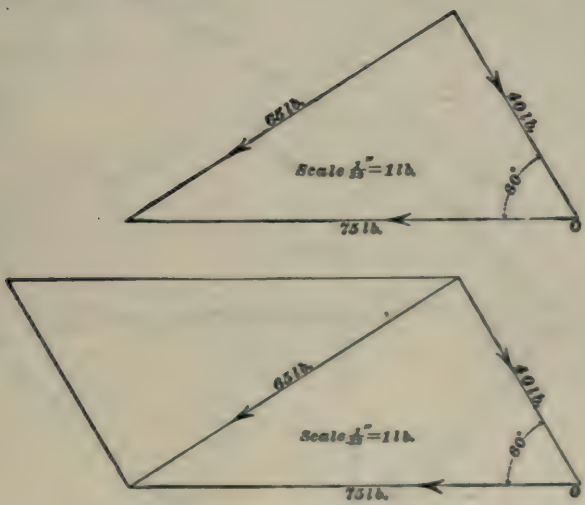

FIG. 24.

(383) $46 - 27 = 19$ lb., acting in the direction of the force of 46 lb. Ans.

(384) (*a*) $18 \times 60 \times 60 = 64{,}800$ miles per hour Ans.
(*b*) $64{,}800 \times 24 = 1{,}555{,}200$ miles. Ans.

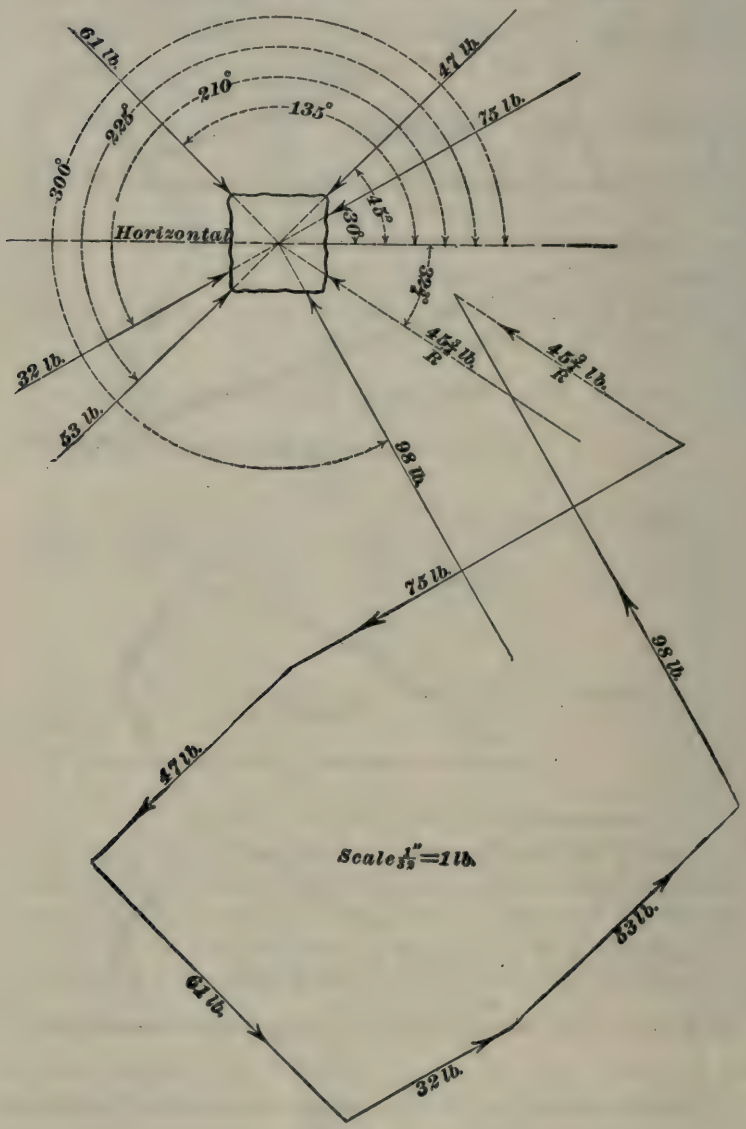

FIG. 25.

(385) (*a*) 15 miles per hour $= \frac{15 \times 5{,}280}{60 \times 60} = 22$ ft. per sec. As the other body is moving 11 ft. per sec., the distance between the two bodies in one second will be $22 + 11 = 33$ ft., and in 8 minutes the distance between them will be $33 \times 60 \times 8 = 15{,}840$ ft., which, divided by the number of feet in one mile, gives $\frac{15{,}840}{5{,}280} = 3$ miles. Ans.

(*b*) As the distance between the two bodies increases 33 ft. per sec., then, 825 divided by 33 must be the time required for the bodies to be 825 ft. apart, or $\frac{825}{33} = 25$ sec. Ans.

(386) See Fig. 25.

(387) (*a*) Although not so stated, the velocity is evidently considered with reference to a point on the shore. $10 - 4 = 6$ miles an hour. Ans.

(*b*) $10 + 4 = 14$ miles an hour. Ans.

(*c*) $10 - 4 + 3 = 9$, and $10 + 4 + 3 = 17$ miles an hour. Ans.

(388) See Fig. 26.

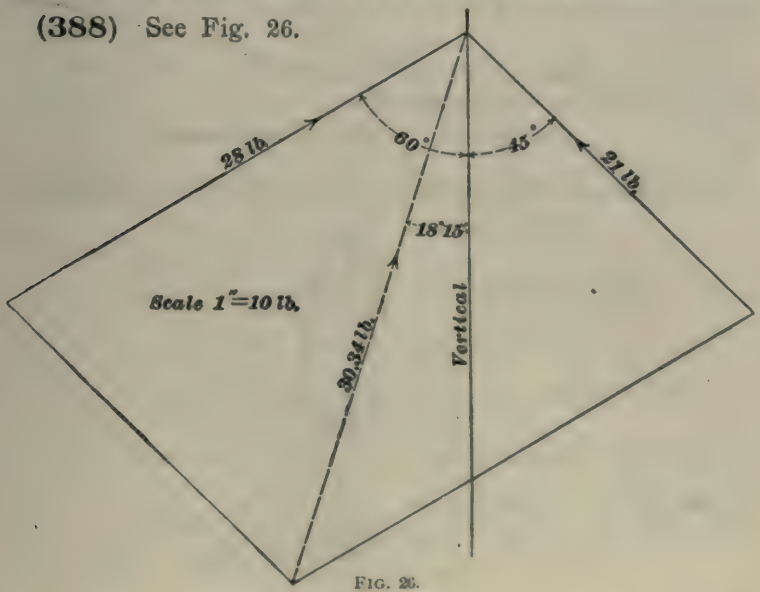

FIG. 26.

(389) See Fig. 27. By rules **2** and **4**, Art. **754**, $bc = 87 \sin 23° = 87 \times .39073 = 33.994$ lb., $ac = 87 \cos 23° = 87 \times .92050 = 80.084$ lb.

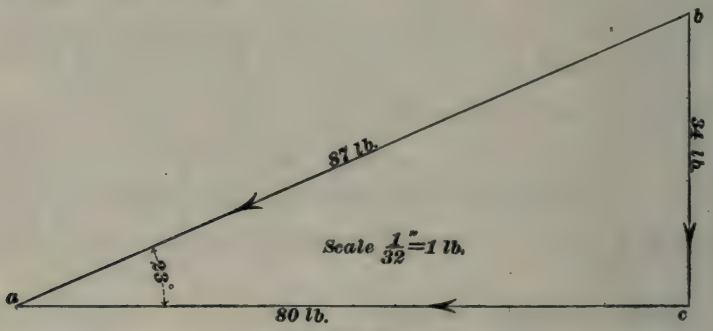

FIG. 27.

(390) See Fig. 28. (*b*) By rules **2** and **4**, Art. **754**, $bc = 325 \sin 15° = 325 \times .25882 = 84.12$ lb. Ans.

(*a*) $ac = 325 \cos 15° = 325 \times .96593 = 313.93$ lb. Ans.

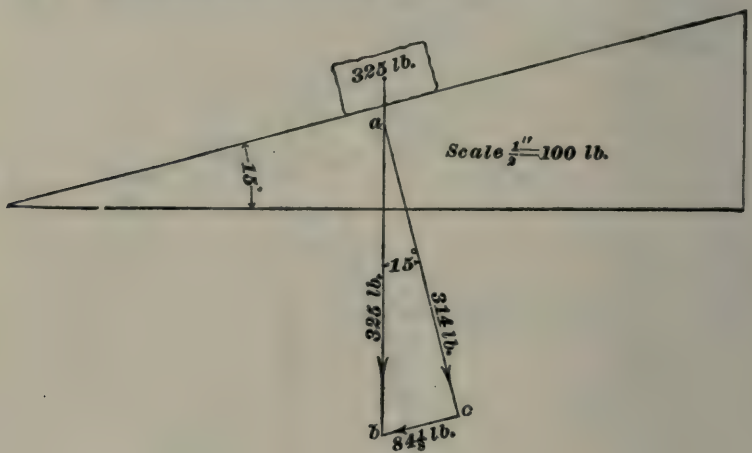

FIG. 28.

(391) Use formula **10.**

$$m = \frac{W}{g} = \frac{125}{32.16} = 3.8868. \quad \text{Ans.}$$

(392) Using formula **10**, $m = \frac{W}{g}$, $W = mg = 53.7 \times 32.16 = 1{,}727$ lb. Ans.

(393) (*a*) Yes. (*b*) 25. (*c*) 25. Ans.

(394) (*a*) Using formula **12**, $d^2 : R^2 :: W : w$, $d = \sqrt{\frac{R^2 W}{w}} = \sqrt{\frac{4{,}000^2 \times 141}{100}} = 4{,}749.736$ miles. $4{,}749.736 - 4{,}000 = 749.736$ miles. Ans.

(*b*) Using formula **11**, $R : d :: W : w$, $d = \frac{Rw}{W} = \frac{4{,}000 \times 100}{141} = 2{,}836.88$ miles. $4{,}000 - 2{,}836.88 = 1{,}163.12$ miles. Ans.

(395) (*a*) Use formula **18**,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5{,}280}{32.16}} = 18.12 \text{ sec.} \quad \text{Ans.}$$

(*b*) Use formula **13**, $v = gt = 32.16 \times 18.12 = 582.74$ ft. per sec., or, by formula **16**, $v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 5{,}280} = 582.76$ ft. per sec. Ans.

The slight difference in the two velocities is caused by not calculating the time to a sufficient number of decimal places, the actual value for t being 18.12065 sec.

(396) Use formula **25**. Kinetic energy $= Wh = \frac{Wv^2}{2g}$.

$$Wh = 160 \times 5{,}280 = 844{,}800 \text{ ft.-lb.}$$

$$\frac{Wv^2}{2g} = \frac{160 \times 582.76^2}{2 \times 32.16} = 844{,}799 \text{ ft.-lb.} \quad \text{Ans.}$$

(397) (*a*) Using formulas **15** and **14**,

$h = \frac{v^2}{2g} = \frac{2{,}360^2}{2 \times 32.16} = 86{,}592$ ft. $= 16.4$ miles. Ans. (*b*) $t = \frac{v}{g} =$ time required to go up or fall back. Hence, total time $= \frac{2v}{g}$ sec. $= \frac{2 \times 2{,}360}{60 \times 32.16} = 2.4461$ min. $= 2$ min. 26.77 sec. Ans.

(398) 1 hour = 60 min., 1 day = 24 hours; hence, 1 day = 60 × 24 = 1,440 min. Using formula **7**, $V = \frac{S}{t}$; whence, $V = \frac{8{,}000 \times 3.1416}{1{,}440} = 17.453+$ miles per min. Ans.

(399) (*a*) Use formula **25.**

Kinetic energy $= \frac{W v^2}{2g} = \frac{400 \times 1{,}875 \times 1{,}875}{2 \times 32.16} = 21{,}863{,}339.55$ ft.-lb. Ans.

(*b*) $\frac{21{,}863{,}339.55}{2{,}000} = 10{,}931.67$ ft.-tons. Ans.

(*c*) See Art. **961.**

Striking force $\times \frac{6}{12} = 21{,}863{,}339.55$ ft.-lb.,

or striking force $= \frac{21{,}863{,}339.55}{\frac{6}{12}} = 43{,}726{,}679$ lb. Ans.

(400) Using formula **18**, $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 200}{32.16}} = 3.52673$ sec., when $g = 32.16$.

$t = \sqrt{\frac{2 \times 200}{20}} = 4.47214$ sec., when $g = 20$.

$4.47214 - 3.52673 = 0.94541$ sec. Ans.

(401) See Art. **910.**

(402) See Art. **963.**

(403) (*a*) See Art. **962.**

$D = \frac{m}{V} = \frac{W}{g v}$. $v = \frac{800}{1{,}728}$. Hence, $D = \frac{W}{g v} = \frac{500}{32.16 \times \frac{800}{1728}} = 33.582$. Ans. (*b*) In Art. **962,** the density of water was found to be 1.941. (*c*) In Art. **963,** it is stated that the specific gravity of a body is the ratio of its density to the density of water. Hence, $\frac{33.582}{1.941} = 17.3 =$ specific gravity. If the weight of water be taken as 62.5 lb. per cu. ft., the specific gravity will be found to be 17.28. Ans.

(404) Assuming that it started from a state of rest, formula **13** gives $v = g t = 32.16 \times 5 = 160.8$ ft. per sec.

(405) Use formulas **17** and **13.** $h = \frac{1}{2} g t^2 = \frac{32.16}{2} \times 3^2 = 144.72$ ft., distance fallen at the end of third second.

$v = g t = 32.16 \times 3 = 96.48$ ft. per sec., velocity at end of third second.

$96.48 \times 6 = 578.88$ ft., distance fallen during the remaining 6 seconds.

$144.72 + 578.88 = 723.6$ ft. = total distance. Ans.

(406) See Art. **961.**

Striking force $\times \frac{\frac{1}{2}}{12} = 8 \times 8 = 64$. Therefore, striking force $= \frac{64}{\frac{\frac{1}{2}}{12}} = 1{,}536$ tons. Ans.

(407) See Arts. **901** and **902.**

(408) Use formula **19.**

Centrifugal force = tension of string $= .00034\ W R N^2 = .00034 \times (.5236 \times 4^2 \times .261) \times \frac{15}{12} \times 60^2 = 13.88+$ lb. Ans.

(409) $(80^2 - 70^2) \times .7854 \times 26 \times .261 \div 2 =$ weight of $\frac{1}{2}$ of rim.

$$R = \frac{80 - 10}{2 \times 12} = \frac{35}{12} \text{ ft.}$$

According to Art. **904,** $F = .00034\ W R N^2 \div 3.1416 = .00034 \times \frac{(80^2 - 70^2) \times .7854 \times 26 \times .261}{2} \times \frac{35}{12} \times 175^2 \div 3.1416 = 38{,}641$ lb. Ans.

(410) (*a*) Use formulas **11** and **12.** $R : d :: W : w$, or $W = \frac{w R}{d} = \frac{1 \times 4{,}000}{100} = 40$ lb. Ans.

(*b*) $d^2 : R^2 :: W : w$, or $w = \frac{4{,}000^2 \times 40}{4{,}100^2} = 38.072$ lb. Ans.

(411) See Art. **955.**

$$\frac{10{,}746 \times 354}{10 \times 33{,}000} = 11.5275 \text{ H. P.} \quad \text{Ans.}$$

(412) Use formula **12.**

$d^2 : R^2 :: W : w$, or $d = \sqrt{\frac{4{,}000^2 \times 2}{\frac{3}{16}}} = 13{,}064$ mi., nearly.

$13{,}064 - 4{,}000 = 9{,}064$ miles. Ans.

(413) Use formula **18.** $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 50}{32.16}} =$ 1.7634 sec., nearly.

$1.7634 \times 140 = 246.876$ ft. Ans.

(414) $\frac{10}{30} = \frac{1}{3}$ sec. Use formula **17.**

$h = \frac{1}{2} g t^2 = \frac{1}{2} \times 32.16 \times \left(\frac{1}{3}\right)^2 = 1.78\frac{2}{3}$ ft. $= 1$ ft. 9.44 in. Ans.

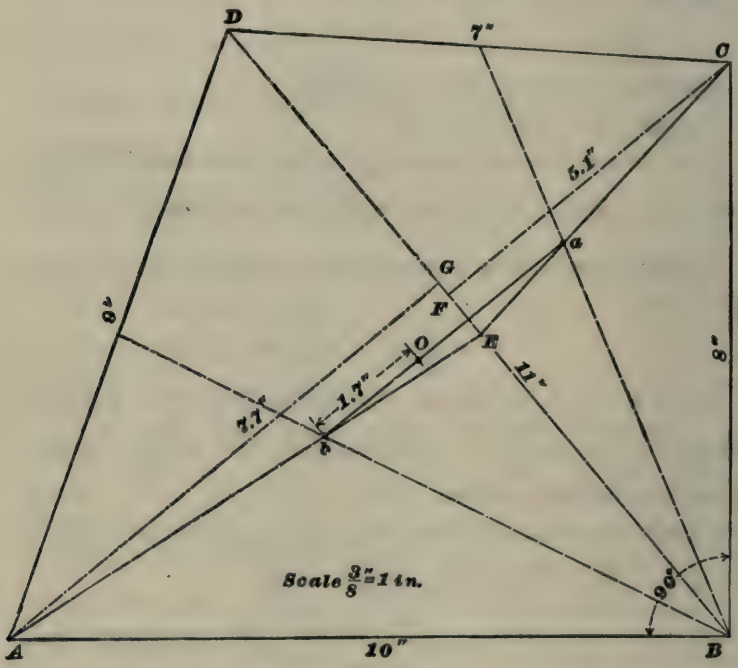

FIG. 29.

(415) See Arts. **906, 907.**

(416) See Arts. **908, 909.**

(417) No. It can only be counteracted by another equal couple which tends to revolve the body in an opposite direction.

(418) See Art. **914.**

(419) Draw the quadrilateral as shown in Fig. 29. Divide it into two triangles by the diagonal $B\ D$. The center of gravity of the triangle $B\ C\ D$ is found to be at a, and the center of gravity of the triangle $A\ B\ D$ is found to be at b (Art. **914**). Join a and b by the line $a\ b$, which, on being measured, is found to have a length of 4.27 inches. From C and A drop the perpendiculars $C\ F$ and $A\ G$ on the diagonal $B\ D$. Then, area of the triangle $A\ B\ D = \frac{1}{2}\ (A\ G \times B\ D)$, and area of the triangle $B\ C\ D = \frac{1}{2}\ (C\ F \times B\ D)$. Measuring these distances, $B\ D = 11''$, $C\ F = 5.1''$, and $A\ G = 7.7''$.

$$\text{Area } A\ B\ D = \frac{1}{2} \times 7.7 \times 11 = 42.35 \text{ sq. in.}$$

$$\text{Area } B\ C\ D = \frac{1}{2} \times 5.1 \times 11 = 28.05 \text{ sq. in.}$$

According to formula **20,** the distance of O, the center of gravity, from b is $\frac{28.05 \times 4.27}{28.05 + 42.35} = 1.7$. Therefore, the center of gravity is on the line $a\ b$ at a distance of $1.7''$ from b.

(420) See Fig. 30. The center of gravity lies at the geometrical center of the pentagon, which may be found as follows: From any vertex draw a line to the middle point of the opposite side. Repeat the operation for any other vertex, and the intersection of the two lines will be the desired center of gravity.

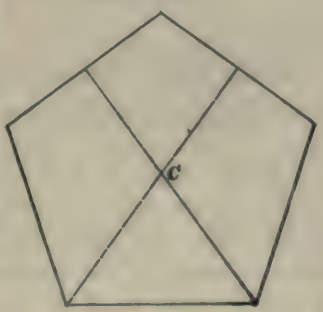

FIG. 30.

(421) See Fig. 31. Since any number of quadrilaterals can be drawn with the sides given, any number of answers can be obtained.

Draw a quadrilateral, the lengths of whose sides are equal to the distances between the weights, and locate a weight on each corner. Apply formula **20** to find the distance $C_1 W_1$; thus, $C_1 W_1 = \frac{9 \times 18}{9 + 21} = 5.4''$. Measure the distance $C_1 W_2$;

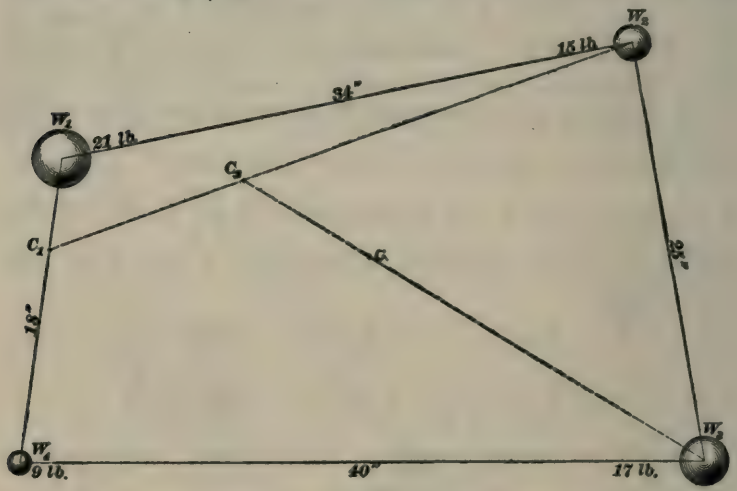

FIG. 31.

suppose it equals say $36''$. Apply the formula again. $C_1 C_2 = \frac{15 \times 36}{15 + (9 + 21)} = 12''$. Measure $C_2 W_3$; it equals say $31.7''$.

Apply the formula again. $C_2 C = \frac{17 \times 31.7}{17 + 15 + 9 + 21} = 8.7''$. C is center of gravity of the combination.

(422) Let $A\ B\ C\ D\ E$, Fig. 32, be the outline, the right-angled triangle cut-off being $E\ S\ D$. Divide the figure into two parts by the line $m\ n$, which is so drawn that it cuts off an isosceles right-angled triangle $m\ B\ n$, equal in area to $E\ S\ D$, from the opposite corner of the square.

The center of gravity of $A\,m\,n\,C\,D\,E$ is then at C_1, its geometrical center. $B\,m = 4$ in.; angle $B\,m\,r = 45°$; there-

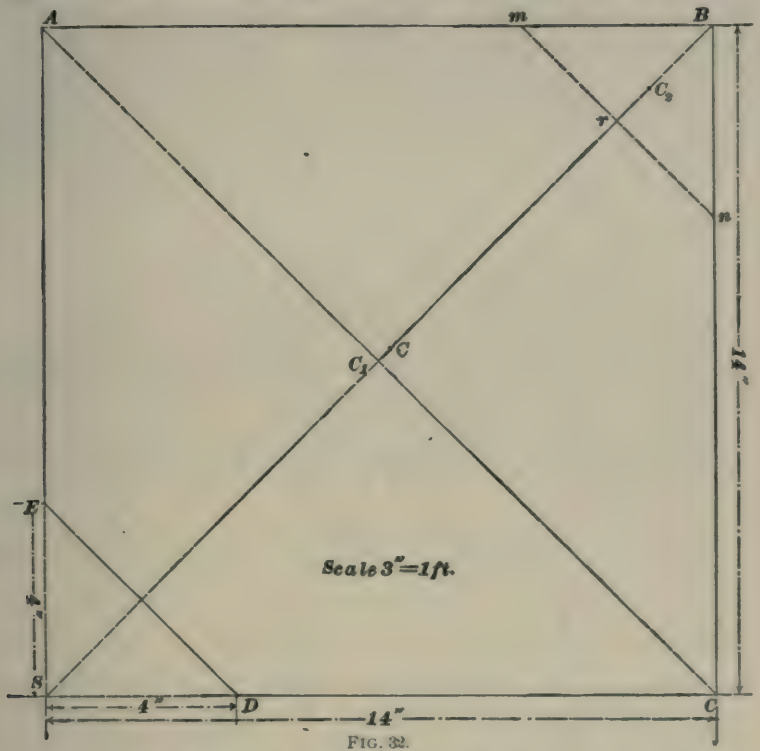

FIG. 32.

fore, $B\,r = B\,m \times \sin B\,m\,r = 4 \times .707 = 2.828$ in. C_2, the center of gravity of $B\,m\,n$, lies on $B\,r$, and $B\,C_2 = \frac{2}{3} B\,r$ $= \frac{2}{3} \times 2.828 = 1.885$ in. $B\,C_1 = A\,B \times \sin B\,A\,C_1 = 14 \times \sin 45° = 14 \times .707 = 9.898$ in. $C_1\,C_2 = B\,C_1 - B\,C_2 = 9.898 - 1.885 = 8.013$ in.

Area $A\,B\,C\,D\,E = 14^2 - \frac{4 \times 4}{2} = 188$ sq. in. Area $m\,B\,n$ $= \frac{4 \times 4}{2} = 8$ sq. in. Area $A\,m\,n\,C\,D\,E = 188 - 8 = 180$ sq. in.

The center of gravity of the combined area lies at C, at

a distance from C_1, according to formula **20** (Art. **911**), equal to $\frac{8 \times C_1 C_2}{180 + 8} = \frac{8 \times 8.013}{188} = .341$ in. $C_1 C = .341$ in. $B C = B C_1 - C_1 C = 9.898 - .341 = 9.557$ inches. Ans.

(423) (*b*) In one revolution the power will have moved through a distance of $2 \times 15 \times 3.1416 = 94.248''$, and the weight will have been lifted $\frac{1''}{4}$. The velocity ratio is then $94.248 \div \frac{1}{4} = 376.992$.

$376.992 \times 25 = 9{,}424.8$ lb. Ans.

(*a*) $9{,}424.8 - 5{,}000 = 4{,}424.8$ lb. Ans.

(*c*) $4{,}424.8 \div 9{,}424.8 = 46.95\%$. Ans.

(424) See Arts. **920** and **922.**

(425) Construct the prism *A B E D*, Fig. 33. From *E*, draw the line *E F*. Find the center of gravity of the

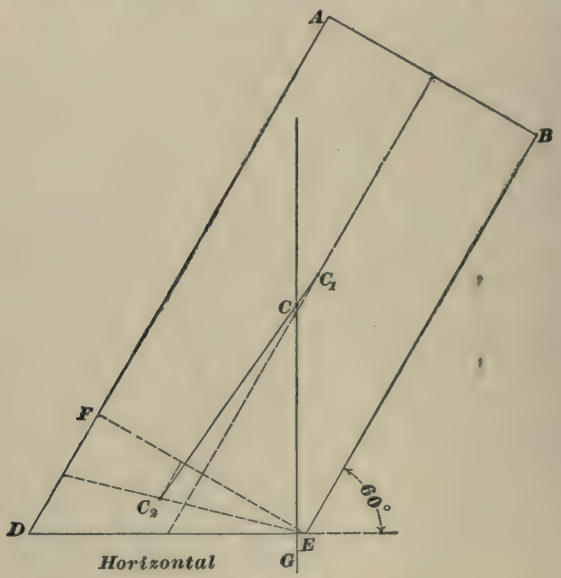

FIG. 33.

rectangle, which is at C_1, and that of the triangle, which is

at C_2. Connect these centers of gravity by the straight line $C_1 C_2$ and find the common center of gravity of the body by the rule to be at C. Having found this center, draw the line of direction $C G$. If this line falls within the base, the body will stand, and if it falls without, it will fall.

(426) (*a*) 5 ft. 6 in. = 66″. $66 \div 6 = 11$ = velocity ratio. Ans.

(*b*) $5 \times 11 = 55$ lb. Ans.

(427) $55 \times .65 = 35.75$ lb. Ans.

(428) Apply formula **20.** 5 ft. = 60″. $\frac{35 \times 60}{180 + 35} =$ 9.7674 in., nearly, = distance from the large weight. Ans.

(429) (*a*) $1{,}000 \div 50 = 20$, velocity ratio. Ans. See Art. **945.** (*b*) 10 fixed and 10 movable. Ans. (*c*) $50 \div 95 = 52.63\%$. Ans.

(430) $P \times$ circumference $= W \times \frac{1}{8}$, or $60 \times 40 \times 3.1416 = W \times \frac{1}{8}$, or $W = 60 \times 40 \times 3.1416 \times 8 = 60{,}318.72$ lb. Since the efficiency of combination is 40%, the tension on the stud would be $.40 \times 60{,}318.72 = 24{,}127.488$ lb. Ans.

(431) (*a*) $\sqrt{20^2 + 5^2} = 20.616$ ft. = length of inclined plane.

$P \times$ length of plane $= W \times$ height, or $P \times 20.616 = 1{,}580 \times 5$.

$P = \frac{1{,}580 \times 5}{20.616} = 383.2$ lb. Ans. (*b*) In the second case, $P \times$ length of base $= W \times$ height, or $P \times 20 = 1{,}580 \times 5$; hence, $P = \frac{1{,}580 \times 5}{20} = 395$ lb. Ans.

(432) $W \times 2 = 42 \times 6$, or $W = \frac{42 \times 6}{2} = 126$ lb.

$126 + 42 = 168$ lb. $168 \times 1 = W' \times 12$, or $W' = \frac{168}{12} = 14$ lb. Ans.

(433) See Fig. 34. $P \times 14 \times 21 \times 19 = 2\frac{1}{2} \times 3\frac{1}{4} \times 2\frac{7}{8} \times 725$, or

$$P = \frac{2\frac{1}{2} \times 3\frac{1}{4} \times 2\frac{7}{8} \times 725}{14 \times 21 \times 19} = 3.032 \text{ lb.} \quad \text{Ans.}$$

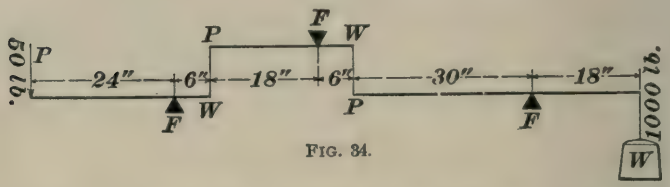

FIG. 34.

(434) See Fig. 35. (*a*) $35 \times 15 \times 12 \times 20 = 5 \times 3\frac{1}{2} \times 3 \times W$, or

$$W = \frac{35 \times 15 \times 12 \times 20}{5 \times 3\frac{1}{2} \times 3} = 2{,}400 \text{ lb.} \quad \text{Ans.}$$

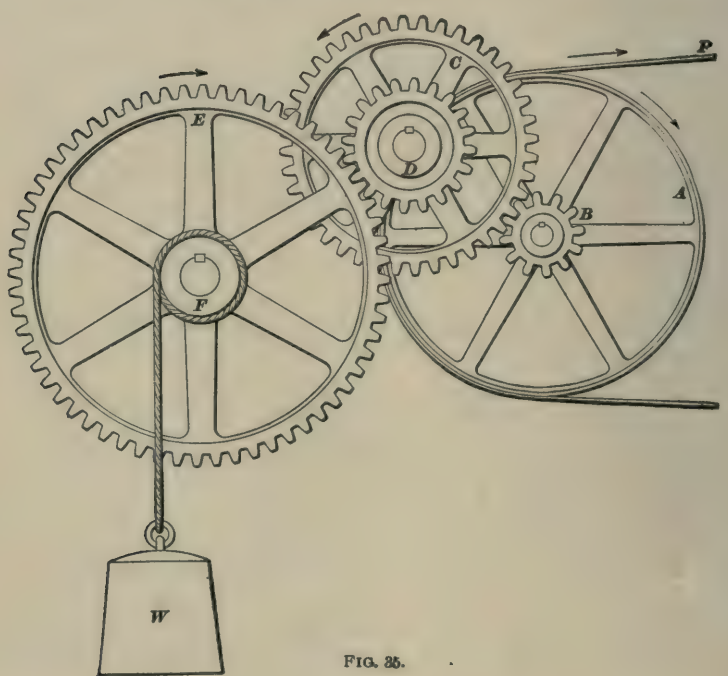

FIG. 35.

(*b*) $2{,}400 \div 35 = 68\frac{4}{7}$ = velocity ratio. Ans.

(*c*) $1{,}932 \div 2{,}400 = .805 = 80.5\%$. Ans.

(435) In Fig. 36, let the 12-lb. weight be placed at A, the 18-lb. weight at B, and the 15-lb. weight at D.

Use formula **20.**

$\frac{12 \times 15}{18 + 12} = 6''$ = distance $C_1 B$ = distance of center of

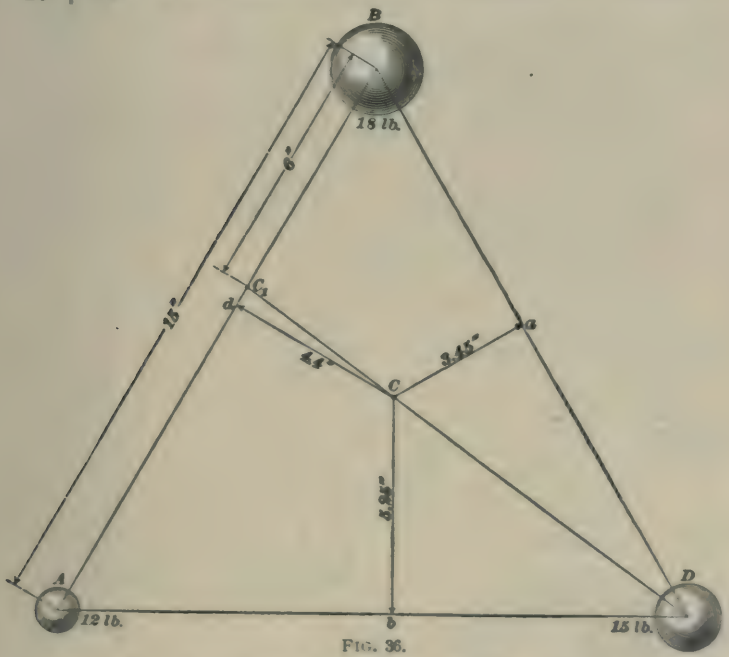

FIG. 36.

gravity of the 12 and 18-lb. weights from B. Drawing $C_1 D$, $C_1 C = \frac{15 \times C_1 D}{(12 + 18) + 15} = \frac{1}{3} C_1 D$. Measuring the distances of C from BD, DA, and AB, it is found that $Ca = 3.45''$, $Cb = 5.25''$, and $Cd = 4.4''$. Ans.

(436) (*a*) Potential energy equals the work which the body would do in falling to the ground = $500 \times 75 =$ 37,500 ft.-lb. Ans.

(*b*) Using formula **18**, $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 75}{32.16}} = 2.1597$ sec. $= .035995$ min., the time of falling.

$$\frac{37{,}500}{33{,}000 \times .035995} = 31.57 \text{ H. P.} \quad \text{Ans.}$$

(437) $127 \div 62.5 = 2.032 =$ specific gravity. Ans.

(438) $\frac{62.5}{1{,}728} \times 9.823 = .35529$ lb. Ans.

(439) Use formula **21.** $W = \left(\frac{2PR}{R-r}\right)$, or $\frac{2 \times 60 \times 6.5}{6.5 - 5.75} \times .48 = 499.2$ lb. Ans. See Fig. 37.

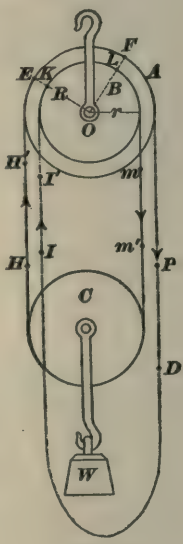

FIG. 37.

(440) See Art. **961.**

$F \times \left(\frac{3}{8} \div 12\right) = \frac{Wv^2}{2g} = \frac{1.5 \times 25^2}{2 \times 32.16}$, or

$F = \frac{\frac{1.5 \times 25^2}{2 \times 32.16}}{\frac{3}{8} \div 12} = 466.42$ lb. Ans.

(441) (*a*) $2{,}000 \div 4 = 500 =$ wt. of cu. ft. $500 \div 62.5 = 8 =$ specific gravity. Ans.

(*b*) $\frac{500}{1{,}728} = .28935$ lb. Ans.

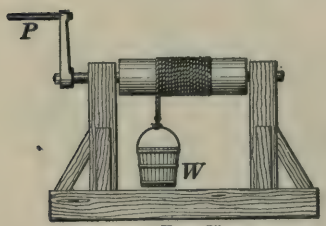

FIG. 38.

(442) See Fig. 38. $14.5 \times 2 = 29$. $30 \times 29 = W \times 5$, or $W = \frac{30 \times 29}{5} = 174$ lb. Ans.

(443) $75 \times .21 = 15.75$ lb. Ans.

(444) (*a*) $900 \times 150 = 135{,}000$ ft.-lb. Ans.

$\frac{135{,}000}{15} = 9{,}000$ ft.-lb. per min. Ans.

(*b*) $\frac{9{,}000}{33{,}000} = \frac{3}{11}$ H.P. Ans.

(445) $900 \times .18 \times 2 = 324$ lb. = force required to overcome the friction. $900 + 324 = 1{,}224$ lb. = total force.

$$\frac{1{,}224 \times 150}{15 \times 33{,}000} = .37091 \text{ H. P.} \quad \text{Ans.}$$

(446) $18 \div 88 = .2045$. Ans.

(447) See Art. **962.** $D = \dfrac{W}{gV} = \dfrac{1{,}200}{32.16 \times 3} = 12.438$. Ans.

(448) See Fig. 39. $125 - 47.5 = 77.5$ lb. = downward pressure.

$77.5 \div 4 = 19.375$ lb. = pressure on each support. Ans.

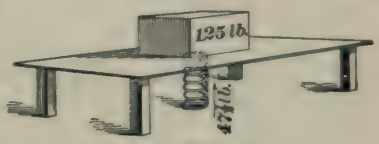

FIG. 39.

(449) See Fig. 40.

FIG. 40.

(450) See Fig. 41. $4.5 \div 2 = 2.25$.

$$\frac{12}{2.25} \times 6 \times 30 = 960 \text{ lb.} \quad \text{Ans.}$$

(451) (*a*) $960 \div 30 = 32$. Ans.

(*b*) $790 \div 960 = .8229 = 82.29\%$. Ans.

(452) (*a*) See Fig. 42. $475 + (475 \times .24) = 589$ lb. Ans.

(*b*) $475 \div 589 = .8064 = 80.64\%$. Ans.

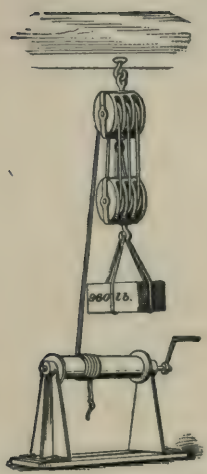

FIG. 41.

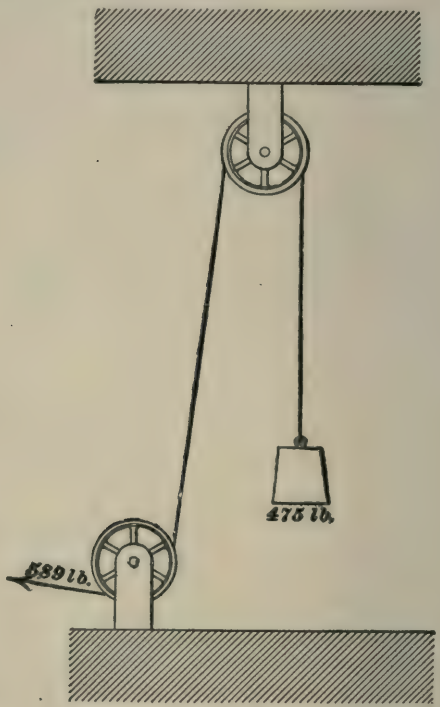

FIG. 42.

(453) (*a*) By formula **23,** $U = FS = 6 \times 25 = 150$ foot-pounds. Ans.

(*b*) $2\frac{1}{2}$ sec. $= \frac{2\frac{1}{2}}{60} = \frac{1}{24}$ min.

Using formula **24,** Power $= \frac{FS}{T} = \frac{150}{\frac{1}{24}} = 3{,}600$ ft.-lb. per min. Ans.

HYDROMECHANICS.

(QUESTIONS 454–503.)

(454) The area of the surface of the sphere is $20 \times 20 \times 3.1416 = 1{,}256.64$ sq. in. (See rule, Art. **817.**)

The specific gravity of sea water is 1.026. (See tables of Specific Gravity.) The pressure on the sphere per square inch is the weight of a column of water 1 sq. in. in cross-section and 2 miles long. The total pressure is, therefore,

$1{,}256.64 \times 5{,}280 \times 2 \times .434 \times 1.026 = 5{,}908{,}971$ lb. Ans.

(455) $125 - 83.5 = 41.5$ lb. = loss of weight in water = weight of a volume of water equal to the volume of the sphere. (See Art. **987.**) 1 cu. in. of water weighs .03617 lb.; hence, 41.5 lb. of water must contain $41.5 \div .03617 = 1{,}147.4$ cu. in. = volume of the sphere. Ans.

(456) $$Q = \frac{225{,}000}{60 \times 60} = 62.5 \text{ gal. per sec.}$$

Substituting the values given in formula **51,**

$$d = 1.229 \sqrt[5]{\frac{2{,}800 \times 62.5^2}{26}} = 16.38'' +.$$

Substituting this value of d in formula **49,**

$$v_m = \frac{24.51 \times 62.5}{16.38^2} = 5.7095.$$

The value of f (from the table) corresponding to $v_m = 5.7095$ is .0216, using but four decimal places. Hence, applying formula **52,**

$$d = 2.57 \sqrt[5]{\frac{(.0216 \times 2{,}800 + \frac{1}{8} \times 16.38)62.5^2}{26}} = 16''. \text{ Ans.}$$

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(457) (*a*) Area of piston $= \left(\frac{7}{8}\right)^2 \times .7854 = .6013$ sq. in.

Pressure per square inch exerted by piston $= \frac{50}{.6013} =$ 83.15 lb.

A column of water 1 foot high and of 1 sq. in. cross-section weighs .434 pound, and therefore exerts a pressure of .434 lb. per sq. in. The height of a column of water to exert a pressure of 83.15 lb. per sq. in. must be $\frac{83.15}{.434} = 191.6$ ft. Consequently, the water will rise 191.6 ft. Ans.

The diameter of the hole in the squirt gun has nothing to do with the height of the water, since the pressure per square inch will remain the same, no matter what the diameter may be.

(*b*) Using formula **34,** range $= \sqrt{4\,h\,y}$, we have

$$\sqrt{4\,h\,y} = \sqrt{4 \times 10 \times 191.6} = 87.54 \text{ ft.} \quad \text{Ans.}$$

(458) Use formulas **44** and **43.**

(*b*) $$Q_a = .41\,b\sqrt{2g}\left[\sqrt{h^3} - \sqrt{h_1^3}\right] =$$

$$.41 \times \frac{30}{12} \times \sqrt{2 \times 32.16}\left[\sqrt{\left(5\tfrac{1}{2}\right)^3} - \sqrt{\left(3\tfrac{1}{2}\right)^3}\right] =$$

52.21 cu. ft. per sec. Ans.

(*a*) Area of weir $= b\,d = 2.5 \times 2 = 5$ sq. ft.

Using formula **43,**

$$v_m = \frac{Q_a}{b\,d} = \frac{52.21}{5} = 10.44 \text{ ft. per sec.} \quad \text{Ans.}$$

(*c*) To obtain the discharge in gallons per hour (*b*) multiply by 60×60 (seconds in an hour) and by 7.48 (gallons in a cu. ft.). Thus $52.21 \times 60 \times 60 \times 7.48 = 1{,}405{,}910.9$ gal. per hour. Ans.

(459) First find the coefficient of friction by using formula **46,** and the table of coefficients of friction.

$$v_m = 2.315\sqrt{\frac{h\,d}{f\,l}} = 2.315\sqrt{\frac{76 \times 7.5}{.025 \times 12{,}000}} = 3.191 \text{ ft. per sec.}$$

In the table $f = .0243$ for $v_m = 3$ and .023 for $v_m = 4$; the difference is .0013. $3.191 - 3 = .191$. Then, $1 : .191 :: .0013 : x$, or $x = .0002$. Therefore, $.0243 - .0002 = .0241 = f$. Use formula **50**; substitute in it the value of f here found, and multiply by 60 to get the discharge per minute.

$$Q = .09445\, d^2 \sqrt{\frac{h\,d}{f\,l}} \times 60 =$$

$$.09445 \times 7.5^2 \sqrt{\frac{76 \times 7.5}{.0241 \times 12{,}000}} \times 60 = 447.6 \text{ gal. per min.}$$

NOTE.—It will be noticed that the term $\frac{1}{3}d$, in formula **50**, has been omitted. This was done because the length of the pipe exceeded 10,000 times its diameter.

(460) (*a*) Use formula **46.**

$$v_m = 2.315 \times \sqrt{\frac{h\,d}{f\,l}} \times 60 =$$

$$2.315 \sqrt{\frac{76 \times 7.5}{.0241 \times 12{,}000}} \times 60 = 195 \text{ ft. per min.} \quad \text{Ans.}$$

(*b*) 447.6 gal. per min. $\div 60 = 7.46$ gal. per sec. $= 1$ cu. ft. per sec., nearly. Ans.

(461) See Art. **1005.**

$v = \sqrt{2\,g\,h} = \sqrt{2 \times 32.16 \times 10} = 25.36$ ft. per sec. Ans.

(462) Use formulas **49** and **47.**

$$v_m = \frac{24.51\,Q}{d^2} = \frac{24.51 \times 42{,}000}{6.5^2 \times 60 \times 60} = 6.768 \text{ ft. per sec.}$$

$$h = \frac{f\,l\,v_m^2}{5.36\,d} + .0233\,v_m^2 =$$

$$\frac{.021 \times 1{,}500 \times 6.768^2}{5.36 \times 6.5} + .0233 \times 6.768^2 = 42.48 \text{ ft.} \quad \text{Ans.}$$

(463) (*b*) Area of top or bottom of cylinder equals $20^2 \times .7854 = 314.16$ sq. in. Area of cross-section of pipe $= \left(\frac{3}{8}\right)^2 \times .7854 = .1104$ sq. in. 25 lb. 10 oz. $= 25.625$ lb. $25.625 \div .1104 = 232.11$ lb., pressure per square inch on top or bottom exerted by the weight and piston.

Pressure due to a head of 10 ft. $= .434 \times 10 = 4.34$ lb. per sq. in.

Pressure due to a head of 13 ft. $= .434 \times 13 = 5.64$ lb. per sq. in.

(Since a column of water 1 ft. high exerts a pressure of .434 lb. per sq. in.)

Pressure on the top = pressure due to weight + pressure due to head of 10 ft. $= 232.11 + 4.34 = 236.45$ lb. per sq. in. Ans.

(*a*) Pressure on bottom = pressure due to weight + pressure due to head of 13 ft. $= 232.11 + 5.64 = 237.75$ lb. per sq. in. Ans.

(*c*) Total pressure, or equivalent weight on the bottom $= 237.75 \times 314.16 = 74{,}691.54$ lb. Ans.

(464) $.434 \times 1\frac{1}{2} = .651$ lb., pressure due to the head of water in the cylinder at the center of the orifice.

236.45, pressure per square inch on top, $+ .651 = 237.101$, total pressure per square inch. Area of orifice $= 1^2 \times .7854 = .7854$ sq. in.

$$.7854 \times 237.101 = 186.22 \text{ lb.} \quad \text{Ans.}$$

(465) (*a*) Use formulas **28** and **27.** Sp.Gr. $= \frac{w}{(W - W') - (W_1 - W_2)} = \frac{11.25}{(91.25 - 41) - (16 \times 5 - 3\frac{1}{8} \times 16)} = .556$. Ans.

(*b*) Sp. Gr. $= \frac{W}{W - W'} = \frac{80}{(16 \times 5) - (3\frac{1}{8} \times 16)} = 2.667$. Ans.

(466) First find the coefficient of friction. Formula **46** gives

$$v_m = 2.315\sqrt{\frac{h\,d}{f\,l}} = 2.315\sqrt{\frac{120 \times 4}{.025 \times 4{,}000}} = 5.0719 \text{ ft. per sec.}$$

From the table in Art. **1033,** $f = .0230$ for $v_m = 4$, and .0214 for $v_m = 6$. $.0230 - .0214 = .0016$. $5.0719 - 4 = 1.0719$. $2 : 1.0719 :: .0016 : x$, or $x = .0009$. $.0230 - .0009 = .0221 = f$ for $v_m = 5.0719$. Hence,

$$v_m = 2.315\sqrt{\frac{120 \times 4}{.0221 \times 4{,}000}} = 5.4 \text{ ft. per sec.} \quad \text{Ans.}$$

(467) Use formulas **46** and **45.**

$$v_m = 2.315\sqrt{\frac{120 \times 4}{.025 \times 2{,}000}} = 7.1728 \text{ ft. per sec.}$$

From the table in Art. **1033,** $f = .0214$ for $v_m = 6$, and .0205 for $v_m = 8$. $8 - 6 = 2$.

$.0214 - .0205 = .0009$. $7.1728 - 6 = 1.1728$.

$2 : 1.1728 :: .0009 : x$, or $x = .0005$.

$.0214 - .0005 = .0209 = f$ for $v_m = 7.1728$.

Hence, the velocity of discharge =

$$v_m = 2.315\sqrt{\frac{120 \times 4}{.0209 \times 2{,}000 + \frac{1}{8} \times 4}} = 7.8 \text{ ft. per sec.} \quad \text{Ans.}$$

(468) (*a*) See Fig. 43. Area of cylinder $= 19^2 \times .7854$; pressure, 90 pounds per sq. in. Hence, the total pressure on the piston is $19^2 \times .7854 \times 90 = 25{,}517.6$ lb. = the load that can be lifted. Ans.

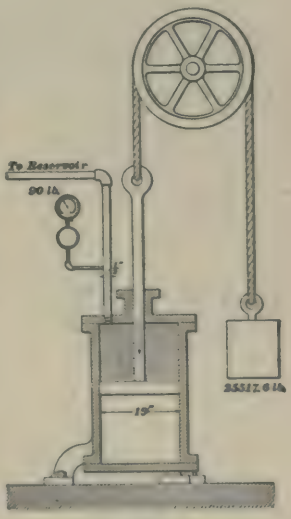

FIG. 43.

(*b*) The diameter of the pipe has no effect on the load which can be lifted, except that a larger pipe will lift the load faster, since more water will flow in during a given time.

(469) (*a*) $f = .0205$ for $v_m = 8$. Therefore, using formula **47,**

$$h = \frac{.0205 \times 5{,}280 \times 8^2}{5.36 \times 10} + .0233 \times 8^2 = 130.73 \text{ ft.} \quad \text{Ans.}$$

(*b*) Using formula **48,** $Q = .0408\, d^2\, v_m = .0408 \times 10^2 \times 8 = 32.64$ gal. per sec. $32.64 \times 60 \times 60 = 117{,}504$ gal. per hour. Ans.

(470) A column of water 1 in. square and 2.304 ft. high weighs 1 lb.; hence, to produce a pressure of 30 lb. per sq. in. would require a column of water $2.304 \times 30 = 69.12$ ft. high = head. Using formula **36,**

$$v = .98\sqrt{2gh} = .98\sqrt{2 \times 32.16 \times 69.12} = 65.34 \text{ ft. per sec.} \quad \text{Ans.}$$

(471) (*a*) 36 in. = 3 ft. A column of water 1 in. square and 1 ft. high weighs .434 lb. $.434 \times 43 = 18.662$ lb. per sq. in., pressure on the bottom of the cylinder. $.434 \times 40 = 17.36$ lb. per sq. in., pressure on the top of the cylinder. Area of base of cylinder $= 20^2 \times .7854 = 314.16$ sq. in. $314.16 \times 18.662 = 5{,}862.85$ lb., total pressure on the bottom. Ans

(*b*) $314.16 \times 17.36 = 5{,}453.82$ lb., total pressure on the top. Ans.

(472) 2 lb. = 32 oz. $32 - 10 = 22$ oz. = loss of weight of the bottle in water. $32 + 16 = 48$ = weight of bottle and sugar in air. $48 - 16 = 32$ oz. = loss of weight of bottle and sugar in water. $32 - 22 = 10$ oz. = loss of weight of sugar in water = weight of a volume of water equal to the volume of the sugar. Then, by formula **27,**

$$\text{specific gravity} = \frac{W}{W - W'} = \frac{16}{10} = 1.6. \quad \text{Ans.}$$

(473) $33 = \sqrt{2gh}$ (see Art. **1005**), or

$$h = \frac{33^2}{64.32} = 16.931 \text{ ft. per sec.} \quad \text{Ans.}$$

(474) (*a*) Use formula **42,** and multiply by $7.48 \times 60 \times 60$ to reduce cu. ft. per sec. to gal. per hour.

$$Q_a = .41 \times \frac{21}{12} \times \sqrt{2 \times 32.16 \times \left(\frac{15}{12}\right)^3} \times 7.48 \times 60 \times 60 =$$

216,551 gal. per hr. Ans.

(*b*) By formula **43,**

$$v_m = \frac{Q_a}{b\,d} = \frac{.41 \times \frac{21}{12} \times \sqrt{2 \times 32.16 \times \left(\frac{15}{12}\right)^3}}{\frac{21}{12} \times \frac{15}{12}} = 3.676$$

ft. per sec. Ans.

(475) $f = .0193$ for $v_m = 12$. Therefore, using formula **47,**

$$h = \frac{f\,l\,v_m^2}{5.36\,d} + .0233\,v_m^2 =$$

$$\frac{.0193 \times 6{,}000 \times 12^2}{5.36 \times 3} + .0233 \times 12^2 = 1{,}040.37 \text{ ft.} \quad \text{Ans}$$

(476) (*a*) 1 cu. in. of water weighs .03617 lb.

$.03617 \times 40 = 1.4468$ lb. = weight of 40 cu. in. of water = loss of weight of lead in water.

$16.4 - 1.447 = 14.953$ lb. weight of lead in water. Ans.

(*b*) $16.4 \div 40 = .41$ = weight of 1 cu. in. of the lead.

$16.4 - 2 = 14.4$ lb. = weight of lead after cutting off 2 lb.

$14.4 \div .41 = 35.122$ cu. in. = volume of lead after cutting off 2 lb.

(477) (*a*) See Fig. 44. $13.5 \times 9 \times .7854 = 95.4261$ sq. in., area of base.

$.03617 \times 20 = .7234$ lb. per sq. in., pressure on the base due to the water only.

$12 + .7234 = 12.7234$ lb., total pressure per square inch on base.

$12.7234 \times 95.4261 = 1{,}214.144$ lb. Ans.

(*b*) $47 \times 12 = 564$ lb., total upward pressure. Ans.

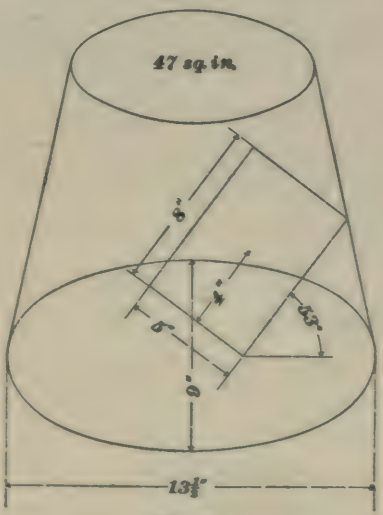

FIG. 44.

(478) (*a*) See Fig. 44. $4 \sin 53° = 3.195''$, nearly.

$20 - 3.195 = 16.805''$ = distance of center of gravity of plate below the surface.

$.03617 \times 16.805 + 12 = 12.60784$ lb. per sq. in. = perpendicular pressure against the plate. $5 \times 8 = 40$ sq. in., area of plate.

$12.60784 \times 40 = 504.314$ lb. = perpendicular pressure on plate. Ans.

(*b*) $504.314 \sin 53° = 402.76$ lb. = horizontal pressure on plate. Ans.

(*c*) $504.314 \cos 53° = 303.5$ lb. = vertical pressure on plate. Ans.

(479) $\frac{5^2 \times .7854}{144}$ = area of pipe in square feet. Using formula **31,**

$Q = A\, v_m = \frac{5^2 \times .7854}{144} \times 7.2$ = discharge in cu. ft. per sec.

$\frac{5^2 \times .7854}{144} \times 7.2 \times 7.48$ = discharge in gal. per sec.

$\frac{5^2 \times .7854}{144} \times 7.2 \times 7.48 \times 60 \times 60 \times 24 = 634{,}478$ gal. discharged in one day. Ans.

(480) 38,000 gallons per hour $\frac{38{,}000}{60 \times 60}$ gal. per sec. $= Q$.

Using formula **49,**

$$v_m = \frac{24.51\, Q}{d^2} = \frac{24.51 \times 38{,}000}{5.5^2 \times 60 \times 60} = 8.5526 \text{ ft. per sec.} \quad \text{Ans.}$$

(481) Area of $2\frac{1}{2}$-in. circle $= 4.9087$ sq. in.; area of a 2-in. circle $= 3.1416$ sq. in. $(4.9087 - 3.1416) \times 12 = 21.2052$ cu. in. of brass.

$21.2052 \times .03617 = .767$ lb. = weight of an equal volume of water.

6 lb. 5 oz. $= 6.3125$ lb. $6.3125 \div .767 = 8.23$ Sp. Gr. of brass. Ans.

(482) (*b*) A column of water 1 ft. high and 1 in. square weighs .434 lb. $.434 \times 180 = 78.12$ lb. per sq. in. Ans.

(*a*) Projected area of 1 foot of pipe $= 6 \times 12 = 72$ sq. in. (See Art. **985.**) $72 \times 78.12 = 5{,}624.64$ lb., nearly. Ans.

(483) Use formula **42.**

(*a*) $Q_a = .41\, b\sqrt{2\, g\, d^3} = .41 \times \frac{27}{12} \times \sqrt{2 \times 32.16 \times \left(\frac{36}{12}\right)^3} =$ 38.44 cu. ft. per sec. Ans.

(*b*) $Q = \frac{Q_a}{.615} = \frac{38.44}{.615} = 62.5$ cu. ft. per sec. Ans.

(484) (*a*) Area of pipe : area of orifice :: 6^2 : 1.5^2; or, area of pipe is 16 times as large as area of orifice. Hence, using formula **35,**

$$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}} = \sqrt{\frac{2 \times 32.16 \times 45}{1 - \frac{(1.5^2 \times .7854)^2}{(6^2 \times .7854)^2}}} = 53.9 \text{ ft. per sec. Ans.}$$

(*b*) $2.304 \times 10 = 23.04$ ft. = height of column of water which will give a pressure of 10 lb. per sq. in. $45 + 23.04 = 68.04$ ft.

$$v = \sqrt{\frac{2 \times 32.16 \times 68.04}{1 - \frac{(1.5^2 \times .7854)^2}{(6^2 \times .7854)^2}}} = 66.28 \text{ ft. per sec. Ans.}$$

(485) Use formula **48.**

$Q = .0408\, d^2 v_m = .0408 \times 6^2 \times 7.5 = 11.016$ gal. per sec. Ans.

(486) $14^2 \times .7854 \times 27 =$ volume of cylinder = volume of water displaced.

$14^2 \times .7854 \times 27 \times .03617 = 150$ lb., nearly, = weight of water displaced.

$(14^2 - 13^2) \times .7854 \times 27 =$ volume of the cylinder walls.

$13^2 \times .7854 \times \frac{1}{4} \times 2 =$ volume of the cylinder ends.

.261 lb. = weight of a cubic inch of cast iron, then,

$[(14^2 - 13^2) \times .7854 \times 27 + 13^2 \times .7854 \times \frac{1}{4} \times 2] \times .261 =$ 167 lb., nearly, = weight of cylinder. Since weight of cylinder is greater than the weight of the water displaced, it will sink. Ans.

(487) 2 lb. − 1 lb. 5 oz. = 11 oz., weight of water.

1 lb. 15.34 oz. − 1 lb. 5 oz. = 10.34 oz., weight of oil.

$10.34 \div 11 = .94 =$ Sp. Gr. of oil. Ans.

(488) Head $= 41 \div .434 = 94.47$ ft. Using formula **36,** $v = .98\sqrt{2gh} = .98\sqrt{2 \times 32.16 \times 94.47} = 76.39$ ft. per sec. Ans. This is not the mean velocity, v_m.

(489) (*a*) Use formula **39.**

$$Q_a = .815\, A \sqrt{2gh}, \text{ or}$$

$$Q_a = .815 \times \frac{1.5^2 \times .7854}{144} \times \sqrt{2 \times 32.16 \times 94.47} \times 60 =$$

46.77 cu. ft. per min. Ans.

(*b*) See Art. **1005.**

The theoretical velocity of discharge is $v = \sqrt{2\,g\,h}$, and as $h = 41 \div .434 = 94.47$ ft., we have $v = \sqrt{2 \times 32.16 \times 94.47} = 77.95$ ft. per sec.

Using formula **31,** $Q = A\,v$, and multiplying by 60 to reduce the discharge from cu. ft. per sec. to cu. ft. per min., we have

$$Q = \frac{1.5^2 \times .7854}{144} \times 77.95 \times 60 = 57.39 \text{ cu. ft. per min.} \quad \text{Ans.}$$

(*c*) $$\frac{Q_a}{Q} = \frac{46.77}{57.39} = .815. \quad \text{Ans.}$$

(490) (*b*) Use formulas **31** and **38.**

$$Q = A\,v = \frac{1.5^2 \times .7854}{144} \times 77.95 = .9566 \text{ cu. ft. per sec.} \quad \text{Ans.}$$

(*a*) $Q_a = .615\,Q = .615 \times .9566 = .5883$ cu. ft. per sec. Ans.

(*c*) $$\frac{Q_a}{Q} = \frac{.5883}{.9566} = .615. \quad \text{Ans.}$$

(491) (*a*) $9 \times 5 \times .7854 = 35.343$ sq. in. = area of base.

$2^2 \times .7854 = 3.1416$ sq. in. = area of hole.

$\frac{3.1416}{35.343} = \frac{1}{11.25}$; hence, the area of the base is less than 20 times the area of the orifice, and formula **35** must be used.

$$v = \sqrt{\frac{2\,g\,h}{1 - \frac{a^2}{A^2}}} = \sqrt{\frac{2 \times 32.16 \times 6}{1 - \frac{(2^2 \times .7854)^2}{(9 \times 5 \times .7854)^2}}} = 19.722 \text{ ft. per sec.} \quad \text{Ans.}$$

(*b*) $.434 \times 6 = 2.604$, or say 2.6 lb. per sq. in. Ans.

(492) $6 \times 4 \times .7854 = 18.85$ sq. in. = area of upper surface.

$15^2 \times .7854 = 176.715$ sq. in. = area of base.

$\frac{132}{18.85} \times 176.715 = 1{,}237.5$ lb., pressure due to weight on upper surface.

$.03617 \times 24 \times 176.715 = 153.4$ lb., pressure due to water in vessel.

$1{,}237.5 + 153.4 = 1{,}390.9$ lb., total pressure. Ans.

(493) Use formula **31** or **32.**

Divide by 60×60 to get the discharge in gallons per second, and by 7.48 to get the discharge in cubic feet per second.

$$\text{Area in sq. ft.} = \frac{4^2 \times .7854}{144}.$$

$$v_m = \frac{Q}{A} = \frac{12{,}000 \times 144}{60 \times 60 \times 7.48 \times 4^2 \times .7854} = 5.106 \text{ ft. per sec.}$$

Ans.

(494) A sketch of the arrangement is shown in Fig. 45.

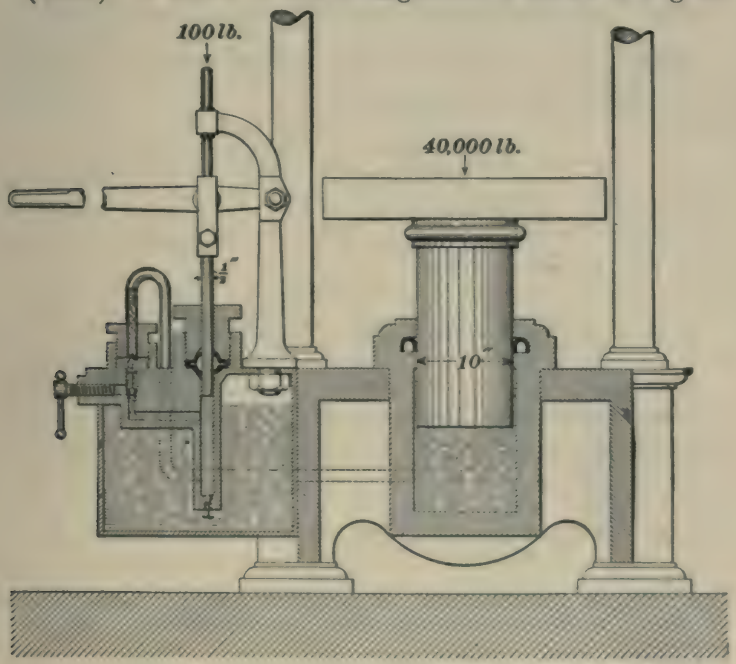

FIG. 45.

(*a*) Area of pump piston $= \left(\frac{1}{2}\right)^2 \times .7854 = .19635$ sq. in.

Area of plunger $= 10^2 \times .7854 = 78.54$ sq. in.

Pressure per square inch exerted by piston $= \frac{100}{.19635}$ lb.

Hence, according to Pascal's law, the pressure on the plunger is $\frac{100}{.19635} \times 78.54 = 40{,}000$ lb. Ans.

(b) Velocity ratio $= 1\frac{1}{2} : .00375 = 400 : 1$. Ans.

(c) According to the principle given in Art. **981,** $P \times 1\frac{1}{2}$ inches $= W \times$ distance moved by plunger, or $100 \times 1\frac{1}{2} = 40{,}000 \times$ required distance; hence, the required distance $= \frac{100 \times 1\frac{1}{2}}{40{,}000} = .00375$ in. Ans.

(495) (a) Use formula **44,** and multiply by 7.48 and 60 to reduce the discharge from cu. ft. per sec. to gal. per min.

$$Q_a = .41\, b\sqrt{2g}\,[\sqrt{h^3} - \sqrt{h_1^3}] \times 60 \times 7.48 =$$

$$.41 \times \frac{14}{12} \times \sqrt{64.32}\left[\sqrt{\left(9 + \frac{20}{12}\right)^3} - \sqrt{9^3}\right] \times 60 \times 7.48 =$$

13,502 gal. Ans.

(b) In the second case,

$$Q_a = .41 \times \frac{20}{12} \times \sqrt{64.32}\left[\sqrt{\left(9 + \frac{14}{12}\right)^3} - \sqrt{9^3}\right] \times 60 \times 7.48 =$$

13,323 gal. Ans.

(496) (a) Area of weir $= 14 \times 20 \div 144$ sq. ft. Use formula **32,** and divide by 60×7.48 to reduce gal. per min. to cu. ft. per sec.

$$v_m = \frac{Q}{A} = \frac{13{,}502 \times 144}{60 \times 7.48 \times 14 \times 20} = 15.47 \text{ ft. per sec.} \quad \text{Ans.}$$

(b) $v_m = \frac{13{,}323 \times 144}{60 \times 7.48 \times 14 \times 20} = 15.27$ ft. per sec. Ans.

(497) (a) See Art. **997.**

$W = 2$ lb. $8\frac{1}{2}$ oz. $= 40.5$ oz.
$w = 12$ oz.
$W' = 1$ lb. 11 oz. $= 27$ oz.

By formula **30,**

$$\text{Sp. Gr.} = \frac{W - w}{W' - w} = \frac{40.5 - 12}{27 - 12} = \frac{28.5}{15} = 1.9. \quad \text{Ans.}$$

(*b*) 15 oz. $= \frac{15}{16}$ lb. $= .9375$ lb. $.9375 \div .03617 = 25.92$ cu. in. = volume of water = volume of slate. Therefore, the volume of the slate = 25.92 cu. in. Ans.

(498) In Art. **1019** it is stated that the theoretical mean velocity is $\frac{2}{3}\sqrt{2gh}$. Hence, $v_m = \frac{2}{3}\sqrt{2 \times 32.16 \times 3} =$ 9.26 ft. per sec. Ans.

(499) (*a*) 4 ft. 9 in. = 4.75 ft. $19 - 4.75 = 14.25$.
Range $= \sqrt{4hy} = \sqrt{4 \times 4.75 \times 14.25} = 16.454$ ft. Ans.

(*b*) $19 - 4.75 = 14.25$ ft. Ans.

(*c*) $19 \div 2 = 9.5$. Greatest range $= \sqrt{4 \times 9.5^2} = 19$ ft. Ans. (See Art. **1009.**)

(500) Use formulas **46** and **50.**

$$v_m = 2.315\sqrt{\frac{hd}{fl}} = 2.315\sqrt{\frac{25 \times 5}{.025 \times 1{,}300}} = 4.5397 \text{ ft. per sec.}$$

From the table, $f = .0230$ for $v_m = 4$ and .0214 for $v_m = 6$. $.0230 - .0214 = .0016$. $6 - 4 = 2$.

$4.5397 - 4 = .5397$. Then, $2 : .5397 :: .0016 : x$, or $x = .0004$. Hence, $.0230 - .0004 = .0226 = f$ for $v_m = 4.5397$.

$$Q = 60 \times 60 \times .09445 \times 5^2 \times \sqrt{\frac{25 \times 5}{.0226 \times 1{,}300 + \frac{1}{8} \times 5}} =$$

17,350 gal. per hr. Ans.

(501) Obtain the values by approximating to those given in Art. **1033.** Thus, for $v_m = 2$, $f = .0265$; for $v_m = 3$, $f = .0243$; $.0265 - .0243 = .0022$. $2.37 - 2 = .37$. Hence, $1 : 37 :: .0022 : x$, or $x = .0008$. Then, $.0265 - .0008 = .0257 = f$ for $v_m = 2.37$. Ans.

For $v_m = 3$, $f = .0243$; for $v_m = 4$, $f = .0230$; $.0243 - .0230 = .0013$. $3.19 - 3 = .19$. Hence, $1 : .19 :: .0013 : x$, or $x = 0002$. Then, $.0243 - .0002 = .0241 = f$ for $v_m = 3.19$. Ans.

For $v_m = 4, f = .0230$; for $v_m = 6, f = .0214$; $.0230 - .0214 = .0016$. $5.8 - 4 = 1.8$. $6 - 4 = 2$. Hence, $2 : 1.8 :: .0016 : x$, or $x = .0014$. Then, $.0230 - .0014 = .0216 = f$ for $v_m = 5.8$. Ans.

For $v_m = 6$, $f = .0214$; for $v_m = 8$, $f = .0205$; $.0214 - .0205 = .0009$. $7.4 - 6 = 1.4$. $8 - 6 = 2$. Hence, $2 : 1.4 = .0009 : x$, or $x = .0006$. Then, $.0214 - .0006 = .0208 = f$ for $v_m = 7.4$. Ans.

For $v_m = 8, f = .0205$; for $v_m = 12, f = .0193$; $.0205 - .0193 = .0012$. $9.83 - 8 = 1.83$. $12 - 8 = 4$. Hence, $4 : 1.83 :: .0012 : x$, or $x = .0005$. Then, $.0205 - .0005 = .02 = f$ for $v_m = 9.83$. Ans.

For $v_m = 8$, $f = .0205$; for $v_m = 12$, $f = .0193$; $.0205 - .0193 = .0012$. $11.5 - 8 = 3.5$. $12 - 8 = 4$. Hence, $4 : 3.5 :: .0012 : x$, or $x = .0011$. $.0205 - .0011 = .0194 = f$ for $v_m = 11.5$. Ans.

(502) Specific gravity of sea-water is 1.026. Total area of cube $= 10.5^2 \times 6 = 661.5$ sq. in. 1 mile $= 5{,}280$ ft. Hence, total pressure on the cube $= 661.5 \times 5{,}280 \times 3.5 \times .434 \times 1.026 = 5{,}443{,}383$ lb. Ans.

(503) $19^2 \times .7854 \times 80 = 22{,}682$ lb. Ans.

PNEUMATICS.

(QUESTIONS 504–553.)

(504) The force with which a confined gas presses against the walls of the vessel which contains it.

(505) (*a*) $4 \times 12 \times .49 = 23.52$ lb. per sq. in. Ans.

(*b*) $23.52 \div 14.7 = 1.6$ atmospheres. Ans.

(506) (*a*) A column of water 1 foot high exerts a pressure of .434 lb. per sq. in. Hence, $.434 \times 19 = 8.246$ lb. per sq. in., the required tension. A column of mercury 1 in. high exerts a pressure of .49 lb. per sq. in. Hence, $8.246 \div .49 = 16.828$ in. = height of the mercury column. Ans.

(*b*) Pressure above the mercury $= 14.7 - 8.246 = 6.454$ lb. per sq. in. Ans.

(507) Using formula **53,** $p_1 = \frac{p v}{v_1} = \frac{(14.7 \times 3) \times 1}{2.5} =$ 17.64 lb. Ans.

(508) (*c*) Using formula **61,**

$$V = \frac{.37052\ W\ T}{p} = \frac{.37052 \times 7.14 \times 535}{22.05} = 64.188 \text{ cu. ft. Ans.}$$

($T = 460° + 75° = 535°$, and $p = 14.7 \times 1.5 = 22.05$ lb. per sq. in.)

(*a*) $7.14 \div .08 = 89.25$ cu. ft., the original volume. Ans.

(*b*) If 1 cu. ft. weighs .08 lb., 1 lb. contains $1 \div .08 = 12.5$ cu. ft. Hence, using formula **60,** $p\ V = .37052\ T$, or

$$T = \frac{p\ V}{.37052} = \frac{22.05 \times 12.5}{.37052} = 743.887°. \quad 743.887 - 460 = 283.887°. \text{ Ans.}$$

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(509) Substituting in formula **59,** $p = 40$, $t = 120$, and $t_1 = 55$,

$$p_1 = 40\left(\frac{460 + 55}{460 + 120}\right) = \frac{40 \times 515}{580} = 35.517 \text{ lb.} \quad \text{Ans.}$$

(510) Using formula **61,** $pV = .37052\, WT$, or

$$W = \frac{pV}{.37052\, T}. \quad T = 460° + 60° = 520°.$$

Therefore, $W = \dfrac{14.7 \times 1}{.37052 \times 520} = .076296$ lb. Ans.

(511) $175{,}000 \div 144 =$ pounds per sq. in.

$(175{,}000 \div 144) \div 14.7 = 82.672 =$ atmospheres. Ans.

(512) Extending formula **63** to include 3 gases, we have $PV = p_1v_1 + p_2v_2 + p_3v_3$, or $40 \times P = 1 \times 12 + 2 \times 10 + 3 \times 8$.

Hence, $P = \dfrac{56}{40} = 1.4$ atmos. $= 1.4 \times 14.7 = 20.58$ lb. per sq. in. Ans.

(513) In the last example, $PV = 56$. In the present case, $P = \dfrac{23}{14.7}$ atmos. Therefore, $V = \dfrac{56}{P} = \dfrac{56}{\frac{23}{14.7}} = 35.79$ cu. ft. Ans.

(514) For $t = 280°$, $T = 740°$; for $t = 77°$, $T = 537°$.

$pV = .37052\, WT$, or $W = \dfrac{pV}{.37052\, T}$. (Formula **61.**)

$$\text{Weight of hot air} = \frac{14.7 \times 10{,}000}{.37052 \times 740} = 536.13 \text{ lb.}$$

$$\text{Weight of air displaced} = \frac{14.7 \times 10{,}000}{.37052 \times 537} = 738.81 \text{ lb.}$$

$738.81 - 536.13 = 202.68$. $202.68 - 100 = 102.68$ lb. Ans.

(515) According to formula **64,**

$$PV = \left(\frac{p_1v_1}{T_1} + \frac{p_2v_2}{T_2}\right)T, \text{ or}$$

$$20 \times 31 = \left(\frac{14.7 \times 13}{533} + \frac{30 \times 18}{513}\right)T = 1.411168\, T.$$

Therefore, $T = \frac{20 \times 31}{1.411168} = 439.35°$. Since this is less than 460°, the temperature is $460 - 439.35 = 20.65°$ below zero, or $-20.65°$. Ans.

(516) A hollow space from which all air or other gas (or gaseous vapor) has been removed. An example would be the space above the mercury in a barometer.

(517) One inch of mercury corresponds to a pressure of .49 lb. per sq. in.

$\frac{1}{40}$ inch of mercury corresponds to a pressure of $\frac{.49}{40}$ lb. per sq. in. $\frac{.49}{40} \times 144 = 1.764$ lb. per sq. ft. Ans.

(518) (*a*) $325 \times .14 = 45.5$ lb. = force necessary to overcome the friction. $6 \times 12 = 72'' =$ length of cylinder. $72 - 40 = 32 =$ distance which the piston must move. Since the area of the cylinder remains the same, any variation in the volume will be proportional to the variation in the length between the head and piston. By formula **53**, $p v = p_1 v_1$. Therefore, $p = \frac{p_1 v_1}{v} = \frac{14.7 \times 40}{72} = 8.1\frac{2}{3}$ lb. per sq. in. = pressure when piston is at the end of the cylinder. Since there is the atmospheric pressure of 14.7 lb. on one side of the piston and only $8.1\frac{2}{3}$ lb. on the other side, the force required to pull it out of the cylinder is $14.7 - 8.1\frac{2}{3} = 6.5\frac{1}{3}$ lb. per sq. in. Area of piston $= 40^2 \times .7854 = 1{,}256.64$ sq. in. Total force $= 1{,}256.64 \times 6.5\frac{1}{3} = 8{,}210.05$. Adding the friction, $8{,}210.05 + 45.5 = 8{,}255.55$ lb. Ans.

(*b*) Proceeding likewise in the second case, $p v = p_1 v_1$, or $p = \frac{p_1 v_1}{v} = \frac{14.7 \times 40}{6} = 98$ lb. $98 - 14.7 = 83.3$ lb. per sq. in.

$1{,}256.64 \times 83.3 + 45.5 = 104{,}723.612$ lb. Ans.

(519) 8.47 = original volume $= v_1$. $8.47 - 4.5 = 3.97$ cu. ft. = new volume $= v$. By formula **53,**

$$p_1 = \frac{p\,v}{v_1} = \frac{3.97 \times 38}{8.47} = 17.812 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(520) Original weight $= W = .5$ lb. $= 8$ oz.; new weight $= W_1 = 1$ lb. 6 oz. $= 22$ oz. According to formula **56,**

$p\,W_1 = p_1\,W$, or $p_1 = \frac{P\,W_1}{W} = \frac{14.7 \times 22}{8} = 40.425$ lb. per sq. in. Ans.

(521) Applying formula **58,**

$$v_1 = v\left(\frac{460 + t_1}{460 + t}\right) = \frac{4,516}{1,728}\left(\frac{460 + 80}{460 + 260}\right) = 1.96 \text{ cu. ft.} \quad \text{Ans.}$$

(522) According to formula **61,** $p\,V = .37052\,W\,T$, or

$$W = \frac{p\,V}{.37052\,T} = \frac{14.7 \times 1.25 \times 55}{.37052 \times 548} = 4.977 \text{ lb.} \quad \text{Ans.}$$

(523) Using formula **63,** $P\,V = p\,v + p_1\,v_1$, or $P \times 7.5 = 14.7 \times 2 \times 7.5 + 40 \times 7.5$, or $P = 69.4$ lb. per sq. in. Ans.

(524) 48″, 36″, and 24″ = 4′, 3′, and 2′, respectively. Hence, $4 \times 3 \times 2 = 24$ cu. ft. = the volume of the block. The block will weigh as much more in a vacuum as the weight of the air it displaces. In example 510, it was found that 1 cu. ft. of air at a temperature of 60° weighed .076296 lb. $.076296 \times 24 + 1,200 = 1,201.83$ lb. Ans.

(525) (*a*) (See Art. **1088.**) $127 + 16 = 143$.

$$\frac{\left(\frac{9}{12}\right)^2 \times .7854 \times 1 \times 125 \times 62.5 \times 143}{33,000} = 14.9563 \text{ H.P.}$$

$$14.9563 \div .75 = 19.942 \text{ H.P.} \quad \text{Ans.}$$

(*b*) Discharge in gallons per hour = volume of cylinder in cu. ft. × number of strokes per minute × 7.48 × 60 =

$$\left(\frac{9}{12}\right)^2 \times .7854 \times 125 \times 7.48 \times 60 = 24,784.3 \text{ gal. per hr.} \quad \text{Ans.}$$

(526) In this example, the number of times that the pump delivers water in 1 minute is $100 \div 2 = 50$; in the last example, 125. Hence, the number of gallons discharged per hour in this case will be $24,786 \times \frac{50}{125} = 9,914.4$ gal. Ans.

(527) See Art. **1043.**

Pressure in condenser $= \frac{30 - 23}{30} \times 14.7 = 3.43$ lb. per sq. in. Ans.

(528) $144 \times 14.7 = 2{,}116.8$ lb. per sq. ft. Ans.

(529) $.27 \div 3 = .09$ = weight of 1 cu. ft. Using formula **56,**

$p\ W_1 = p_1\ W$, or $30\ W_1 = 65 \times .09$. $W_1 = .195$ lb. Ans.

(530) Using formula **61,**

$p\ V = .37052\ W\ T$, or $30 \times 1 = .37052 \times .09 \times T$.

$T = \frac{30}{.37052 \times .09} = 899.6°$. $899.6° - 460° = 439.6°$. Ans.

(531) $460° + 32° = 492°$; $460° + 212° = 672°$; $460° + 62° = 522°$, and $460° + (-40°) = 420°$.

(532) Using formula **61,** $p\ V = .37052\ W\ T$, and substituting,

$(14.7 \times 10) \times 4 = .37052 \times 3.5 \times T$, or $T = \frac{14.7 \times 10 \times 4}{.37052 \times 3.5} = 453.417°$. $453.417 - 460° = -6.583°$. Ans.

(533) Using formula **63,** $V\ P = v\ p + v_1\ p_1$, we find

$P = \frac{15 \times 63 + 19 \times 14.7 \times 3}{25} = 71.316$ lb. Ans.

(534) Using formula **60,** $p\ V = .37052\ T$, or $P = \frac{.37052 \times 540}{10} = 20$ lb. per sq. in., nearly. Ans.

(535) One inch of mercury represents a pressure of .49 lb. Therefore, the height of the mercury column is $12.5 \div .49 = 25.51$ in. Ans.

(536) Thirty inches of mercury corresponds to 34 ft. of water. (See Art. **1043.**) Therefore,

$30'' : 34$ ft. $:: 27'' : x$ ft., or $x = 30.6$ ft. Ans.

A more accurate way is $(27 \times .49) \div .434 = 30.5$ ft.

(537) (*a*) $30 - 17.5 = 12.5$ in. = original tension of gas in inches of mercury. $30 - 5 = 25$ in. = new tension in inches of mercury.

$VP = vp + v_1 p_1$ (formula **63**), or $6.7 \times 25 = 6.7 \times 12.5 + v_1 \times 30$.

$$v_1 = \frac{6.7 \times 25 - 6.7 \times 12.5}{30} = 2.79\frac{1}{6} \text{ cu. ft.} \quad \text{Ans.}$$

(*b*) To produce a vacuum of 0 inches,

$$v_1 = \frac{6.7 \times 30 - 6.7 \times 12.5}{30} = 3.908 \text{ cu. ft.} \quad \text{Ans.}$$

(538) $11 + 25 = 36$, final volume of gas. $2.4 \div 36 = \frac{1}{15}$ lb. Ans.

(539) Using formula **59,**

$$p_1 = p\left(\frac{460 + t_1}{460 + t}\right) = 12 \times \left(\frac{460 + 300}{460 + 60}\right) = 17.54 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(540) $T = 460 + 212 = 672°$. Using formula **61,** $pV = .37052\ WT$, we have $14.7 \times 1 = .37052 \times W \times 672$, or $W = \frac{14.7}{.37052 \times 672} = .059039$ lb. Ans.

(541) (*a*) $\frac{20^2 \times .7854 \times 32}{1,728} = 5.8178$ cu. ft. = volume of cylinder.

$32 - 26 = 6$ in., length of stroke unfinished.

$$5.8178 \times \frac{6}{32} = 1.0908 \text{ cu. ft.} \quad \text{Ans.}$$

(*b*) By formula **61,** taking the values of p, V, and T at the beginning of stroke,

$pV = .37052\ WT$, or $W = \frac{pV}{.37052\ T} = \frac{14.7 \times 5.8178}{.37052 \times 535} = .43143$ lb. Ans.

(*c*) Now, substituting in formula **61** the values of V, W, and T at time of discharge,

$$p = \frac{.37052\ WT}{V} = \frac{.37052 \times .43143 \times 585}{1.0908} = 85.727 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(542) Using formula **63,** $VP = vp + v_1 p_1$, or $30 \times 35 = 19 \times 12 + 21\, p_1$, or $p_1 = \dfrac{30 \times 35 - 19 \times 12}{21} = 39.14$ lb. per sq. in. Ans.

(543) Use formula **64.** $PV = \left(\dfrac{p_1 v_1}{T_1} + \dfrac{p_2 v_2}{T_2}\right) T.$

$$T = 460 + 72 = 532.$$

Therefore, $P = \dfrac{\left(\dfrac{13 \times 45}{520} + \dfrac{17 \times 60}{540}\right) 532}{60} = 26.723$ lb. per sq. in. Ans.

(544) See Art. **1043.** Since the barometer column stands at 30 inches for a perfect vacuum, the height when the air is admitted will be $30'' - 4\frac{1}{2}'' = 25\frac{1}{2}''$. Ans.

(545) Sp. Gr. of alcohol = .8. Therefore, $16 \times .434 \times .8 =$ pressure exerted by the column of alcohol. $\dfrac{16 \times .434 \times .8}{.49} =$ 11.337 in. = height of a column of mercury that will give the same pressure as 16 ft. of alcohol = number of inches shown by the gauge. Ans.

(546) (*a*) $14.7 + 9 = 23.7$ lb. per sq. in. Using formula **53,**

$$v_1 = \frac{p\, v}{p_1} = \frac{14.7 \times 80}{23.7} = 49.62 \text{ in.} =$$

distance between piston and end of stroke. Since the area of the piston remains constant, the volume at any point of the stroke is proportional to the distance passed over by the piston. Hence, we may use the latter for the former in the formula. $80 - 49.62 = 30.38$ in. Ans.

(*b*) Area of piston $= 80^2 \times .7854$. The volume of air at point of discharge is $80^2 \times .7854 \times 49.62$ cu. in. =

$$\frac{80^2 \times .7854 \times 49.62}{1{,}728} = 144.34 \text{ cu. ft.} \quad \text{Ans.}$$

(547) Using formula **56,** $p\, W_1 = p_1 W$, or $3.5 \times 14.7 \times 2 = p_1 \times 13$; hence, $p_1 = \dfrac{14.7 \times 3.5 \times 2}{13} = 7.915+$ lb. per sq. in. Ans.

(548) $60'' - 50'' = 10''$. Since the volumes are proportional to the lengths of the spaces between the piston and the end of the stroke, we may apply formula **62,**

$$\frac{p\,V}{T} = \frac{p_1\,V_1}{T_1}\text{; or } \frac{14.7 \times 60}{460 + 60} = \frac{p_1 \times 10}{460 + 130}.$$

Therefore, $p_1 = \dfrac{14.7 \times 60 \times 590}{520 \times 10} = 100.07$ lb. per sq. in. Ans.

(549) $T = 127° + 460° = 587°$. Using formula **60,** $p\,V = .37052\ T$, or $V = \dfrac{.37052 \times 587}{27} = 8.055$ cu. ft. Ans.

(550) $T = 100° + 460° = 560°$.

Substituting in formula **61,** $p\,V = .37052\ W\,T$, or

$$V = \frac{.37052\ W\,T}{p} = \frac{.37052 \times .5 \times 560}{\frac{4{,}000}{144}} = 3.735 \text{ cu. ft.} \quad \text{Ans.}$$

(551) Use formula **64.** $PV = \left(\dfrac{p_1 v_1}{T_1} + \dfrac{p_2 v_2}{T_2}\right) T$.

$T = 110° + 460° = 570°$; $T_1 = 100° + 460° = 560°$; $T_2 = 130° + 460° = 590°$.

Therefore, $V = \dfrac{\left(\dfrac{90 \times 40}{560} + \dfrac{80 \times 57}{590}\right) 570}{120} = 67.248$ cu. ft. Ans.

(552) The pressure exerted by squeezing the bulb may be found from formula **53,** in which p is 14.7, v, the original volume = 20 cu. in., and v_1, the new volume, = 5 cu. in. $p_1 = \dfrac{p\,v}{v_1} = \dfrac{14.7 \times 20}{5} = 58.8$ lb. The pressure due to the atmosphere must be deducted, since there is an equal pressure on the outside which balances it. $58.8 - 14.7 = 44.1$ lb. per sq. in. = pressure due to squeezing the bulb. $3^2 \times .7854 = 7.0686$ sq. in. = area of bottom. $7.0686 \times 44.1 = 311.725$ lb. $7.0686 \times .434 = 3.068$ lb. = pressure due to weight of water. $311.725 + 3.068 = 314.793$ lb. Ans.

(553) Use formula **58.**

$$v_1 = v\left(\frac{460 + t_1}{460 + t}\right) = 4\left(\frac{460 + 115}{460 + 40}\right) = 4.6 \text{ cu. ft.} \quad \text{Ans.}$$

HEAT.

(QUESTIONS 554–613.)

(554) 9 hr. 25 min. = 565 min.

$55 \div 2 = 27.5$ min. = time it would take to heat 1 lb. of the substance to this temperature under the same conditions. $27.5 \div 565 = .04867$ = specific heat of the substance. Ans.

(555) Increase in diameter $= 4.001 - 3.9985 = .0025''$. $C_1 = .00000599$, from Table 19. Hence, using formula **67**, $t = \frac{1}{L C_1} = \frac{.0025}{3.9985 \times .00000599} = 104.4°$, nearly. Then, $104.4 + 80 = 184.4°$, nearly. Ans.

(556) (*a*) Substituting in formula **72** $W = 360$, $t_1 - t = 104.4$, and $s = .1165$; $n = W\ (t_1 - t)\ s = 360 \times 104.4 \times .1165 = 4{,}378.536$ B. T. U.

Since 12% of the total heat is lost by radiation, the above number of B. T. U. must be the number remaining after subtracting the heat units lost by radiation from the total number; or it is $100 - 12 = 88\%$ of the total number. Hence, dividing by .88 we obtain $4{,}378.536 \div .88 = 4{,}975.6$ B. T. U., total heat required. Ans.

(*b*) $4{,}975.6 \times 778 = 3{,}871{,}017$ ft.-lb. Ans.

(557) (*a*) $80^2 \times .7854 = 5{,}026.56$ sq. in. = area of piston.

$p\ v = p_1\ v_1$. At the beginning of the stroke $p = 14.7$, and the volume may be represented by the length of the cylinder, 80 in.; at the point of discharge p_1 is 120, therefore, $80 \times 14.7 = 120 \times v$, or $v = 9.8$ in., the portion of the stroke uncompleted at discharge.

$\frac{5{,}026.56 \times 9.8}{1{,}728} = 28.507$ cu. ft., volume of air discharged. Ans.

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(b) By formula **80**, $L = 331.5744\, p\, V \log \frac{p_1}{p} = 331.5744 \times 120 \times 28.507 \log \frac{120}{14.7} = 331.5744 \times 120 \times 28.507 \times .91186 =$ 1,034,289 ft.-lb. Ans.

(558) (a) Volume of cylinder = area of piston × length of stroke $= \frac{5,026.56}{144} \times \frac{80}{12} = 232.71$ cu. ft., nearly.

From formula **81**, $p_1 v_1^{1.41} = p_2 v_2^{1.41}$, or $v_2 = \sqrt[1.41]{\frac{p_1 v_1^{1.41}}{p_2}}$.

Substituting for p_1 and v_1 their values at the beginning of the stroke, and for p_2 its value at discharge, we have

$$v_2 = \sqrt[1.41]{\frac{14.7 \times 232.71^{1.41}}{120}};$$

$$\log v_2 = \frac{\log 14.7 + 1.41 \log 232.71 - \log 120}{1.41}, \text{ or}$$

$$\log v_2 = \frac{1.16732 + 1.41 \times 2.36682 - 2.07918}{1.41} = 1.72011.$$

$v_2 = 52.494$ cu. ft., the volume discharged. Ans.

(b) To find the work, use formula **84**,

$$L = 351.36\, p\, V \left[1 - \left(\frac{V}{V_1}\right)^{.41}\right] =$$

$$351.36 \times 14.7 \times 232.71 \times \left[1 - \left(\frac{232.71}{52.494}\right)^{.41}\right].$$

$$\text{Log} \left(\frac{232.71}{52.494}\right)^{.41} = .41 (\log 232.71 - \log 52.494) =$$

$.41 (2.36682 - 1.72011) = .26515$. Therefore, $\left(\frac{232.71}{52.494}\right)^{.41} =$ 1.8414.

$1 - 1.8414 = -.8414$, the minus sign indicating that the work done by the air is negative; that is, that work is done upon the air instead of by it, or that the air is compressed.

$L = 351.36 \times 14.7 \times 232.71 \times .8414 =$ 1,011,317 ft.-lb. Ans.

(559) See Arts. **1090** and **1135.**

(560) Using formula **65,**

(*a*) $t_f = \frac{9}{5} \times 2{,}917 + 32 = 5{,}282.6°$ F. Ans.

(*b*) $t_f = \frac{9}{5} \times 637 + 32 = 1{,}178.6$ F. Ans.

(*c*) $t_f = \frac{9}{5} \times (-260) + 32 = -436°$ F. Ans.

(561) See Table 21, Art. **1134,** and Table 22, Art. **1141.**

$-50 - (-37.8) \times 12 \times .0333 = 4.875$ = units of heat necessary to raise the temperature to the fusion point. $5.09 \times 12 = 61.08$ B. T. U. required to melt the mercury. $[662 - (-37.8)] \times .0333 \times 12 = 279.64$ B. T. U. required to raise the temperature to the point of vaporization. $157 \times 12 = 1{,}884$ B. T. U. required for vaporizing it. $4{,}875 + 61.08 + 279.64 + 1{,}884 = 2{,}229.595$ B. T. U. Ans.

(562) Using formula **71,** $pV = WRT$, or

$$W = \frac{pv}{RT} = \frac{18 \times 100}{5.34946 \times 540} = .6231 \text{ lb.} \quad \text{Ans.}$$

(563) See Tables 21 and 22 for specific heat and temperature of fusion of copper. 7 lb. 5 oz. = 7.3125 lb. $2{,}100° - 78° = 2{,}022°$. $2{,}022 \times 7.3125 \times .0951 = 1{,}406.13$ B. T. U. Ans.

(564) By formula **73,** $t = \dfrac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3} =$

$$\frac{2.25 \times .0314 \times 40 + 4 \times 65 + 1.75 \times .1298 \times 62}{2.25 \times .0314 + 4 + 1.75 \times .1298} = 64.43°. \quad \text{Ans.}$$

(565) Use formulas **69, 67,** and **68.** (*a*) $v = VC_3t = 4 \times 4 \times 20 \times 1{,}200 \times .00002058 = 7.903$ cu. in. Ans.

(*b*) $l = LC_1t = 20 \times 1{,}200 \times .00000686 = .16464$ in. Ans.

(*c*) $a = AC_2t = 4 \times 4 \times 1{,}200 \times .00001372 = .2634$ sq. in. Ans.

(566) Using formula **71,**

$$pV = WRT, \text{ or } T = \frac{pV}{WR} = \frac{73 \times 61}{10 \times .38143} = 1{,}167.45°.$$

$1{,}167.45° - 460° = 707.45°$. **Ans.**

(567) See Art. **1124.**

(568) By formula **73**, $t = \dfrac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3}$, or

$$t_1 = \frac{(W_1 s_1 + W_2 s_2 + W_3 s_3)\, t - W_2 s_2 t_2 - W_3 s_3 t_3}{W_1 s_1} =$$

$$\frac{(1 \times .1298 + 3.25 \times 1 + 1.5 \times .0951)\, 128 - 3.25 \times 85 - 1.5 \times .0951 \times 85}{1 \times .1298} =$$

1,252°. Ans.

(569) Draw the lines $O\ X$ and $O\ Y$ (Fig. 46) perpendicular to each other. On $O\ X$ take any convenient distance, say $\frac{1}{2}$ inch, to represent one unit of volume (1 cu. ft.), and on $O\ Y$ take a distance of say 3 inches to represent the required pressure of 84 pounds. Our scale of pressure is,

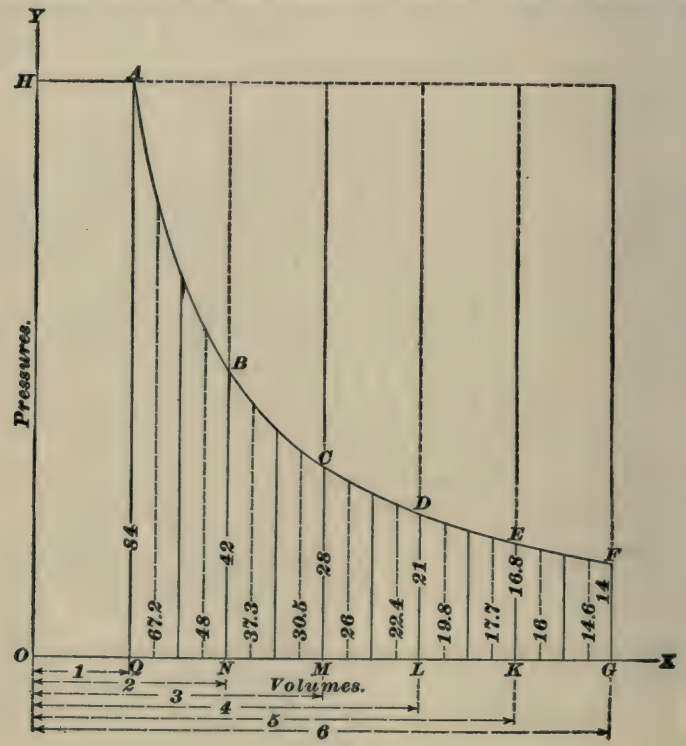

FIG. 46.

then, 28 pounds to the inch. The ordinates are now erected as shown in Art. **1156.** Thus, at A the volume is 1, and

the pressure 84 pounds; hence, $p\,v = 84$. But $p\,v = p_1\,v_1 = p_2\,v_2$, etc., $= 84$; hence, $p_1 = \frac{84}{v_1}$, $p_2 = \frac{84}{v_2}$, etc. When $v = 1$, $p = \frac{84}{1} = 84 = Q\,A$.

When $v = 2$, $p = \frac{84}{2} = 42 = N\,B$.

When $v = 3$, $p = \frac{84}{3} = 28 = M\,C$.

When $v = 4$, $p = \frac{84}{4} = 21 = L\,D$.

When $v = 5$, $p = \frac{84}{5} = 16.8 = K\,E$.

When $v = 6$, $p = \frac{84}{6} = 14 = G\,F$.

A curve through A, B, C, D, E, and F will be the one required.

(570) Divide the space $Q\,G$ into 10 equal parts, as shown in the figure, and draw the ordinate at the middle of each space. The average ordinate will be found to be 29.95 lb. The volume represented by the length $Q\,G$ is 5 cu. ft. Hence, the work is $29.95 \times 5 \times 144 = 21{,}564$ foot-pounds. Ans.

Calculating the work by formula **79**$_1$,

$$L = 331.5744\,p\,V \log \frac{V}{V_1};$$

$$L = 331.5744 \times 84 \times 1 \times \log \frac{6}{1} = 21{,}673 \text{ ft.-lb.} \quad \text{Ans.}$$

(571) (*a*) $460 + 96 = 556°$ F. (*b*) $273\frac{1}{3} + 32 = 305\frac{1}{3}°$ C. (*c*) $273\frac{1}{3} + 180 = 453\frac{1}{3}°$ C. (*d*) $460 + 650 = 1{,}110°$ F. (*e*) $273\frac{1}{3} - 40 = 233\frac{1}{3}°$ C.

(572) $32° - 20° = 12°$. $12 \times 7 \times .504 = 42.336$ B. T. U. required to heat the ice to 32°. $142.65 \times 7 = 998.55$ B. T. U. required to melt the ice. $212° - 32° = 180°$. $180 \times 7 \times 1 = 1{,}260$ B. T. U. required to heat the *water* to 212°. $966.6 \times 7 = 6{,}766.2$ B. T. U. required to change 7 lb. of water at 212° into steam at 212°. $42.336 + 998.55 + 1{,}260 + 6{,}766.2 = 9{,}067.086$ B. T. U.

$$\frac{9{,}067.086 \times 778}{43 \times 33{,}000} = 4.971 \text{ horsepower.} \quad \text{Ans.}$$

(573) Using formula **84,** and substituting the values of p and V at beginning of compression, we have

$$L = 351.36\, p\, V\left[1 - \left(\frac{V}{V_1}\right)^{.41}\right] =$$

$$351.36 \times 14.7 \times 1\left[1 - \left(\frac{1}{.25}\right)^{.41}\right].$$

$$\text{Log}\left(\frac{1}{.25}\right)^{.41} = \log 4^{.41} = .41 \log 4 = .41 \times .60206 = .24684.$$

The number whose logarithm is .24684 is 1.7654.

Therefore, $\left(\frac{1}{.25}\right)^{.41} = 1.7654$. $1 - \left(\frac{1}{.25}\right)^{.41} = 1 - 1.7654 = -.7654$, the minus sign indicating that the air is compressed. Hence, $L = 351.36 \times 14.7 \times .7654 = 3{,}953.28$ ft.-lb. Ans.

(574) On account of the great irregularity in outline of this figure, a division into ten parts will not give a sufficiently close approximation to the mean ordinate; hence, it is divided into 20 equal parts, as shown by the full lines.

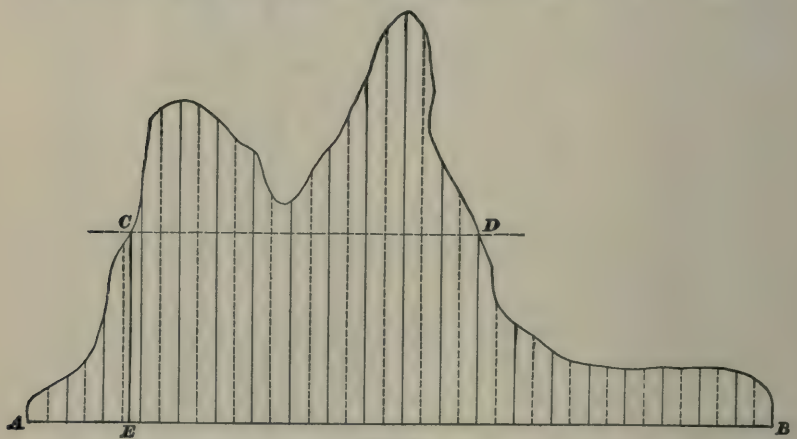

FIG. 47.

(See Fig. 47.) The dotted lines, situated midway between the full lines, represent the ordinates which are to be measured. The sum of all the dotted lines, divided by the number of divisions, gives .9365″ for the mean ordinate. Draw

the line $C\ D$ parallel to $A\ B$ at a distance from $A\ B$ equal to .9365″, and where it cuts the curve will be the points from which to draw the mean ordinate, as $C\ E$.

(575) Prolong $A\ B$ in both directions as shown in Fig. 48. Draw the tangents $h\ i$, $f\ A$, $e\ d$, and $c\ b$, perpendicular to $A\ B$. Since the outline $f\ A\ g$ is very nearly triangular, and is quite small compared with the rest of the figure, consider it a triangle, and draw $m\ n$ half way

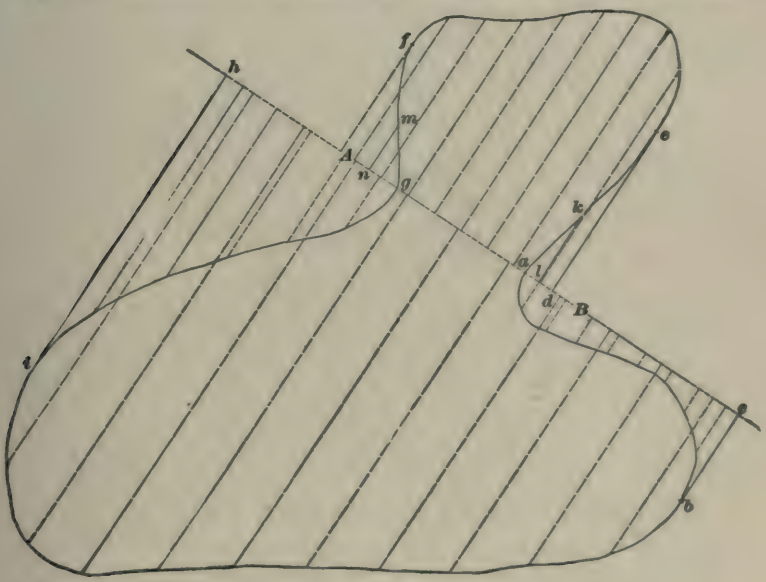

FIG. 48.

between A and g. Consider $m\ n$ as the mean ordinate. Then, $A\ g \times m\ n = .34 \times .28 = .0952$ sq. in. In a similar manner, area of $a\ d\ e = .17 \times .4 = .068$ sq. in. Dividing $A\ f\ e\ d$ into 8 equal parts, and drawing the ordinates at the middle points of these divisions (for convenience the full lines, similar to those in the last figure, have been omitted, and the ordinates at the middle points only have been drawn), the mean ordinate is found to be 1.228″. The length $A\ d = 1.31''$; hence, area of $A\ f\ e\ d = 1.31 \times 1.228 = 1.6087$ sq. in. Dividing $a\ c\ b$ into 8 equal parts, the mean

ordinate is found to be .154; the length $a\,c$ is 1.34; hence, area $a\,c\,b = 1.34 \times .154 = .2064$ sq. in. Dividing $h\,g\,i$ into 5 equal parts, the mean ordinate is found to be .82″; the length $h\,g$ is 1.07″; hence, area $h\,g\,i = 1.07 \times .82 = .8774$ sq. in. Dividing $h\,i\,b\,c$ into 10 equal parts, the mean ordinate is found to be 1.925″; its length $h\,c = 3.21$″; hence, area $h\,i\,b\,c = 3.21 \times 1.925 = 6.179$ sq. in.

$1.6087 + 6.179 = 7.7877$ sq in. = area $h\,i\,b\,c\,d\,e\,f\,A\,h$.

$.0952 + .068 + .2064 + .8774 = 1.247$ sq. in.

$7.7877 - 1.247 = 6.5407$ sq. in. = required area $g\,i\,b\,a\,e\,f\,g$. Ans.

(576) Using formula **73**, $t = \dfrac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3}$.

For water, $s_1 = 1$; therefore, $W_1 t + (W_2 s_2 + W_3 s_3)\, t = W_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3$, and

$$W_1 = \frac{W_2 s_2 t_2 + W_3 s_3 t_3 - t(W_2 s_2 + W_3 s_3)}{t - t}.$$

Substituting the given values for W and t and the values of s_2, s_3, etc., from Table 21, we have

$$W_1 = \frac{4 \times .426 \times 80 + .5 \times .0939 \times 73 - 75.61(4 \times .426 + .5 \times .0939)}{75.61 - 73}$$

$$= \frac{7.35802}{2.61} = 2.819 \text{ lb.} \quad \text{Ans.}$$

(577) See Arts. **1138, 1139, 1131, 1132,** and **1152.**

(578) $K = \dfrac{Wv^2}{2g} = \dfrac{120 \times 1{,}200^2}{2 \times 32.16}$ = kinetic energy in foot-pounds.

(See Art. **957.**) $\dfrac{120 \times 1{,}200^2}{2 \times 32.16} \times .15$ = kinetic energy expended in heat. Now, dividing by 778 to reduce the foot-pounds to B.T.U., $\dfrac{120 \times 1200^2 \times .15}{2 \times 32.16 \times 778} = 517.975$ B.T.U. Ans.

(579) See Art. **1126.**

(580) Using formula **73**, $t = \dfrac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3}$.

(Since s_2, the specific heat of water, is 1, it may be left out.)

Transposing,

$$s_1 = \frac{W_2(t_2 - t) + W_3 s_3(t_3 - t)}{W_1(t - t_1)} = \frac{1.875(91-86)+1.25\times.0562\times(91-86)}{5(86-40)} =$$

.0423. Ans.

(See example, Art. **1137.**)

(581) $T_1 = 460° + 450° = 910°$; $T_2 = 460° + 70° = 530°$.

$$\text{Efficiency} = \frac{T_1 - T_2}{T_1} = \frac{910 - 530}{910} = 41.76\%. \quad \text{Ans.}$$

(582) Use formula **71**, $p V = W R T$. $T = 460° + 200 = 660°$; and R, from Table 20, is 5.34946; therefore, $W = \frac{p V}{R T} = \frac{20 \times 700}{5.34946 \times 660} = 3.9653$ lb. Ans.

(583) (See Arts. **1091** to **1098.**)

(*a*) $C : R = 100 : 80$, or $C = \frac{100}{80} R = \frac{5}{4} R$.

Hence, $44 \times \frac{5}{4} = 55°$ C. **Ans.**

(*b*) $F : R = 180 : 80$, or $F = \frac{180}{80} R = \frac{9}{4} R$.

Hence, $44 \times \frac{9}{4} + 32 = 131°$ F. Ans.

(584) By formula **81**, $p_1 v_1^{1.41} = p_2 v_2^{1.41}$. The volume of 1 lb. of air at atmospheric pressure, and having a temperature of 60°, is $\frac{1}{.076296} = 13.107$ cu. ft. (See question 510.)

Substituting in the above formula, $14.7 \times 13.107^{1.41} = 235 \times v_2^{1.41}$, or

$$v_2 = \sqrt[1.41]{\frac{14.7 \times 13.107^{1.41}}{235}} = 1.8356 \text{ cu. ft.} \quad \text{Ans.}$$

From formula **71**, remembering that $W = 1$, we have $p V = R T$, or $T = \frac{p V}{R} = \frac{235 \times 1.8356}{.37052} = 1{,}164.2°$. $1{,}164.2° - 460° = 704.2°$. Ans.

(585) (See Arts. **1107, 1054,** and **1099.**)

(586) (*a*) Cubical contents of cylinder $= (10^2 - \overline{9\frac{1}{4}}^2) \times .7854 \times 72 = 816.48$ cu. in. $= \frac{816.48}{1,728}$ cu. ft. Specific gravity of copper is 8.79. Then, the weight of the cylinder is $\frac{816.48}{1,728} \times 62.5 \times 8.79 = 259.58$ lb. Specific heat of copper, from Table 21, is .0951. Substituting the values of n, W, and s, in formula **72**, $n = W(t_1 - t)s$, we have $7,000 = 259.58\,(t_1 - t) \times .0951$, or $t_1 - t = \frac{7,000}{259.58 \times .0951} = 283.56° =$ increase of temperature.

By formula **67**, $l = L\ C_1\ t = 72 \times .00000955 \times 283.56 =$ .195 in. Ans.

(*b*) By formula **69**, $v = V\ C_3\ t = 816.48 \times .00002864 \times 283.56 = 6.63$ cu. in. Ans.

(*c*) By formula **67**, $l = L\ C_1\ t = 10 \times .00000955 \times 283.56 = .027$ in. Ans.

(587) From Table 19, the coefficient of expansion for gases is .00203252. Substituting in formula **70**,

$V_2 = \left[\frac{1 + C_3(t_2 - 32)}{1 + C_3(t_1 - 32)}\right] V_1 = \left[\frac{1 + .00203252(390 - 32)}{1 + .00203252(65 - 32)}\right] \times 12 = 19.428$ cu. ft. $19.428 - 12 = 7.428$ cu. ft. Ans.

The same answer may be obtained by using formula **58.**

(588) Foot-pounds per minute $= 75 \times 2 \times 20 \times 2 \times .18$.
Foot-pounds per hour $= 75 \times 2 \times 2 \times 20 \times .18 \times 60$.

Heat units per hour $= \frac{75 \times 2 \times 2 \times 20 \times .18 \times 60}{778} = 83.29$ B. T. U. Ans.

(589) (*a*) Using formula **87**, $T_1 = T\left(\frac{V}{V_1}\right)^{.41} = 610 \times \left(\frac{1}{4.5}\right)^{.41} = -130.76°$. Ans.

(*b*) Substituting in formula **71**, the initial values of W, R, T, and V, we obtain $p = \frac{WRT}{V} = \frac{.5 \times .37052 \times 610}{.9} =$ 125.565 lb. per sq. in., the initial pressure. Ans.

(*c*) Using formula **86**,

$$p_1 = p\left(\frac{T_1}{T}\right)^{\frac{\gamma}{\gamma-1}} = 125.565 \times \left(\frac{329.24}{610}\right)^{\frac{1.3}{1.3-1}} = 15.06 \text{ lb. per sq. in.}$$

Ans.

(590) $12' \times .5236 = 904.78$ cu. in. = volume of sphere. Sp. Gr. of zinc = 6.86; therefore, $\frac{904.78}{1,728} \times 62.5 \times 6.86 =$ 224.5 lb. = weight of sphere. (See example 586.) Using formula **73**, $t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2}{W_1 s_1 + W_2 s_2} = \frac{8 \times 212 + 224.5 \times .0956 \times 70}{8 + 224.5 \times .0956}$ $= 108.49°$. $108.49° - 70° = 38.49°$ = increase in temperature of the zinc sphere after dipping in the boiling water.

Using formula **69**, $v = V C_3 t = 904.78 \times .00004903 \times 38.49 = 1.71$ cu. in., nearly. Ans.

(591) By formula **67**, $l = L\ C_1\ t$. $C_1 = .00000599$; $L = 900 \times 12$; $t = 90° - 28°$; therefore, $1 = 900 \times 12 \times .00000599 \times (90 - 28) = 4.01$ in. Ans.

(592) According to formula **72**, $n = W(t_1 - t)s$, or $s = \frac{n}{W(t_1 - t)} = \frac{n}{W} = \frac{5}{26} = .1923$. Ans.

(593) See Art. **1161.**

(594) (*a*) Temperature of vaporization of sulphur, from Table 22, is 228.3°.

Specific heat of sulphur, from Table 21, is .2026.

Latent heat of fusion of sulphur, from Table 22, is 13.26.

$228.3 - 40 = 188.3$. $13 \times .2026 \times 188.3 = 495.94454$ B. T. U. to raise sulphur to melting point. $13.26 \times 13 = 172.38$ B. T. U. to melt the sulphur. $(495.94454 + 172.38) \times 778 \times 519{,}956.5$ ft.-lb., total work required to perform both of the above operations. Ans.

(*b*) $\frac{519{,}956.5}{10 \times 33{,}000} = 1.5756$ horsepower. Ans.

(595) $124 \times 3 = 372$ B. T. U., heat given up by the turpentine in changing from gas to liquid. $75 \times 4 = 300$ B. T. U., heat contained in the water. $300 + 372 = 672$

B. T. U., heat contained in the mixture at the instant the turpentine has become liquid. $672 \div 4 = 168°$ = temperature of water at the instant that the turpentine has become a liquid.

Applying formula **73**, $t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2}{W_1 s_1 + W_2 s_2} =$

$\frac{.3 \times .426 \times 313 + 168 \times 4}{3 \times .426 + 4} = 203.11°$, final temperature. Ans.

(596) Specific heat of melted lead, from Table 21, = .0402.

Specific heat of solid lead, from Table 21, = .0314.

Temperature of fusion of lead, from Table 22, = 626°.

Latent heat of fusion of lead, from Table 22, = 9.67.

$626° - 46° = 580°$. $800° - 626° = 174°$.

$25 \times .0314 \times 580 = 455.3$ B. T. U., heat required to raise the lead to melting point. $9.67 \times 25 = 241.75$ B. T. U., heat required to melt the lead. $25 \times .0402 \times 174 = 174.87$ B. T. U., heat required to raise the temperature of the melted lead from fusion point to 800°. $455.3 + 241.75 + 174.87 = 871.92$ B. T. U., total heat required. $871.92 \times 778 = 678{,}353.76$ ft.-lb. Ans.

(597) $32° - 10° = 22°$. $22 \times 10 \times .504 = 110.88$ B. T. U., heat required to raise ice from 10° to 32°. Latent heat of fusion of ice, from Table 22, is 142.65; then $142.65 \times 10 = 1{,}426.5$ B. T. U., heat units to melt the ice. $1{,}426.5 + 110.88 = 1{,}537.38$ B. T. U., total heat units to be taken from the mixture to melt the ice. Hence, applying formula **73**,

$$t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 t_3 + W_4 t_4 + W_5 s_5 t_5 - 1{,}537.38}{W_1 s_1 + W_2 s_2 + W_3 + W_4 + W_5 s_5} =$$

$$\frac{11.5 \times .1298 \times 180 + 42 \times .0939 \times 240 + 10 \times 32 + 50 \times 120 + 20 \times .0314 \times 80 - 1{,}537.38}{11.5 \times .1298 + 42 \times .0939 + 10 + 50 + 20 \times .0314}$$

$= 91.55°$. Ans.

(598) (*a*) $p_1 v_1^{1.41} = p_2 v_2^{1.41}$, or $p_2 = \frac{140 \times 3^{1.41}}{16^{1.41}} = 13.215$

$$\text{Area} = \frac{p_1 v_1 - p_2 v_2}{.41} = \frac{140 \times 3 - 13.215 \times 16}{.41} = 508.69.$$

$508.69 \div 20 = 25.434$ sq. in. Ans.

(*b*) $25.434 \div 13 = 1.9565$. Ans.

(*c*) $1.9565 \times 20 = 39.13$ lb. per sq. in. Ans.

(599) Use formula **66**, $t_c = (t_f - 32)\frac{5}{9}$.

(*a*) $t_c = (-10 - 32) \times \frac{5}{9} = -23\frac{1}{3}°$ C. Ans.

(*b*) $t_c = (25 - 32) \times \frac{5}{9} = -3\frac{8}{9}°$ C. Ans.

(*c*) $t_c = (2{,}200 - 32) \times \frac{5}{9} = 1{,}204\frac{4}{9}°$ C. Ans.

(600) This must be answered by the student.

(601) Using formula **80**, $L = 331.5744\, p\, v \log \frac{p_1}{p} =$

$331.5744 \times 15 \times 10 \times \log \frac{85}{15} = 37{,}467.74$ ft.-lb. Ans.

(602) 520 H. P. $= 520 \times 33{,}000$ ft.-lb. per minute $=$ $520 \times 33{,}000 \times 60$ ft.-lb. per hour $=$

$\frac{520 \times 33{,}000 \times 60}{778}$ B. T. U. $= 1{,}323{,}393.31$ B. T. U. Ans.

(603) Let l = the required latent heat;
W_1 = given weight of zinc;
W_2 = given weight of water;
s_1 = specific heat of zinc;
s_2 = specific heat of water $(= 1)$;
t_1 = temperature of melting zinc;
t_2 = temperature of water.

Then, the total heat of the mixture is $W_1 s_1 t_1 + W_2 s_2 t_2 + W_1 l$, and from formula **73**, the temperature

$$t = \frac{W_1 s_1 t_1 + W_2 t_2 + W_1 l}{W_1 s_1 + W_2}, \text{ or}$$

$$l = \frac{t(W_1 s_1 + W_2) - W_1 s_1 t_1 - W_2 t_2}{W_1} =$$

$$\frac{102\frac{1}{3}(4 \times .0956 + 10) - 4 \times .0956 \times 680 - 10 \times 60}{4} = 50.61.$$

Ans.

(604) See Arts. **1117** and **1120.**

(605) (*a*) and (*b*) See Art. **1118.**

(*c*) A non-conductor is one which will not conduct heat. There really is no such substance, but some substances are such bad conductors that they are termed non-conductors.

(606) When answering this question, consult Art. **1123.**

(607) In Art. **1130** it is stated that 1 calorie = 3.96 B. T. U. Hence, (*a*) 798 B. T. U. = $798 \div 3.96 = 201.515$ calories. Ans.

(*b*) $40 \times 3.96 = 158.4$ B. T. U. Ans.

(608) (*a*) First calculate the weight by formula **71.**

$$W = \frac{p\,V}{R\,T} = \frac{18 \times 7.68}{.33552 \times 500} = .824 \text{ lb., nearly.}$$

Units of heat required $= s_v\ W\ (T_1 - T) = s_v\ W(t_1 - t)$ $= .15507 \times .824 \times (416 - 40) = 48.0444$ B. T. U.

Therefore, $48.0444 \times 778 = 37{,}379$ ft.-lb., nearly. Ans.

(*b*) $.21751 \times .824\,(416 - 40) \times 778 = 52{,}429$ ft.-lb., nearly. Ans.

(609) See Arts. **1144** to **1147.**

(610) See Arts. **1155** and **1164.**

(611) Using formula **87,**

$$T_1 = T\left(\frac{V}{V_1}\right)^{.41} = 528\left(\frac{3.72}{1.2}\right)^{.41} = 839.64° = 380°,$$ nearly, above 0° F. Ans.

(612) (See Arts. **1149** and **1177.**)

(613) Use formula **81,** $p\,v^{1.41} = p_1\,v_1^{1.41}$.

$$14.7 \times 48^{1.41} = p_1 \times (48 - 38)^{1.41}, \text{ or}$$

$$p_1 = 14.7\left(\frac{48}{10}\right)^{1.41} = 134.24 \text{ lb. per sq. in.} \quad \text{Ans.}$$

Now, using formula **86,**

$$T_1 = 500\left(\frac{134.24}{14.7}\right)^{.29078} = 951°,$$ nearly, $= 491°$ above 0° F. Ans.

STEAM AND STEAM ENGINES.

(QUESTIONS 614–688.)

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(614) See Arts. **1191** and **1192.**

(615) See Arts. **1189** to **1195.**

(616) See Arts. **1198** to **1201.**

(617) Log $p = 6.1007 - \frac{2,719.78}{T} - \frac{400,215}{T^2} = 6.1007 - \frac{2,719.78}{290+460} - \frac{400,215}{(290+460)^2} = 1.76284$. Hence, $p = 57.92$ lb. per sq. in. Ans.

(618) Using formula **91,**

$H = 1,081.94 + 305\ t = 1,081.94 + .305 \times 318 = 1,178.93$ B. T. U. Ans.

(619) See Arts. **1217** and **1218.**

(620) Using formula **92,**

$$p\,v^{\frac{16}{17}} = 475, \text{ or } v^{\frac{16}{17}} = \frac{475}{p} = \frac{475}{81}.$$

Hence,

$$\tfrac{16}{17} \log v = \log 475 - \log 81 = 2.67669 - 1.90849 = .76820$$

$$\log v = .76820 \times \frac{16}{17} = .72301.$$

$v = 5.2846$ cu. ft. = volume of one pound

$5.2846 \times 6 = 31.7076$ cu. ft. Ans.

(621) Using formula **92,**

$$p\,v^{\frac{16}{17}} = 475, \text{ or } p = \frac{475}{v^{\frac{16}{17}}} = \frac{475}{\gamma^{\frac{16}{17}}}.$$

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$$\text{Hence, } \log p = \log 475 - \frac{17}{16} \log 7 = 1.77877.$$

$$p = 60.086 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(622) (*a*) Looking in the table of Properties of Saturated Steam, it is found that for a temperature of 320.094° the pressure is 90 pounds, and for the next lower pressure of the table, 88 pounds, the temperature is 318.510°. Hence, the difference in temperature for a difference of one pound in pressure is

$$\frac{320.094 - 318.510}{2} = .792°.$$

$320.094° - 320° = .094°$ = actual difference.

Then 1 lb. : .792° :: x lb. : .094, or $x = .12$ lb., nearly = difference in pressure between 320° and 320.094°.

$90 - .12 = 89.88$ lb. per sq. in., absolute pressure.

$89.88 - 14.7 = 75.18$ lb. per sq. in., gauge pressure. Ans.

(*b*) 110 lb. gauge pressure $= 110 + 14.7 = 124.7$ lb., absolute.

Temperature corresponding to 125 lb. = 344.136°.

Temperature corresponding to 120 lb. = 341.058°.

Difference, per pound difference in pressure =

$$\frac{344.136° - 341.058°}{5} = .616°.$$

$125 - 124.7 = .3$ lb., difference in pressure; $.616 \times .3 = .185°$.

$344.136 - .185 = 343.951°$. Ans.

(*c*) 70 lb., gauge pressure, $= 70 + 14.7 = 84.7$ lb., absolute.

Heat of liquid for 84 lb. = 285.414 B. T. U.

Heat of liquid for 86 lb. = 287.096 B. T. U.

$$\text{Difference per pound} = \frac{287.096 - 285.414}{2} = .841 \text{ B. T. U.}$$

$.841 \times .7 = .589$. $285.414 + .589 = 286.003$. B. T. U. Ans.

Total heat at 86 lb. = 1,178.592 B. T. U.

Total heat at 84 lb. = 1,178.091 B. T. U.

$$\text{Difference per pound} = \frac{1{,}178.592 - 1{,}178.091}{2} = .25 \text{ B. T. U.}$$

$.25 \times .7 = .175$. $1{,}178.091 + .175 = 1{,}178.266$ B. T. U. Ans.

(*d*) Latent heat per pound at 66 lb. pressure = 904.443 B. T. U.

Latent heat per pound at 68 lb. pressure = 903.02 B. T. U.

$$\text{Difference per pound} = \frac{904.443 - 903.02}{2} = .712 \text{ B. T. U.}$$

$904.443 - .712 = 903.731$ B. T. U., latent heat per pound at 67 lb. pressure.

$$903.731 \times 3 = 2{,}711.193 \text{ B. T. U.} \quad \text{Ans.}$$

(623) (*a*) The isothermal of saturated steam is a straight line parallel to the axis of volumes.

(*b*) The equilateral hyperbola.

(624) From the table of the Properties of Saturated Steam, the weight of one cubic foot of steam at a pressure of 44 lb. is .106345 lb., and at a pressure of 42 lb. it is .101794 lb.

$$\text{Difference per pound} = \frac{.106345 - .101794}{2} = .002276 \text{ lb.}$$

The weight of a cubic foot at 43 lb. pressure is

$$.101794 + .002276 = .10407 \text{ lb.}$$

Weight of 38 cu. ft. $= .10407 \times 38 = 3.95466$ lb. Ans.

(625) See Arts. **1214** and **1215.**

(626) See Art. **1206.**

To raise 1 pound of water from 55° to 140° requires $140 - 55 = 85$ B. T. U. Then, to raise 300 pounds of water

through the same range of temperature requires $85 \times 300 = 25{,}500$ B. T. U.

60 pounds gauge pressure $= 60 + 14.7 = 74.7$ pounds, absolute.

Total heat of 1 pound of steam at 74 pounds pressure $= 1{,}175.431$ B. T. U.

Total heat of 1 pound of steam at 76 pounds pressure, $= 1{,}175.985$ B. T. U.

Difference per pound pressure $= \frac{1{,}175.985 - 1{,}175.431}{2} =$ .277 B. T. U.

$.277 \times .7 = .194$. $1{,}175.431 + .194 = 1{,}175.625$ B. T. U., total heat of 1 pound of steam at an absolute pressure of 74.7 pounds per sq. in.

The total heat (above 32°) of a pound of water at 140° is $140 - 32 = 108$ B. T. U.

Hence, each pound of steam on being mixed with water lowers in temperature to 140°, and in so doing gives up $1{,}175.625 - 108 = 1{,}067.625$ B. T. U.

The required amount of steam is

$$\frac{25{,}500}{1{,}067.625} = 23.885 \text{ lb.} \quad \text{Ans.}$$

(627) The temperature 256° does not appear in the table of the Properties of Saturated Steam. The next lower temperature is found to be 254.002°; and the next higher, 257.523°. The difference is $257.523 - 254.002 = 3.521°$. The difference between the lower temperature and 256° is $256 - 254.002 = 1.998°$. The volume of a pound of steam at the lower temperature is 12.68 cu. ft.; and at the higher temperature, 11.98 cu. ft. The difference is $12.68 - 11.98 = .7$ cu. ft.

The difference between the volume at the lower temperature and the volume at 256° may now be found by proportion

$$3.521 : 1.998 :: .7 : x, \text{ or } x = .397.$$

Hence, the required volume of one pound of steam at 256° is

$$12.68 - .397 = 12.283 \text{ cu. ft.} \quad \text{Ans.}$$

$$12.283 \times 4\tfrac{1}{3} = 53.226 \text{ cu. ft.} \quad \text{Ans.}$$

(628) See Art. **1192.** Specific heat of superheated steam is .48. Hence, to raise 1 pound of superheated steam from 310° to 342° requires

$$.48 \times (342 - 310) = 15.36 \text{ B. T. U.}$$

To raise 6 pounds through the same range of temperature requires

$$15.36 \times 6 = 92.16 \text{ B. T. U.} \quad \text{Ans.}$$

(629) (*a*) $$\log p = 6.1007 - \frac{2{,}719.78}{T} - \frac{400{,}215}{T^2}$$

$$= 6.1007 - \frac{2{,}719.78}{254 + 460} - \frac{400{,}215}{(254 + 460)^2}$$

$$= 1.50643.$$

$$p = 32.094 \text{ lb. per sq. in.} \quad \text{Ans.}$$

From the tables, $p = 32$ lb. per sq. in. for a temperature of 254°.

(*b*) $$\log p = 6.1007 - \frac{2{,}719.78}{377 + 460} - \frac{400{,}215}{(377 + 460)^2} = 2.27999,$$

or $p = 190.54$ lb. per sq. in. Ans.

From the tables, $p = 189.212$ lb. per sq. in. for a temperature of 377°.

(630) See Arts. **1220** and **1221.**

(631) (*a*) $H = 1{,}081.94 + .305\ t$. From the table, $t = 385.759°$.

Hence, $H = 1{,}081.94 + .305 \times 385.759 = 1{,}199.596$ B. T. U. Ans.

From the tables, $H = 1{,}199.597$.

(*b*) $p = 88$; $t = 318.51°$.

Hence, $H = 1{,}081.94 + .305 \times 318.51 = 1{,}179.086$ B. T. U. Ans.

From the tables, $H = 1{,}179.085$.

(*c*) $p = 37$; $t = 260.883 + \frac{264.093 - 260.883}{2} = 262.488°$.

$H = 1{,}081.94 + .305 \times 262.488 = 1{,}161.999$ B.T.U. Ans.

From the tables, $H = 1{,}161.999$.

(632) See Art. **1293.**

(633) See Fig. 49. The lines $O\,X$ and $O\,Y$ are drawn at right angles. The scale of volumes chosen is 20 cu. ft. to the inch, and the scale of pressures 20 lb. to the inch. The vol-

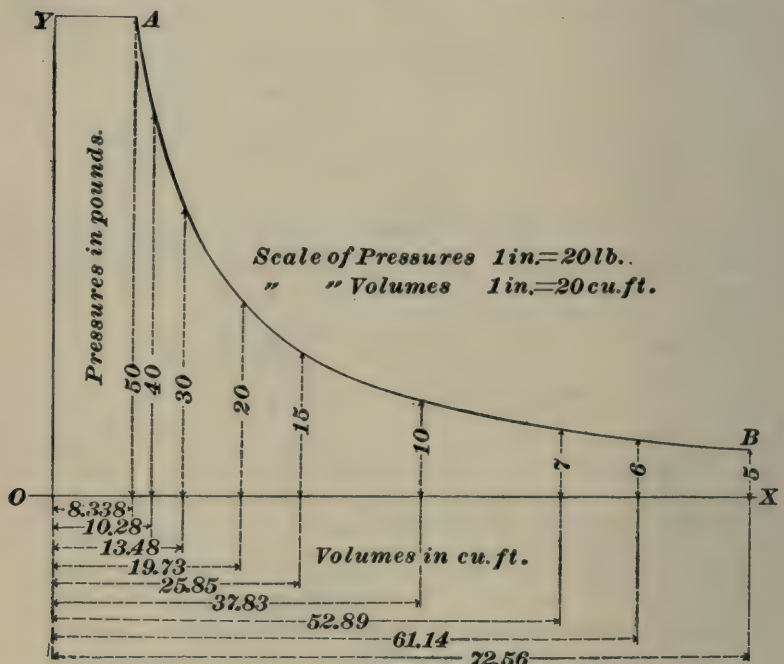

FIG. 49.

umes corresponding to 50, 40, 30 lb., etc., are found from the steam table, and laid off along $O\,X$. The pressures are then laid off vertically, as shown, forming the required curve $A\,B$.

(634) See Art. **1224.**

(635) (*a*) See Arts. **1252** to **1254.**

(*b*) Clearance $i = .07$; apparent cut-off $k_1 = \frac{3}{8} = .375$.

Then, by formula **97,** the real cut-off =

$$k = \frac{k_1 + i}{1 + i} = \frac{.375 + .07}{1 + .07} = \frac{.445}{1.07} = .416. \quad \text{Ans.}$$

By formula **96,** ratio of expansion, $e = \frac{1}{k} = \frac{1}{.416} = 2.4.$ Ans.

(636) By formula **97,** $k = \frac{k_1 + i}{1 + i} = \frac{.625 + .06}{1 + .06} = \frac{.685}{1.06}.$

By formula **96,** $e = \frac{1}{k} = \frac{1}{\frac{.685}{1.06}} = \frac{1.06}{.685} = 1.547.$

By formula **99,** M. E. P. $= \frac{.9\,P(1 + 2.3 \log e)}{e} - .9p.$

$P = 60 + 14.7 = 74.7$ lb.; $p = 17$ lb.

Then, M. E. P. $= \frac{.9 \times 74.7\,(1 + 2.3 \log 1.547)}{1.547} - .9 \times 17 =$ 47.1 lb. per sq. in. Ans.

(637) (*a*) Using formula **98,**

I. H. P. $= \frac{PLAN}{33{,}000} = \frac{47.1 \times \frac{1}{12} \times 11^2 \times .7854 \times 210 \times 2}{33{,}000} =$ 85.45 H. P. Ans.

(*b*) $85.45 \times .83 = 70.92$ actual H. P. Ans.

(*c*) $85.45 - 70.92 = 14.53$ friction H. P. Ans.

(638) See Arts. **1232, 1236, 1244,** and **1245.**

(639) (*a*) See Arts. **1262** and **1276.**

(*b*) The three principal uses are the following:

1. To find the I. H. P. of the engine.
2. To detect defects in valve setting, and to serve as a guide in setting the valves.
3. To determine, approximately, the steam consumption of the engine.

(640) (*a*) Work per stroke per sq. in. of piston =

$$3.47 \times \frac{6}{12} \times 40 = 69.4 \text{ ft.-lb.}$$

Area of piston $= 16^2 \times .7854 = 201.0624$ sq. in.

Then, total work per stroke $= 69.4 \times 201.0624 = 13{,}953.73$ ft.-lb. Ans.

(*b*) 120 rev. per min. $= 120 \times 2 = 240$ strokes per min.

$$\text{Then, H. P.} = \frac{13{,}953.73 \times 240}{33{,}000} = 101.48. \quad \text{Ans.}$$

(641) Admission, cut-off, release, and compression are all too late, and the back pressure is excessive. Admission, release, and compression may be made earlier by increasing the angular advance; cut-off may be made earlier by adding lap to the valve; the back pressure can be reduced only by reducing the resistance to the passage of the exhaust steam; that is, by enlarging the exhaust port or the exhaust pipe, or both.

(642) Assuming a mechanical efficiency of 80%, the I. H. P. of the engine must be $\frac{120}{.80} = 150$.

The probable M. E. P. may be found from formula **99.**

$$\text{M. E. P.} = \frac{.9\,P(1 + 2.3 \log e)}{e} - .9p =$$

$$\frac{.9 \times 84.7\,(1 + 2.3 \log 4)}{4} - .9 \times 3 = 42.75 \text{ lb. per sq. in.}$$

Assume 550 feet per minute as a fair piston speed. We have, then, $P = 42.75$, and $L\,N =$ piston speed $= 550$. Substituting, in formula **98,** I. H. P. $= \frac{P\,L\,A\,N}{33{,}000}$, $150 = \frac{42.75 \times 550 \times A}{33{,}000}$, or $A = \frac{150 \times 33{,}000}{42.75 \times 550} = 210.52$ sq. in., the area of the piston. The diameter is, consequently, $\sqrt{\frac{210.52}{.7854}} = 16\frac{1}{2}''$, nearly. Assuming the stroke to be 42 in., the number of revolutions, from formula **100,** is

$$R = \frac{6\,S}{L} = \frac{6 \times 550}{42} = 78.57.$$

This is a fair number of revolutions for a Corliss engine. Hence, an engine $16\frac{1}{2}'' \times 42''$, making $78\frac{1}{2}$ rev. per min., will do the required work.

(643) (*a*) 60 pounds, gauge pressure, $= 60 + 14.7 = 74.7$ lb., absolute; 2 lb. above atmosphere $= 2 + 14.7 = 16.7$ lb., absolute.

Temperature of steam at 74.7 lb. pressure is, from the table of the Properties of Saturated Steam,

$$306.526 + \left(\frac{308.344 - 306.526}{2}\right).7 = 307.162°.$$

Temperature of steam at 16.7 is

$216.347 + (219.452 - 216.347).7 = 218.521°$. Then,

$$T_1 = 307.162° + 460° = 767.162°$$

and $$T_2 = 218.521° + 460° = 678.521°.$$

Thermal efficiency $= \frac{T_1 - T_2}{T_1} = \frac{767.162 - 678.521}{767.162} = .1155 = 11.55\%$. Ans.

(*b*) In this case, the absolute pressure of the entering steam is $90 + 14.7 = 104.7$ pounds per sq. in., and the pressure of the exhaust steam 3 pounds per sq. in., absolute.

The temperature corresponding to the former pressure is from the table $= 331.169 - \left(\frac{331.169 - 327.625}{5}\right).3 = 330.956°$.

The temperature corresponding to the latter pressure is 141.654°. Hence, T_1 is $330.956° + 460° = 790.956°$, and T_2 is $141.654° + 460° = 601.654°$.

Thermal efficiency $= \frac{790.956 - 601.654}{790.956} = .2393 = 23.93\%$. Ans.

(644) (*a*) Using formula **100,**

$$L = \frac{6\,S}{R} = \frac{6 \times 540}{150} = 21.6''. \quad \text{Ans.}$$

(*b*) $R = \frac{6\,S}{L} = \frac{6 \times 900}{2\frac{1}{2} \times 12} = 180$ rev. per min. Ans.

(645) The M. E. P. is found as shown in Fig. 50.

The middle ordinates are drawn as explained in Art. **1159.**

These ordinates are measured and multiplied by the scale of the spring, which reduces them from inches to pounds.

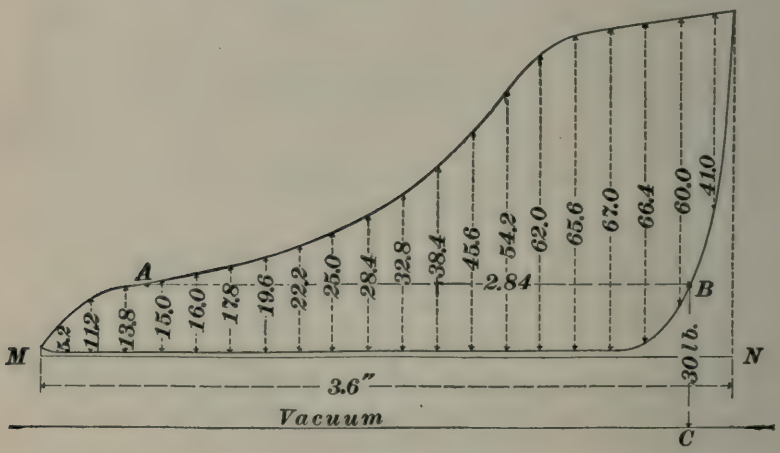

FIG. 50.

In the figure the sum of the 20 ordinates is 707.2 pounds. The mean ordinate, or, what is the same thing, the M. E. P., is, therefore, $\frac{707.2}{20} = 35.36$ lb. Draw the vacuum line at a distance of $\frac{14.7}{40} = .3675''$ below the atmospheric line MN. Choose the point A near the point of release, and draw AB parallel to MN. The height BC of this line above the vacuum line is $\frac{3''}{4}$; therefore, the absolute pressure at A or B is $\frac{3}{4} \times 40 = 30$ pounds. The length $AB = l = 2.84''$, and the length of the diagram L is $3.6''$. The weight of a cu ft of steam at 30 pounds pressure, absolute, is .074201 lb.

Substituting these values in formula **101,**

$$Q = \frac{13{,}750\, l\, W}{P L} = \frac{13{,}750 \times 2.84 \times .074201}{35.36 \times 3.6} =$$

22.76 lb. per I. H. P. per hour. Ans.

(646) (*a*) See Arts. **1293** and **1294.**

(*b*) See Arts. **1314** and **1315.**

(647) Area of high-pressure cylinder $= 19^2 \times .7854$ sq.in.

Area of low-pressure cylinder $= 32^2 \times .7854 = 804.25$ sq. in.

Ratio of area of high-pressure cylinder to area of low-pressure cylinder $= \frac{19^2 \times .7854}{32^2 \times .7854} = \frac{19^2}{32^2} = \frac{361}{1{,}024}$.

M. E. P. of high-pressure cylinder reduced to low-pressure cylinder $= 52 \times \frac{361}{1{,}024} = 18.332$ lb. per sq. in.

Total M. E. P. reduced to low-pressure cylinder $=$
$18 + 18.332 = 36.332$ lb. per sq. in.

(*a*) Substituting now in formula **98,**

$$\text{I. H. P.} = \frac{P L A N}{33{,}000} = \frac{36.332 \times \frac{3}{12} \times 804.25 \times 120 \times 2}{33{,}000} = 531.27.$$

Ans.

(*b*) Since the stroke is the same for each cylinder, the ratio of the work done by the two cylinders is proportional to the ratio of the two M. E. P.'s reduced to the low-pressure cylinder. Hence, the ratio of the work done in the high-pressure cylinder to that done in the low-pressure cylinder is $18.332 : 18 = 1.0184$. Ans.

(648) (*a*) See Art. **1276.**

(*b*) 1. Decrease the angular advance.

2. Increase the angular advance.

3. Lower the boiler pressure or decrease the number of revolutions.

4. Raise the boiler pressure or increase the number of revolutions.

(649) (*a*) The points desired are shown in Fig. 51. To locate the point of cut-off, prolong the steam and expansion

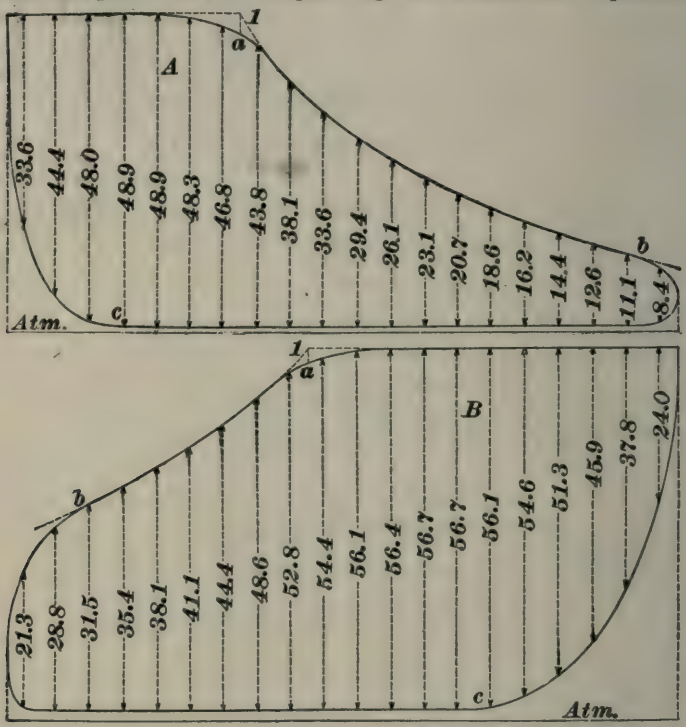

FIG. 51.

lines till they intersect, as at *1*. From *1* drop the perpendicular *1 a*, and *a* will be the point of cut-off.

The point of release may be located by prolonging the expansion curve, and noting the point where the actual curve departs from it, as shown at *b*. The point of compression *c* is easily located.

(*b*) The M. E. P. of the two diagrams are found as shown in the figure. The length of each diagram is divided into 20 equal parts, and ordinates are erected at the middle points of the divisions. The lengths of these ordinates multiplied by the scale of the spring used, in this case, $1'' = 30$ lb., added together and divided by 20 gives the required M. E. P. The

M. E. P. of diagram A is found to be 30.75 lb. per sq. in., and the M. E. P. of diagram B is found to be 44.6 lb. per sq. in. Ans.

(650) Using formula **106,**

$$S = .0944\,W = .0944 \times 675 \times 26\tfrac{1}{2} = 1{,}688.58 \text{ sq. ft.} \quad \text{Ans}$$

(651) Area of piston $= .7854 \times 30^2 = 706.86$ sq. in. The piston speed is, from formula **100,** $S = \dfrac{L\,R}{6} = \dfrac{48 \times 80}{6} =$ 640 ft. per min.; hence, the velocity of the crank-pin ($= v_0$) is $\dfrac{640 \times 3.1416}{2} = 1{,}005.3$ ft. per min. $= \dfrac{1{,}005.3}{60} = 16.755$ ft. per sec. The ratio n between the radius of the fly-wheel and length of crank is $\dfrac{10}{2} = 5$. Substituting now in formula **107,**

$$W = \frac{A\,H\,g}{n^2\,E\,V_0^2} = \frac{706.86 \times 88 \times 32.16}{5^2 \times \frac{1}{30} \times 16.755^2} = 14{,}251.92 \text{ lb.} \quad \text{Ans.}$$

(652) (*a*) Use formula **98.**

I. H. P. $= \dfrac{P\,L\,A\,N}{33{,}000}$, or $A = \dfrac{33{,}000 \text{ I. H. P.}}{P\,L\,N} = \dfrac{33{,}000 \times 1{,}200}{42 \times 1\frac{2}{3} \times 70 \times 2} =$ 1,924.2 sq. in., area of low-pressure piston. Diameter of low-pressure cylinder $= \sqrt{\dfrac{1{,}924.2}{.7854}} = 49\dfrac{1''}{2}$. Ans.

(*b*) Since the stroke is the same for both cylinders, their volumes are proportional to the areas of their pistons.

Hence, using formula **103,**

$$\frac{V}{v} = \frac{\text{area of low-pressure piston}}{\text{area of high-pressure piston}} = \frac{E}{2.72}, \text{ or area of high-pressure piston} = \frac{2.72 \times 1{,}924.2}{5.5} = 951.6 \text{ sq. in.}$$

The corresponding diameter is

$$\sqrt{\frac{951.6}{.7854}} = 34\frac{13''}{16}, \text{ or say } 34\frac{7''}{8}. \quad \text{Ans.}$$

Using formula **104,**

$$\frac{V}{v} = \frac{\text{area of low-pressure piston}}{\text{area of high-pressure piston}} = \sqrt{E} = \sqrt{5.5}.$$

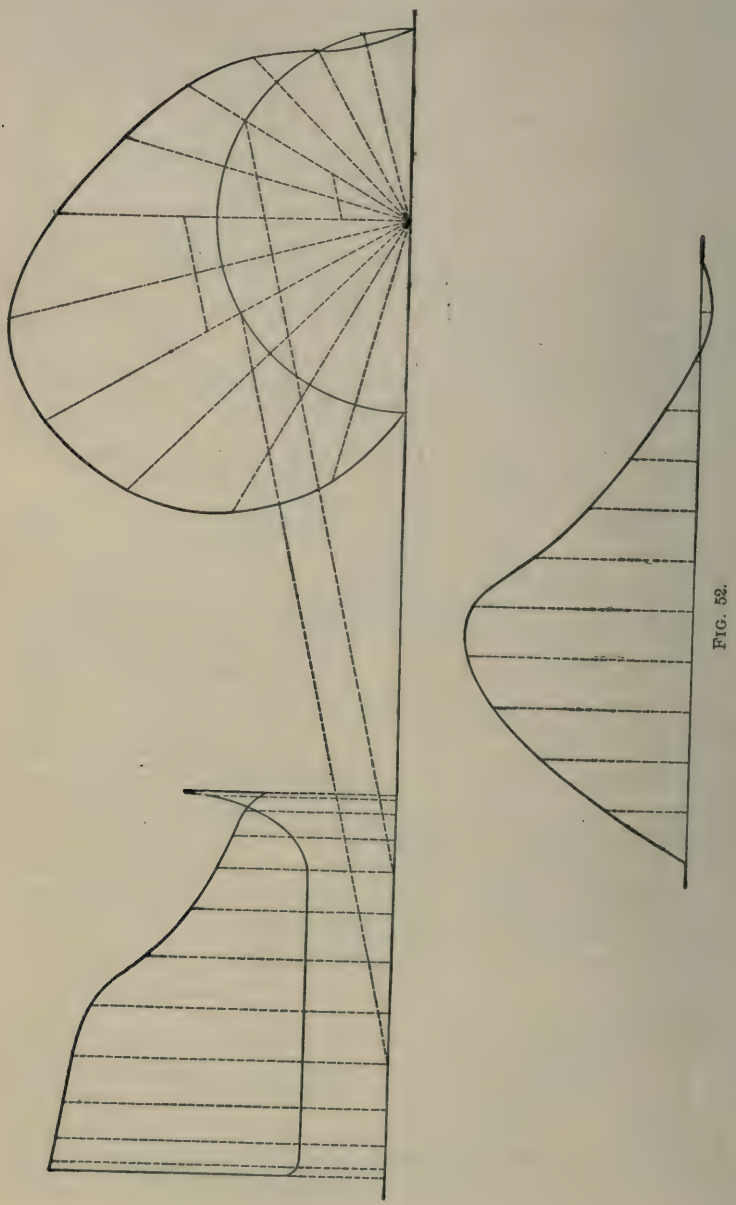

FIG. 52.

Hence, area of high-pressure piston $= \frac{1,924.2}{\sqrt{5.5}} = 820.48$ sq. in.

The corresponding diameter is $32\frac{5''}{16}$, say $32\frac{3''}{8}$. Ans.

(653) (*a*) By formula **102,** $E = \frac{eV}{v}$, or $e = \frac{Ev}{V}$; but, by formula **103,** $\frac{V}{v} = \frac{E}{2.72}$, or $E = 2.72\frac{V}{v}$. Substituting this value of E in **102,** we have, $e = 2.72\frac{V}{v}\ \frac{v}{V} = 2.72$. Hence, the real cut-off $= \frac{1}{e} = \frac{1}{2.72} = .368$. Ans.

(*b*) From formula **104,** $\frac{V}{v} = \sqrt{E}$, or $\frac{v}{V} = \frac{1}{\sqrt{E}}$.

Substituting this value of $\frac{v}{V}$ in formula **102,** above,

$$e = \frac{E}{\sqrt{E}} = \sqrt{E} = \sqrt{5.5} = 2.345.$$

Real cut-off $= \frac{1}{e} = \frac{1}{2.345} = .426$. Ans.

(654) See Art. **1330.**

(655) The construction is shown in Fig. 52. It is precisely similar to the construction shown in Art. **1313,** and need not be further described.

(656) The total heat of steam at $4\frac{1}{2}$ pounds pressure is, from the steam tables,

$$\frac{1,128.641 + 1,131.462}{2} = 1,130.051 \text{ B. T. U.}$$

Now, using formula **105,**

$$W = \frac{H - t_1 + 32}{t_1 - t_2} = \frac{1,130.051 - 130 + 32}{130 - 55} = 13.76 \text{ lb.}$$ Ans.

(657) See Art. **1291.**

(658) (*b*) Using formula **98,** I. H. P. $= \frac{PLAN}{33,000}$, or

$$LA = \frac{33,000 \text{ I. H. P.}}{PN} = \frac{33,000 \times 42}{36.3 \times 155 \times 2}.$$

L = stroke in feet.

$12\,L$ = stroke in inches.

From the conditions of the problem, $12\,L = 1\frac{1}{3}\,d$, when d is taken in inches, or $L = \frac{1}{9}\,d$. $A = .7854\,d^2$. Substituting these values of L and A,

$$\frac{1}{9}\,d \times .7854\,d^2 = \frac{.7854\,d^3}{9} = \frac{33{,}000 \times 42}{2 \times 36.3 \times 155},$$

or $d = \sqrt[3]{\dfrac{33{,}000 \times 42 \times 9}{36.3 \times 155 \times 2 \times .7854}} = 11.22''$ nearly, or, say $11\frac{1''}{4}$. Ans.

(*a*) Stroke $= 1\frac{1}{3}\,d = 11\frac{1}{4} \times 1\frac{1}{3} = 15''$, nearly. Ans.

(*c*) Using formula **100**,

$$S = \frac{L\,R}{6} = \frac{15 \times 155}{6} = 387.5 \text{ ft. per min.} \quad \text{Ans.}$$

(659) (*a*) Area of piston, $.7854 \times 22^2 = 380.1336$ sq. in.
Total pressure on piston $= 380.1336 \times 72 = 27{,}369.62$ lb.
At the beginning of the stroke, the piston rod, connecting-rod, and crank lie in the same straight line, and, consequently, the total pressure on the piston is transmitted to the crank-shaft.

(*b*) In this case the total pressure on the piston is $380.1336 \times 65 = 24{,}708.68$ lb., which is also the horizontal pressure on the crank-pin.

Then, the pressure on the crank-shaft is $24{,}708.68 \times \cos 60° = 12{,}354.34$ lb. Ans.

(*c*) The tangential pressure is $24{,}708.68 \times \sin 60° = 24{,}708.68 \times .86603 = 21{,}398.46$ lb. Ans.

(660) See Art. **1311.**

The mean ordinate should be $34.6 \times \dfrac{2}{3.1416} = 22.03$ lb. per sq. in. Ans.

(661) Diameter of drivers $= 80'' = \frac{80}{12} = 6\frac{2}{3}$ ft.

Circumference of drivers $= 6\frac{2}{3} \times \pi = 20.944$ ft.

60 miles per hour=1 mile per min.

One mile contains 5,280 ft. Hence, the number of revolutions of the driver per min. is $\frac{5,280}{20.944}$.

Area of piston $= .7854 \times 19^2 = 283.53$.

Now, using formula **98,**

$$\text{I.H.P.} = \frac{PLAN}{33,000} = \frac{52.5 \times \frac{24}{12} \times 283.53 \times \frac{5,280}{20.944} \times 2}{33,000} = 454.86.$$

Since there are two cylinders, the total horsepower $= 454.86 \times 2 = 909.72$ I. H. P. Ans.

(662) (*a*) As shown in Fig. 53, the diameter of the path described by the eccentric is the travel of the valve, **5″.**

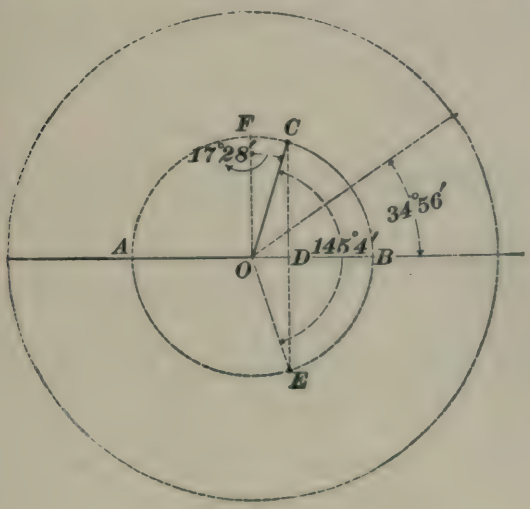

FIG. 53.

The eccentric crank must be in a position *O C*, so that, upon

dropping the perpendicular $C\,D$ upon $A\,B$, the distance $O\,D$ will equal the lap $\frac{3''}{4}$. Then, $\cos C\,O\,D = \frac{O\,D}{O\,C} = \frac{\frac{3}{4}}{2\frac{1}{2}} = .3$.

The angle whose cosine is .3 is 72° 32′, nearly. Hence, $C\,O\,D = 72°\ 32'$, and the angular advance $= C\,O\,F = 90° - C\,O\,D = 90 - 72°\ 32' = 17°\ 28'$.

(*b*) When cut-off occurs the valve must be in the same position as it was at the beginning of the stroke, but moving in the opposite direction. Hence, the center of the eccentric must be at E vertically below C, and must have passed through the angle $C\,O\,E = 2\,C\,O\,B = 2 \times 72°\ 32'' = 145°\ 4'$. Therefore, the crank has moved 145° 4′ from the dead-center position, and now makes an angle of $\frac{145°\ 4'}{2} = 72°\ 32'$, with $A\,B$, the diameter through the dead points.

(663) (*a*) Efficiency $= \frac{12.325}{15.36} = .8024 = 80.24\%$. Ans.

(*b*) Area of piston $= .7854 \times 9^2 = 63.617$ sq. in.

Stroke $= 12'' = 1$ foot.

Using formula **98**, I.H.P. $= \frac{PLAN}{33{,}000}$, or

$$P = \frac{\text{I.H.P.} \times 33{,}000}{LAN} = \frac{15.36 \times 33{,}000}{1 \times 63.617 \times 240 \times 2} = 16.6 \text{ lb.}$$

(664) See Arts. **1223** and **1224.**

(665) The construction is shown in Fig. 54. Choose a suitable length for the diagram, say 3 inches. Draw the atmospheric line $M\,N$. Choose a scale of pressures, say 40 pounds per inch, and draw the vacuum line $O\,X$ parallel to $M\,N$ and $\frac{14.7}{40} = .3675''$ below it. On $O\,X$ lay off the length of the diagram $A\,B = 3''$. This length represents to some scale the volume of the cylinder, and 7% of this length must then represent to the same scale the clearance. Therefore,

from A, lay off $A\,O$ equal to 7% of $A\,B = 3 \times .07 = .21''$, and from O draw $O\,Y$ perpendicular to $O\,X$. Lay off $M\,C = \frac{70}{40} = 1\frac{3''}{4}$ to represent the boiler pressure, and through C draw $C\,E$ parallel to $O\,B$. The number of expansions is 3. Hence, lay off $C\,D = \frac{1}{3}\,O\,B$, and D will be the point of cut-off. Through D, draw the equilateral hyperbola $D\,F$,

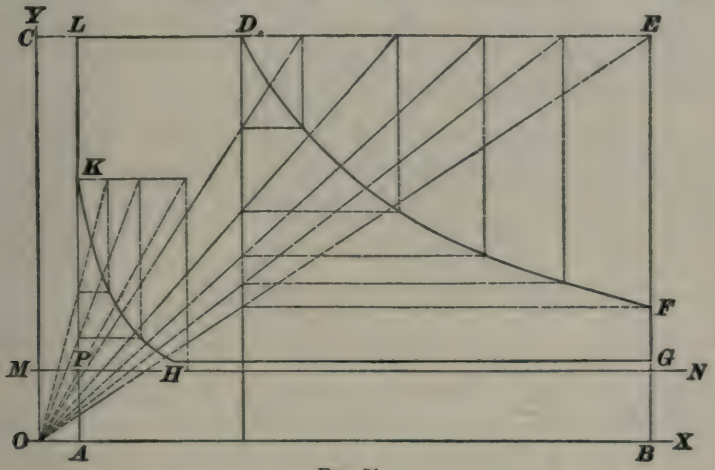

FIG. 54.

as explained in Art. **1161,** taking O as the point from which to draw the radial lines. $D\,F$ is the expansion line. The back pressure is 2 lb. above the atmosphere. Hence, the line $G\,H$ drawn parallel to $M\,N$ and $\frac{2}{40} = \frac{1''}{20}$ above 1 will represent the back-pressure line of the diagram. Since the steam is compressed to 40 lb., the point where compression ends may be found by measuring $P\,K = \frac{40}{40} = 1''$ above $M\,N$. Through K, the equilateral hyperbola $K\,H$ is drawn in the same manner as $D\,F$. Draw $K\,L$ perpendicular to $M\,N$, and the diagram is finished.

(666) See Art. **1308.**

(667) To obtain an idea of the condensation of the steam in the cylinder. There is generally more condensation at cut-off than at release, on account of reevaporation, and if the steam consumption be calculated from both points, the difference will tell more or less, approximately, the amount of condensation at cut-off.

(668) The data assumed will, of course, depend largely upon circumstances. Suppose the engine to be non-condensing, and to run with a piston speed of 600 feet per minute, and use steam at a boiler pressure of 75 lb. The mechanical efficiency will probably be about 80%, which requires the engine to develop $\frac{240}{.80} = 300$ I. H. P. Assuming the number of expansions to be $2\frac{1}{2}$, the probable M. E. P. is, from formula **99**, M. E. P. $= \frac{.9\,P(1 + 2.3 \log e)}{e} - .9p =$ $\frac{.9 \times 89.7\,(1 + 2.3 \log 2.5)}{2.5} - .9 \times 17 = 46.55.$

Now, using formula **98**, I. H. P. $= \frac{PLAN}{33{,}000}$, or $A = \frac{33{,}000 \text{ I. H. P.}}{PLN}$. But $LN =$ piston speed $= 600$ ft.

Hence, $A = \frac{33{,}000 \times 300}{46.55 \times 600} = 354.46$ sq. in. $=$ area of piston.

The diameter corresponding to this area is $21\frac{1}{4}''$.

For this diameter a fair stroke would be 30″, the number of revolutions being from formula **100**,

$$R = \frac{6\,S}{L} = \frac{6 \times 600}{30} = 120.$$

The engine would be made $21\frac{1}{4}'' \times 30''$, and run at 120 rev. per min. Ans.

(669) The scale of the spring should be in general not less than $\frac{1}{2}$ of the boiler pressure.

(*a*) $\frac{54}{2} = 27$; hence, a 30 spring should be used.

(*b*) $\frac{115}{2} = 57\frac{1}{2}$; hence, a 60 spring should be used.

(*c*) $1.834'' \times 40 = 73.36$ lb. per sq. in. Ans.

(670) Weight of condensing water per pound of steam $= \frac{13,580}{906}$.

Total heat of steam at an absolute pressure of 7 pounds is, from the table, 1,135.908. Substituting in formula **105,**

$W = \frac{H - t_2 + 32}{t_1 - t_2}$, or $t_2 = H + 32 - W(t_1 - t_2)$, we obtain

$t_2 = 1,135.908 + 32 - \frac{13,580}{906}(120 - 52) = 148.66°$. Ans.

(671) The areas of the three pistons are proportional to the squares of their respective diameters. Hence, the M. E. P. of the high-pressure cylinder reduced to the low-pressure cylinder is $72 \times \frac{27^2}{66^2} = 12.05$ lb. per sq. in. The M. E. P. of the intermediate cylinder reduced to the low-pressure cylinder is $40 \times \frac{42^2}{66^2} = 16.2$ lb. per sq. in.

The total M. E. P. reduced to the low-pressure cylinder is, consequently, $12.05 + 16.2 + 16.5 = 44.75$ lb. per sq. in.

Area of low-pressure piston $= .7854 \times 66^2 = 3,421.2$ sq. in.

(*a*) Substituting, now, in formula **98,**

$$\text{I. H. P.} = \frac{PLAN}{33,000} = \frac{44.75 \times 4 \times 3,421.2 \times 70 \times 2}{33,000} = 2,598.$$

Ans.

(*b*) The work done in each cylinder is proportional to the M. E. P. of the cylinder reduced to the low-pressure cylinder. Hence, the percentage of the total work done in the high-pressure cylinder is $\frac{12.05}{44.75} = .269 = 26.9\%$. Ans. The percentage of the work done in the intermediate cylinder is $\frac{16.2}{44.75} = .362 = 36.2\%$. Ans. Lastly, the percentage of the work done in the low-pressure cylinder is $\frac{16.5}{44.75} = .369 = 36.9\%$. Ans.

(672) See Art. **1287.**

(673) The number of expansions is determined from formulas **97** and **96,**

$$k = \frac{k_1 + i}{1 + i} = \frac{.25 + .025}{1 + .025} = \frac{.275}{1.025}.$$

$$e = \frac{1}{k} = \frac{1}{\frac{.275}{1.025}} = \frac{1.025}{.275} = 3.727.$$

(*a*) The M. E. P. may now be found from formula **99.**

$$\text{M. E. P.} = \frac{.9\,P(1 + 2.3 \log e)}{e} - .9p =$$

$$\frac{.9 \times 98.7\,(1 + 2.3 \log 3.727)}{3.727} - .9 \times 3 = 52.455 \text{ lb. per sq. in.}$$

Ans.

(*b*) Using formula **98,** $\text{I. H. P.} = \frac{PLAN}{33{,}000}$, or $A = \frac{33{,}000 \times \text{I. H. P.}}{PLN} = \frac{33{,}000 \times 120}{52.455 \times 500} = 151$ sq. in. The required diameter is, therefore, $\sqrt{\frac{151}{.7854}} = 13\frac{7''}{8}$. Ans.

(674) Since the gauge pressure is 93 lb., the absolute pressure is $93 + 14.7 = 107.7$ lb. Substituting the logarithm of 107.7 in formula **90,**

$$2.03222 = 6.1007 - \frac{2{,}719.78}{T} - \frac{400{,}215}{T^2}.$$

Clearing of fractions and transposing,

$$4.06848\ T^2 - 2{,}719.78\ T = 400{,}215.$$

As this is an affected quadratic, it may be solved by the rule given in algebra; hence, dividing by the coefficient of T^2, $T^2 - 668.5\ T = 98{,}369.66$. Completing the square $T^2 - 668.5\ T + 111{,}723.2 = 210{,}092.86$. Extracting square root, $T - 334.25 = 458.359$, or $T = 792.609°$. Therefore, $792.609 - 460 = 332.609°$. Ans.

(675) See rule in Art. **1246,** and the answer to question 662.

(676) The volume discharged in 1 stroke is

$$\frac{18^2 \times .7854 \times 24}{1{,}728} \text{ cu. ft.,}$$

and the volume discharged in 1 minute is

$$\frac{18^2 \times .7854 \times 24 \times 175 \times 2}{1{,}728}.$$

Formula **93** gives the work in foot-pounds which is done in 1 stroke; hence, if multiplied by the number of strokes per minute and divided by 33,000, the result will be the horsepower. The pressure is 62.4 lb. Substituting these values of the pressure and volume, in formula **93,** and dividing by 33,000, we have

$$\frac{144 \times 62.4 \times 18^2 \times .7854 \times 24 \times 175 \times 2}{1{,}728 \times 33{,}000} = 336.825 \text{ horsepower.}$$

Ans.

(677) Dividing the diagram into 10 equal parts, and measuring the ordinates drawn at the middle of these divisions, the sum of their lengths for diagram A is 7.16″, for diagram B, 7.24″, and for diagrams C and D, 4.68″ and 4.72″, respectively. Dividing these sums for A and B by 10 and multiplying by 60, the scale of spring, we have,

for A, $7.16 \times \frac{60}{10} = 42.96$ lb. per sq. in.;

for B, $7.24 \times \frac{60}{10} = 43.44$ lb. per sq. in.

Adding and dividing by 2, the mean effective pressure for the high-pressure cylinder is found to be $\frac{42.96 + 43.44}{2} = 43.2$ lb. per sq. in.

The M. E. P. for the low-pressure cylinder is,

for C, $4.68 \times \frac{30}{10} = 14.04$ lb. per sq. in.;

for D, $4.72 \times \frac{30}{10} = 14.16$ lb. per sq. in.

Adding and dividing by 2, the M. E. P. for the low-pressure cylinder is $\frac{14.04 + 14.16}{2} = 14.1$ lb. per sq. in.

Reducing the M. E. P. of the high-pressure cylinder to the area of the low-pressure piston, we have $43.2 \times \frac{13^2 \times .7854}{20^2 \times .7854} = 18.252$ lb. per sq. in.

Hence, $14.1 + 18.252 = 32.352$ lb. per sq. in. = the M. E. P. if the work was all done in the low-pressure cylinder.

Therefore, I. H. P. $= \frac{PLAN}{33{,}000} =$

$$\frac{32.352 \times \frac{15}{12} \times 20^2 \times .7854 \times 230 \times 2}{33{,}000} = 177.1 \text{ I. H. P.} \quad \text{Ans.}$$

(678) Fig. 55 shows diagram A and diagram D drawn as mentioned in Art. **1305;** that is, the scale of pressures

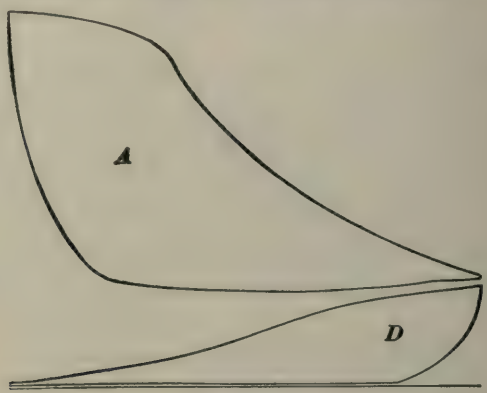

FIG. 55.

on diagram D has been reduced to 60, the same as on diagram A. In Fig. 56 the length of diagram A has been reduced in the proportion of the ratio of the volume of the high-pressure cylinder to that of the low-pressure cylinder. By reading Arts. **1304** and **1305,** the student should have no trouble in drawing the diagrams by aid of the above figures.

(679) Using a scale divided into thirtieths of an inch, and locating for this case a point $\frac{28''}{30}$ above the vacuum line

(which should be drawn previous to this), we draw through this point a line parallel to the atmospheric line. The length of the portion of this line included between the

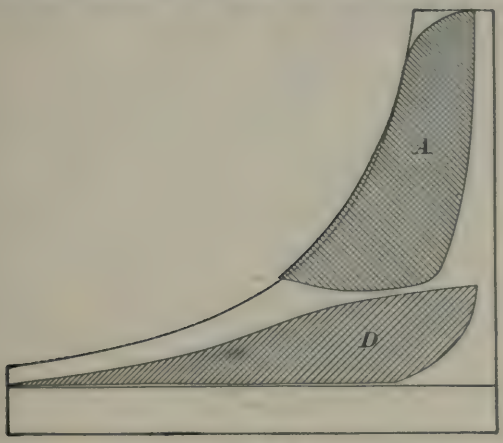

FIG. 56.

bounding lines of the diagram is 2.95″, and the absolute pressure which it represents is 28 lb. The whole length of the diagram is $3\frac{1}{2}''$. The weight of a cubic foot of steam at this pressure is, from the table of the Properties of Saturated Steam, .069545 lb. The M. E. P. was found to be 30.75 lb. per sq. in., in example 649.

Substituting these values in formula **101**,

$$Q = \frac{13{,}750 \times 2.95 \times .069545}{30.75 \times 3.5} = 26.21 \text{ lb. per I. H. P. per hour.}$$ Ans.

(680) For (*a*) and (*b*) see Art. **1292.** (*c*) No. The stroke may be so short that the piston speed will be low, although the rotative speed is high.

(681) The absolute pressure = 123 + 14.7 = 137.7 lb. Logarithm of 137.7 = 2.13893. Substituting in formula **90**,

$$2.13893 = 6.1007 - \frac{2{,}719.78}{T} - \frac{400{,}215}{T^3}$$

Clearing of fractions and transposing, $3.96177\ T^2 - 2{,}719.78\ T = 400{,}215$.

Solving this equation for T in the same manner as was done in example 674, $T = 811.49°$, or $t = 811.49 - 460 = 351.49°$. Ans.

(682) Since, according to formula **98,**

$$\text{I. H. P.} = \frac{PLAN}{33{,}000},\ P = \frac{\text{I. H. P.} \times 33{,}000}{LAN}.$$

Substituting the values given in the present example,

$$P = \frac{300 \times 33{,}000}{\frac{18}{12} \times 22^2 \times .7854 \times 200 \times 2} = 43.406 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(683) See answer to question 675.

(684) Drawing and uniting the lines $s\ t\ u$ and $v\ t\ r$ of diagrams A and B, respectively, the diagram shown at (a), Fig. 57, is drawn. (Fig. 57 is not drawn to scale.) Dividing the part $r\ s\ t$ into 10 equal parts, and measuring the ordinates drawn at the middle of those parts, the sum of their lengths is 7.78″, and the average length (mean ordinate) is $7.78 \div 10 = .778$″. The length of a line parallel to the atmospheric line included between the point t and the line $s\ r$ is 2.23″. Hence, area of $s\ t\ r = .778 \times 2.23 = 1.7349$ sq. in. Dividing $v\ t\ u$ into 5 equal parts, the sum of the middle ordinates is 1.8″. $1.8 \div 5 = .36$″ = mean ordinate of $v\ t\ u$. Length from point t to line $v\ u = .37$″. Area of $v\ t\ u = .36 \times .37 = .1332$ sq. in. Subtracting area $v\ t\ u$ from area $r\ s\ t$, $1.7349 - .1332 = 1.6017$ sq. in. The length $r\ u = 2.6$″. Hence, the mean pressure urging the piston ahead is $\frac{1.6017 \times 60}{2.6} = 36.96$ lb. per sq. in. Ans.

(685) This example is worked like the previous examples where the horsepower is to be found from the diagrams. Dividing the length of the diagrams into ten equal parts, and measuring the middle ordinates, their sum for diagram

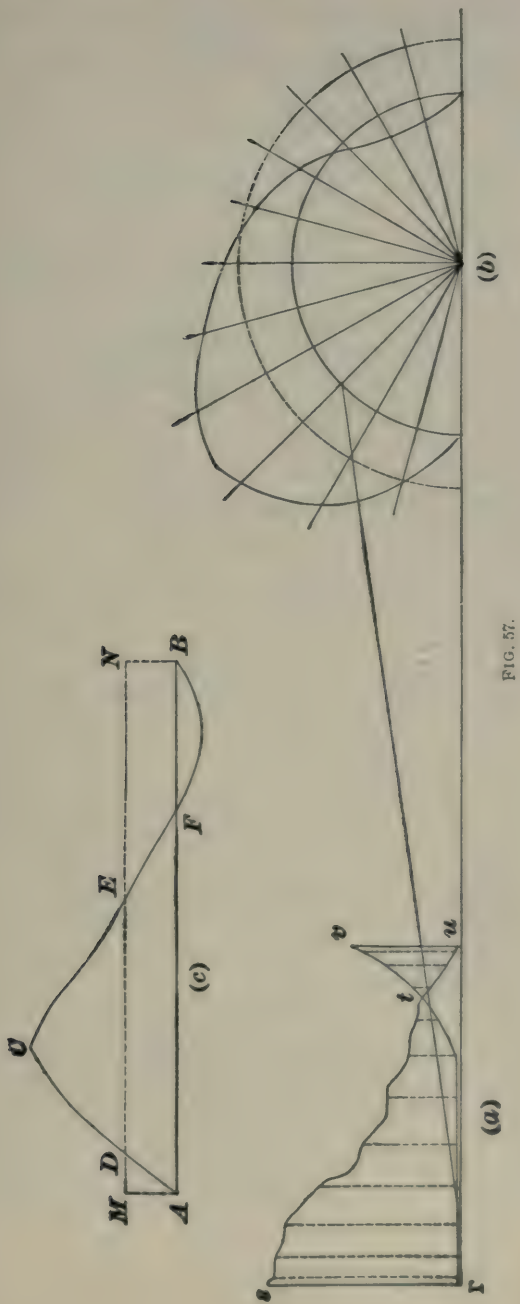

FIG. 57.

A is found to be 6.04″, and for B, 6.62″. The mean ordinate for $A = \frac{6.04}{10} = .604$, and for B, $\frac{6.62}{10} = .662$. Hence, the total M. E. P. $= \frac{.604 + .662}{2} \times 60 = 37.98$ lb. per sq. in.

$$\text{I. H. P.} = \frac{37.98 \times 1 \times 13^2 \times .7854 \times 300 \times 2}{33{,}000} = 91.66. \quad \text{Ans.}$$

(686) Drawing a line parallel to the atmospheric line, and at a distance from the vacuum line corresponding to a pressure of 40 lb., absolute, the length included between the bounding lines of diagram A is 1.46″, and the length between the bounding lines of diagram B is 1.72″. The length of both diagrams is 2.6″. The weight of a cubic foot of steam at an absolute pressure of 40 lb. is .097231 lb. The M. E. P. for card $A = .604 \times 60 = 36.24$ lb., and for card B, $.662 \times 60 = 39.72$ lb. (See example 685.) Substituting in formula **101,** we have, for card A,

$$Q = \frac{13{,}750 \times 1.46 \times .097231}{36.24 \times 2.6} = 20.716 \text{ lb.},$$

and for card B,

$$Q = \frac{13{,}750 \times 1.72 \times .097231}{39.72 \times 2.6} = 22.267 \text{ lb.}$$

Taking the average of both cards,

$$Q = \frac{20.716 + 22.267}{2} = 21.49 \text{ lb. per I. H. P. per hour.} \quad \text{Ans.}$$

(687) In Fig. 57 is shown the solution to this question. The cut is not drawn to scale, and the student should redraw it full size. By referring to Art. **1313,** the student should have no difficulty.

Find the areas of $A\ C\ F$ and $F\ B$; subtract the smaller from the larger, and divide by the length $A\ B$; the result is the height $A\ M$. Draw $M\ N$ parallel to $A\ B$; also the dotted semicircle, at a distance from the full semicircle equal to $A\ M$.

(688) Compute the area of $D\ C\ E\ D$ in the usual manner; for the present case it equals .78 sq. in. The stroke

of the engine $= 12'' = 1$ ft. and the crank-pin travels in one stroke a distance $= \frac{1 \times 3.1416}{2} = 1.5708$ feet. The length $AB = 4.08''$. Consequently, $1''$ of length on $AB = \frac{1.5708}{4.08} =$ .385 ft. of crank-pin travel. Since the vertical scale of pressures is $1'' = 60$, H, in formula **107**, $= .78 \times 60 \times .385 =$ 18.018 lb. per sq. in. of piston area. Substituting in formula **107**, we have $W = \frac{AHg}{n^2 E v_e^2}$

$$= \frac{13^2 \times .7854 \times 18.018 \times 32.16}{5^2 \times \frac{1}{50} \times \left(\frac{300 \times 2 \times 1 \times 3.1416}{2 \times 60}\right)^2} = 623 \text{ lb.} \quad \text{Ans.}$$

STRENGTH OF MATERIALS.

(QUESTIONS 689–748.)

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(689) See Arts. **1336, 1339,** and **1338.**

(690) See Arts. **1344, 1345, 1352,** and **1354.**

(691) See Art. **1347.**

(692) Use formula **110.**

$$E = \frac{Pl}{A e}; \text{ therefore, } e = \frac{Pl}{AE}.$$

$A = .7854 \times 2^2$; $l = 10 \times 12$; $P = 40 \times 2{,}000$; $E = 25{,}000{,}000$.

Therefore, $e = \dfrac{40 \times 2{,}000 \times 10 \times 12}{.7854 \times 4 \times 25{,}000{,}000} = .12223''$. Ans.

(693) Using formula **110,**

$$E = \frac{Pl}{A e} = \frac{7{,}000 \times 7\frac{1}{2}}{.7854 \times (\frac{1}{2})^2 \times .009} = 29{,}708{,}853.2 \text{ lb. per sq. in.}$$

Ans.

(694) Using formula **110,**

$$E = \frac{Pl}{A e}, \text{ or } P = \frac{A e E}{l} = \frac{1\frac{1}{2} \times 2 \times .006 \times 15{,}000{,}000}{9 \times 12} =$$

2,500 lb. Ans.

(695) By formula **110,**

$$E = \frac{Pl}{A e}, \text{ or } l = \frac{A e E}{P} = \frac{.7854 \times 3^2 \times .05 \times 1{,}500{,}000}{2{,}000} = 265.07''.$$

Ans.

(696) Using a factor of safety of 4 (see Table 28), formula **108** becomes

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$$P = \frac{A\,S_1}{4}, \text{ or } A = \frac{4\,P}{S_1} = \frac{4 \times 6 \times 2{,}000}{55{,}000} = .8727272 \text{ sq. in.}$$

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{.8727272}{.7854}} = 1.054''. \quad \text{Ans.}$$

(697) From Table 23, the weight of a piece of cast iron 1″ square and 1 ft. long is 3.125 lb.; hence, each foot of length of the bar makes a load of 3.125 lb. per sq. in. The breaking load—that is, the ultimate tensile strength—is 20,000 lb. per sq. in. Hence, the length required to break the bar is $\frac{20{,}000}{3.125} = 6{,}400$ ft. Ans.

(698) Let t = the thickness of the bolt head;

d = diameter of bolt.

Area subject to shear $= \pi\,d\,t$.

Area subjected to tension $= \frac{1}{4}\pi\,d^2$.

$S_1 = 55{,}000. \quad S_2 = 50{,}000.$

Then, in order that the bolt shall be equally strong in both tension and shear, $\pi\,d\,t\,S_2 = \frac{1}{4}\pi\,d^2\,S_1$,

$$\text{or } t = \frac{\pi\,d^2\,S_1}{4\pi\,d\,S_2} = \frac{d\,S_1}{4\,S_2} = \frac{\frac{3}{4} \times 55{,}000}{4 \times 50{,}000} = .206''. \quad \text{Ans.}$$

(699) Using a factor of safety of 15 for brick, formula **108** gives

$$P = \frac{A\,S_2}{15}.$$

$A = (2\frac{1}{2} \times 3\frac{1}{2})$ sq. ft. $= 30 \times 42 = 1{,}260$ sq. in.; $S_2 = 2{,}500$.

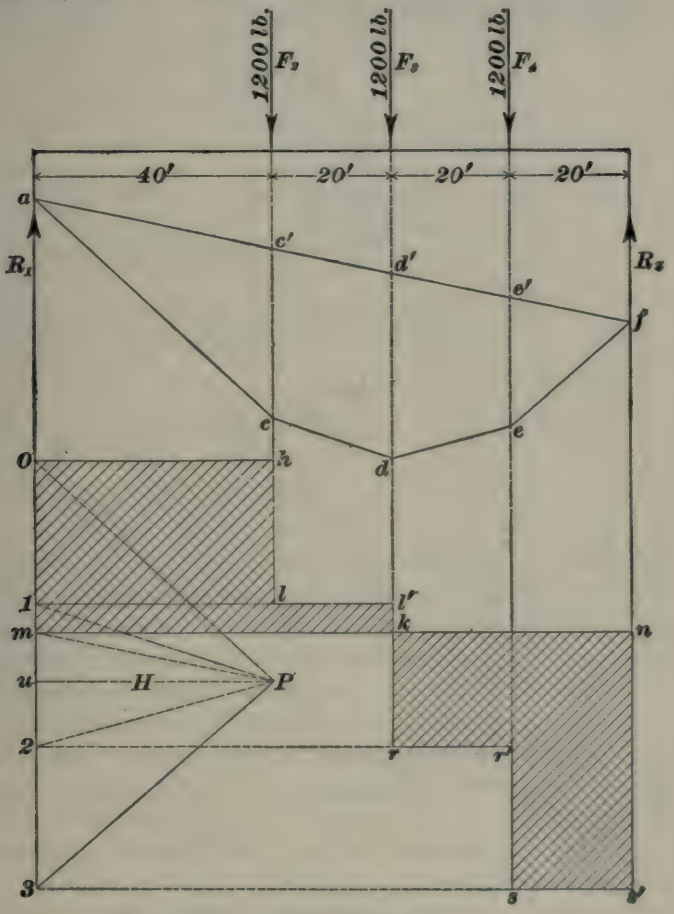
Therefore, $P = \frac{1{,}260 \times 2{,}500}{15} = 210{,}000$ lb. $= 105$ tons. Ans.

(700) The horizontal component of the force P is $P \cos 30° = 3{,}500 \times .866 = 3{,}031$ lb. The area A is $4\,a$, the ultimate shearing strength, S_s, 600 lb., and the factor of safety, 8.

Hence, from formula **108,**

$$P = \frac{A S_2}{8} = \frac{4 a S_2}{8} = \frac{a S_2}{2}. \quad a = \frac{2P}{S_2} = \frac{2 \times 3{,}031}{600} = 10.1''. \quad \text{Ans.}$$

(701) See Art. **1366.**



Scale of forces 1″=1600 lb.
Scale of distance 1″=32′

FIG. 58.

(702) Using formula **111,** with the factor of safety 4

$$p d = \frac{2 t S_1}{4} = \frac{t S_1}{2}, \text{ or } t = \frac{2 p d}{S_1} = \frac{2 \times 120 \times 48}{55{,}000}.$$

Since 40% of the plate is removed by the rivet holes, 60% remains, and the actual thickness required is

$$\frac{t}{.60} = \frac{2 \times 120 \times 48}{.60 \times 55{,}000} = .349''. \quad \text{Ans.}$$

(703) Using a factor of safety of 6, in formula **111**,

$$p\,d = \frac{2\,t\,S_1}{6} = \frac{t\,S_1}{3}.$$

Hence, $t = \frac{3\,p\,d}{S_1} = \frac{3 \times 6 \times 200}{20{,}000} = .18''$. Ans.

(704) Using formula **114**, with a factor of safety of 10,

$$p = \frac{9{,}600{,}000\,t^{2.18}}{10\,l\,d} = 960{,}000\frac{t^{2.18}}{l\,d}.$$

Hence, $t = \sqrt[2.18]{\frac{p\,l\,d}{960{,}000}} = \sqrt[2.18]{\frac{130 \times 12 \times 12 \times 3}{960{,}000}} = .272''$. Ans.

(705) From formula **113**,

$$p = \frac{S\,t}{r + t}, \text{ or } t = \frac{p\,r}{S - p} = \frac{2{,}000 \times \frac{4}{2}}{2{,}800 - 2{,}000} = \frac{4{,}000}{800} = 5''. \quad \text{Ans.}$$

(706) See Fig. 58. (*a*) Upon the load line, the loads *O*, *1*, *1-2*, and *2-3* are laid off equal, respectively, to F_2, F_3, and F_4; the pole *P* is chosen, and the rays drawn in the usual manner; the pole distance $H = 2{,}000$ lb. The equilibrium polygon is constructed by drawing *a c*, *c d*, *d e*, and *e f* parallel to *P O*, *P 1*, *P 2*, and *P 3*, respectively, and finally drawing the closing line *f a* to the starting point *a*. *P m* is drawn parallel to the latter line, dividing the load line into the reactions $m\,O = R_1$, and $3\,m = R_2$. The shear axis *m n* is drawn through *m*, and the shear diagram *O h l* .... *s' n m O* is constructed in the usual manner. To the scale of forces $m\,O = 1{,}440$ lb., and $3\,m = 2{,}160$ lb. To the scale of distances the maximum vertical intercept $y = d'\,d = 31.2$ ft., which, multiplied by H, $= 31.2 \times 2{,}000 = 62{,}400$ ft.-lb. $= 748{,}800$ in.-lb. Ans.

(*b*) The shear at a point 30 ft. from the left support $= O\,m = 1{,}440$ lb. Ans.

(*c*) The maximum shear $= n\,s' = -2{,}160$ lb. Ans.

(707) See Fig. 59. Draw the force polygon *O 1 2 3 4 5 O* in the usual manner, *O 1* being equal to and parallel to F_1, *1-2* equal to and parallel to F_2, etc. *O 5* is the resultant.

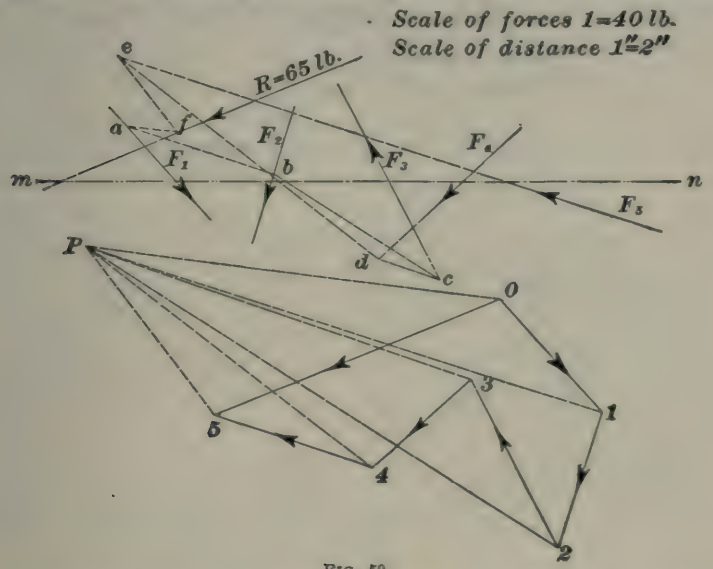


FIG. 59.

Choose the pole *P*, and draw the rays *P O*, *P 1*, *P 2*, etc. Choose any point, *a* on F_1, and draw through it a line parallel to the ray *P 1*. From the intersection *b* of this line with F_2, draw a line parallel to *P 2*; from the intersection *c* of the latter line with F_3 produced draw a parallel to *P 3*, intersecting F_4 produced in *d*. Finally, through *d*, draw a line parallel to *P 4*, intersecting F_5 produced in *e*. Now, through *a* draw a line parallel to *P O*, and through *e* a line parallel to *P 5*; their intersection *f* is a point on the resultant. Through *f* draw the resultant *R* parallel to *O 5*. It will be found by measurement that $R = 65$ lb., that it makes an angle of $22\frac{1}{2}°$ with *m n*, and intersects it at a distance of $1\frac{1}{4}''$ from the point of intersection of F_1 and *m n*.

(708) See Fig. 60. The construction is entirely similar to those given in the text. *O 1*, *1-2*, and *2-3* are laid off to represent F_1, F_2, and F_3; the pole *P* is chosen, and the rays

drawn. Parallel to the rays are drawn the lines of the equilibrium polygon *a b c d g a*. The closing line *g a* is found to be parallel to *P 1*. Consequently, *O 1* is the left reaction and *1-3* the right reaction, the former being 6 tons

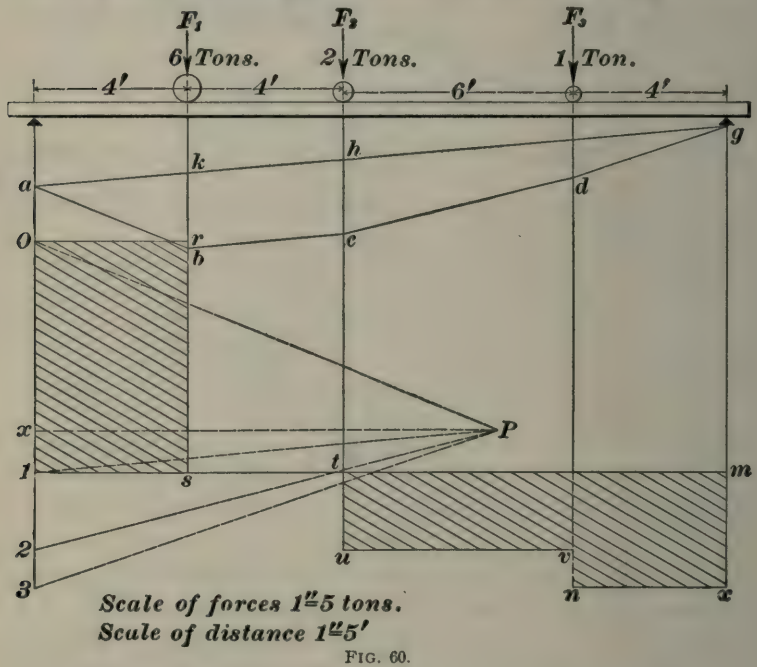


FIG. 60.

and the latter 3 tons. The shear diagram is drawn in the usual manner; it has the peculiarity of being zero between F_1 and F_2.

(709) The maximum moment occurs when the shear line crosses the shear axis. In the present case the shear line and shear axis coincide with *s t*, between F_1 and F_2; hence, the bending moment is the same (and maximum) at F_1 and F_2, and at all points between. This is seen to be true from the diagram, since *k h* and *b c* are parallel. Ans.

(*b*) By measurement, the moment is found to be $24 \times 12 = 288$ inch-tons. Ans.

(*c*) $288 \times 2{,}000 = 576{,}000$ inch-pounds. Ans.

(710) See Arts. **1375** to **1379.**

(711) See Fig. 61. The force polygon *O 1 2 3 4 O* is drawn as in Fig. 59, *O 4* being the resultant. The equilibrium polygon *a b c d g a* is then drawn, the point *g* lying on the resultant. The resultant *R* is drawn through *g*, parallel to and equal to *O 4*. A line is drawn through *C*, parallel to *R*. Through *g* the lines *g e* and *g f* are drawn parallel, respectively, to *P O* and *P 4*, and intersecting the parallel to *R*, through *C* in *e* and *f*; then, *e f* is the intercept, and *P u*, perpendicular to *O 4*, is the pole distance. *P u* = 33 lb.; *e f* = 1.32″. Hence, the resultant moment is 33 × 1.32 = 43.6 in.-lb. Ans.

Scale of forces 1″=50 lb.
Scale of distance 1″=2″

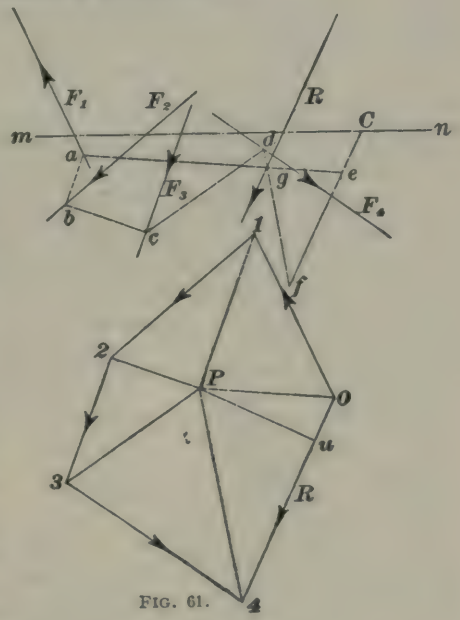


FIG. 61.

(712) The maximum bending moment, $M = W\frac{l_1 l_2}{l}$ (see Fig. 6 of table of Bending Moments) $= 4 \times 2{,}000 \times \frac{14 \times 8}{22} =$ $40{,}727\frac{3}{11}$ ft.-lb. = 488,727 in.-lb. Then, according to formula **117,**

$$\frac{S_t I}{f c} = 488{,}727.$$

$$\frac{I}{c} = \frac{488{,}727 f}{S_t} = \frac{488{,}727 \times 8}{9{,}000} = 434.424.$$

But, $\frac{I}{c} = \frac{\frac{1}{12} b d^3}{\frac{1}{2} d} = \frac{1}{6} b d^2$, and, according to the conditions of the problem, $b = \frac{1}{2} d$.

Therefore, $\frac{I}{c} = \frac{1}{6} b d^2 = \frac{1}{12} d^3 = 434.424.$

$$d^3 = 5{,}213.088.$$

$$\left.\begin{aligned} d &= 17\tfrac{1}{3}''. \\ b &= 8\tfrac{2}{3}''. \end{aligned}\right\} \text{Ans.}$$

(713) The beam, with the moment and shear diagrams, is shown in Fig. 62. On the line, through the left reaction,

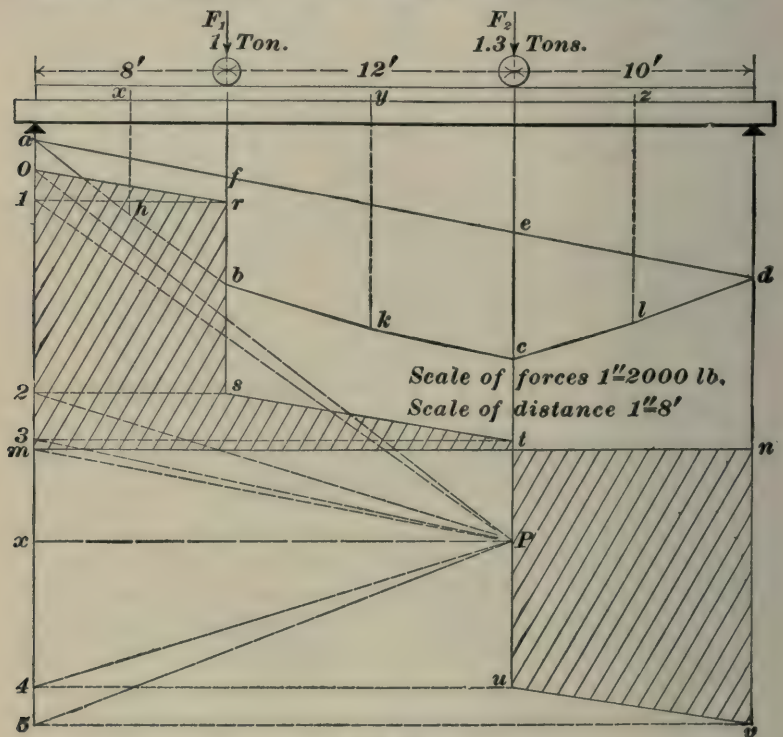


FIG. 62.

are laid off the loads in order. Thus, $O\,1 = 40 \times 8 = 320$ lb., is the uniform load between the left support and F_1; *1-2* is

$F_1 = 2{,}000$ lb.; $2\text{-}3 = 40 \times 12 = 480$ lb., is the uniform load between F_1 and F_2; $3\text{-}4 = 2{,}000 \times 1.3 = 2{,}600$ lb., is F_2, and $4\text{-}5 = 40 \times 10 = 400$ lb., is the uniform load between F_2 and the right support. The pole P is chosen and the rays drawn. Since the uniform load is very small compared with F_1 and F_2, it will be sufficiently accurate to consider the three portions of it concentrated at their respective centers of gravity x, y, and z. Drawing the equilibrium polygon parallel to the rays, we obtain the moment diagram $a\ h\ b\ k\ c\ l\ d\ a$. From P, drawing $P\,m$ parallel to the closing line $a\,d$, we obtain the reactions $O\,m$ and $m\,5$, equal, respectively, to 2,930 and 2,870 lb. Ans. The shear axis $m\,x$, and the shear diagram $O\ r\ s\ t\ u\ v\ n\ m$, are drawn in the usual manner. The greatest shear is $O\,m$, 2,930 lb. The shear line cuts the shear axis under F_2. Hence, the maximum moment is under F_2. By measurement, $e\,c$ is 64″, and $P\,x$ is 5,000 lb.; hence, the maximum bending moment is $64 \times 5{,}000 = 320{,}000$ in.-lb. Ans.

(714) From the table of Bending Moments, the greatest bending moment of such a beam is $\frac{w\,l^2}{8}$, or, in this case, $\frac{w \times 240^2}{8}$.

By formula **117**,

$$M = \frac{w \times 240^2}{8} = \frac{S_t I}{f\,c} = \frac{45{,}000}{4} \times \frac{280}{12 \div 2}.$$

Therefore, $w = \frac{45{,}000 \times 280 \times 8}{240 \times 240 \times 4 \times 6} = 72.92$ lb. per inch of length $= 72.92 \times 12 = 875$ lb. per foot of length. Ans.

(715) From the table of Bending Moments, the maximum bending moment is

$$\frac{W\,l}{4} = \frac{W \times 96}{4} = 24\,W.$$

From formula **117**,

$$M = 24\,W = \frac{S_t I}{f\,c}.$$

$I = \frac{\pi}{64}(d^4 - d_1^4) = 56.945$; $c = \frac{1}{2}\,d = \frac{6\frac{1}{2}}{2} = 3\frac{1}{4}$; $S_t = 38{,}000$; $f = 6$.

Hence, $24\,W = \dfrac{38{,}000 \times 56.945}{6 \times 3.25}$.

$$W = \frac{38{,}000 \times 56.945}{24 \times 6 \times 3.25} = 4{,}624 \text{ lb.} \quad \text{Ans.}$$

(716) (*a*) From the table of Bending Moments,

$$M = \frac{w\,l^2}{8} = \frac{w \times 192^2}{8}.$$

From formula **117,**

$$M = \frac{w \times 192^2}{8} = \frac{S_1 I}{f c}.$$

$S_1 = 7{,}200$; $f = 8$; $I = \dfrac{1}{12} b\,d^3 = \dfrac{2{,}000}{12}$; $c = \dfrac{1}{2} d = 5$.

Then, $\dfrac{w \times 192^2}{8} = \dfrac{7{,}200}{8} \times \dfrac{2{,}000}{12 \times 5}$.

$$w = \frac{7{,}200 \times 2{,}000 \times 8}{8 \times 12 \times 5 \times 192 \times 192} =$$

6.51 lb. per in. $= 6.51 \times 12 = 78.12$ lb. per ft. Ans.

(*b*) $I = \dfrac{1}{12} b\,d^3 = \dfrac{10 \times 2^3}{12} = \dfrac{80}{12}$. $c = 1''$.

$$\frac{w \times 192^2}{8} = \frac{7{,}200}{8} \times \frac{80}{12 \times 1}.$$

$$w = \frac{7{,}200 \times 80 \times 8}{8 \times 12 \times 192 \times 192} = 1.3 \text{ lb. per in.}$$

$$= 1.3 \times 12 = 15.6 \text{ lb. per ft.} \quad \text{Ans.}$$

(717) (*a*) From the table of Bending Moments, the deflection of a beam uniformly loaded is $\dfrac{5}{384} \dfrac{W l^3}{E I}$. In example 714, $W = 875 \times 20 = 17{,}500$ lb.; $l = 240''$, $E = 25{,}000{,}000$, and $I = 280$.

Hence, deflection $s = \dfrac{5 \times 17{,}500 \times 240^3}{384 \times 25{,}000{,}000 \times 280} = .45$ in. Ans.

(*b*) From the table of Bending Moments, $s = \dfrac{1}{48} \dfrac{W l^3}{E I}$.

In example 715, $W = 4{,}624$ lb.; $l = 96$ in.; $E = 15{,}000{,}000$, and $I = 56.945$.

Hence, $s = \dfrac{4{,}624 \times 96^3}{48 \times 15{,}000{,}000 \times 56.945} = .1''$, nearly. Ans.

(*c*) $s = \dfrac{5}{384}\dfrac{Wl^3}{EI}$. In example 716 (*a*), $W = 78.12 \times 16$; $l = 192$; $E = 1{,}500{,}000$, and $I = \dfrac{2{,}000}{12}$.

Hence, $s = \dfrac{5 \times 78.12 \times 16 \times 192^3}{384 \times 1{,}500{,}000 \times \frac{2000}{12}} = .461''$. Ans.

(718) Area of piston $= \frac{1}{4}\pi d^2 = \frac{1}{4}\pi \times 14^2$.

$W =$ pressure on piston $= \frac{1}{4}\pi \times 14^2 \times 80$.

From the table of Bending Moments, the maximum bending moment for a cantilever uniformly loaded is $\dfrac{wl^2}{2} = \dfrac{Wl}{2} = \dfrac{\frac{1}{4}\pi \times 14^2 \times 80 \times 4}{2} = \dfrac{S_1}{f}\dfrac{I}{c}$. (See formula **117.**)

$$\frac{S_1}{f} = \frac{45{,}000}{10} = 4{,}500. \quad \frac{I}{c} = \frac{\frac{1}{64}\pi d^4}{\frac{1}{2}d} = \frac{1}{32}\pi d^3.$$

$$\text{Hence, } \frac{\frac{1}{4}\pi \times 14^2 \times 80 \times 4}{2} = \frac{4{,}500\,\pi d^3}{32},$$

$$\text{or } d^3 = \frac{14^2 \times 80 \times 4 \times 32}{4 \times 2 \times 4{,}500} = 55.75.$$

$$d = \sqrt[3]{55.75} = 3.82''. \quad \text{Ans.}$$

(719) Substituting in formula **119,** $S_2 = 90{,}000$; $A = 6^2 \times .7854$; $f = 6$; $l = 14 \times 12 = 168$; $g = 5{,}000$; $I = \dfrac{\pi}{64} \times 6^4$, we obtain

$$W = \frac{S_2 A}{f\left(1 + \dfrac{Al^2}{gI}\right)} = \frac{90{,}000 \times 6^2 \times .7854}{6\left(1 + \dfrac{6^2 \times .7854 \times 168^2}{5{,}000 \times \dfrac{3.1416 \times 6^4}{64}}\right)} = 120{,}872 \text{ lb.}$$

Ans.

(720) For timber, $S_2 = 8{,}000$ and $f = 8$; hence, $\dfrac{S_2}{f} = \dfrac{8{,}000}{8} = 1{,}000$.

Substituting in formula **108,**

$$P = A\frac{S_2}{f} = 1{,}000\,A.$$

$A = \frac{P}{1{,}000} = \frac{7 \times 2{,}000}{1{,}000} = 14$ sq. in., necessary area of a *short* column to support the given load. Since the column is quite long, assume it to be 6″ square. Then, $A = 36$, and $I = \frac{1}{12}b^4 = \frac{6^4}{12} = 108$.

Formula **119** gives

$$W = \frac{S_2 A}{f\left(1 + \frac{A\,l^2}{g\,I}\right)}, \text{ or } \frac{S_2}{f} = \frac{W}{A}\left(1 + \frac{A\,l^2}{g\,I}\right).$$

$l = 30 \times 12 = 360$, and $g = 3{,}000$.

$$\frac{S_2}{f} = \frac{14{,}000}{36}\left(1 + \frac{36 \times 360^2}{3{,}000 \times 108}\right) = 5{,}990, \text{ nearly.}$$

Since this value is much too large, the column must be made larger. Trying 9″ square, $A = 81$, $I = 546\frac{3}{4}$.

Then, $\frac{S_2}{f} = \frac{14{,}000}{81}\left(1 + \frac{81 \times 360 \times 360}{3{,}000 \times 546\frac{3}{4}}\right) = 1{,}279.$

This value of $\frac{S_2}{f}$ is much nearer the required value, 1,000.

Trying 10″ square, $A = 100$, $I = \frac{10{,}000}{12} = 833\frac{1}{3}$.

$$\frac{S_2}{f} = \frac{14{,}000}{100}\left(1 + \frac{100 \times 360 \times 360}{3{,}000 \times 833\frac{1}{3}}\right) = 866, \text{ nearly.}$$

Since this value of $\frac{S_2}{f}$ is less than 1,000, the column is a little too large; hence, it is between 9 and 10 inches square. $9\frac{5}{8}''$ will give 997.4 lb. as the value of $\frac{S_2}{f}$; hence, the column should be $9\frac{5}{8}''$ square.

This problem may be more readily solved by formula **120,** which gives

$$c = \sqrt{\frac{7 \times 2000 \times 8}{2 \times 800} + \sqrt{\frac{7 \times 2000 \times 8}{8000}\left(\frac{7 \times 2000 \times 8}{4 \times 8000} + \frac{12 \times 360^2}{3000}\right)}} =$$

$$\sqrt{7 + \sqrt{14\,(3.5 + 518.4)}} = \sqrt{92.479} = 9.61'' = 9\tfrac{5}{8}'', \text{ nearly.}$$

(721) Here $W = 21{,}000$; $f = 10$; $S_2 = 150{,}000$; $g = 6{,}250$; $l = 7.5 \times 12 = 90''$. For using formula **121**, we have

$$\frac{.3183\ Wf}{S_2} = \frac{.3183 \times 21{,}000 \times 10}{150{,}000} = .4456.$$

$$\frac{16\,l^2}{g} = \frac{16 \times 8100}{6250} = 20.7360.$$

Therefore,

$$d = 1.4142\sqrt{.4456 + \sqrt{.4456\,(.4456 + 20.7360)}} =$$

$$1.4142\sqrt{.4456 + 3.0722} = 2.65'', \text{ or say } 2\tfrac{5}{8}''.$$

(722) For this case, $A = 3.1416$ sq. in.; $l = 4 \times 12 = 48''$; $S_2 = 55{,}000$; $f = 10$; $I = .7854$; $g = 20{,}250$.

Substituting these values in formula **119**,

$$W = \frac{S_2 A}{f\left(1 + \frac{A\,l^2}{g\,I}\right)} = \frac{55{,}000 \times 3.1416}{10\left(1 + \frac{3.1416 \times 48^2}{20{,}250 \times .7854}\right)} =$$

$$\frac{5{,}500 \times 3.1416}{1.4551}.$$

Steam pressure = 60 lb. per sq. in.

Then, area of piston $= .7854\,d^2 = \dfrac{W}{60} = \dfrac{5{,}500 \times 3.1416}{1.4551 \times 60}$.

Hence, $d^2 = \dfrac{5{,}500 \times 3.1416}{.7854 \times 1.4551 \times 60} = 252$, nearly,

and $d = \sqrt{252} = 15\tfrac{7}{8}''$, nearly. Ans.

(723) (*a*) The strength of a beam varies directly as the width and square of the depth and inversely as the length.

Hence, the ratio between the loads is

$$\frac{6 \times 8^2}{10} : \frac{4 \times 12^2}{16} = 16 : 15, \text{ or } 1\tfrac{1}{15}. \quad \text{Ans.}$$

(*b*) The deflections vary directly as the cube of the lengths, and inversely as the breadths and cubes of the depths.

Hence, the ratio between the deflections is

$$\frac{10^3}{6 \times 8^3} : \frac{16^3}{4 \times 12^3} = .549. \quad \text{Ans.}$$

(724) Substituting the value of c_1, from Table 31, in formula **123**, we obtain

$$(a)\quad d = c_1\sqrt[4]{\frac{H}{N}} = 4.92\sqrt[4]{\frac{40}{120}} = 3.739''.\quad \text{Ans.}$$

$$(b)\quad d = c_1\sqrt[4]{\frac{H}{N}} = 4.92\sqrt[4]{\frac{80}{100}} = 4.65''.\quad \text{Ans.}$$

(725) Using formula **123**,

$$d = 4.7\sqrt[4]{\frac{4{,}000}{50}} = 14.06''.$$

Since this result is greater than 13.6″, formula **124** must be used, in which

$$d = k_1\sqrt[3]{\frac{H}{N}} = 3.3\sqrt[3]{\frac{4{,}000}{50}} = 14.22''.\quad \text{Ans.}$$

(726) From formula **123**,

$$d = c_1\sqrt[4]{\frac{H}{N}},\ \text{or}\ H = \frac{d^4N}{c_1^4}.\quad c_1 = 4.11.\quad \text{(Table 31.)}$$

$$\text{Hence,}\ H = \frac{4^4 \times 80}{4.11^4} = 71.775\ \text{H.P.}\quad \text{Ans.}$$

(727) Using formula **126**,

$$H = q_1N\left(\frac{d_1^4 - d_2^4}{d_1}\right) = .0212 \times 100\left(\frac{(7\tfrac{1}{2})^4 - 5^4}{7\tfrac{1}{2}}\right) = 717.7\ \text{H.P.}\quad \text{Ans.}$$

(.0212 is the value of q_1 from Table 32.)

(728) (*a*) Using formula **127**,

$$(P = 100\,C^2 = 100 \times 8^2 = 6{,}400\ \text{lb.}\quad \text{Ans.}$$

(*b*) Using formula **128**,

$$P = 600\,C^2,\ \text{or}\ C^2 = \frac{P}{600} = \frac{6{,}000}{600} = 10.$$

$$C = \sqrt{10} = 3.162''.$$

$$d = \frac{1}{3}C = 1.054''.\quad \text{Ans.}$$

(*c*) Using formula **129**,

$$P = 1{,}000\,C^2;\ C^2 = \frac{P}{1{,}000} = \frac{6\tfrac{2}{3} \times 2{,}000}{1{,}000} = 13\tfrac{1}{3}.$$

$$C = \sqrt{13\tfrac{1}{3}} = 3.651''.\quad \text{Ans.}$$

(729) (*a*) Using formula **130**,

$$P = 12{,}000\,d^2 = 12{,}000 \times \left(\frac{7}{8}\right)^2 = 9{,}187.5 \text{ lb.} \quad \text{Ans.}$$

(*b*) Formula **131** gives $P = 18{,}000\,d^2$.

Therefore, $d = \sqrt{\dfrac{P}{18{,}000}} = \sqrt{\dfrac{8{,}000}{18{,}000}} = \sqrt{\dfrac{4}{9}} = .667''$. Ans.

(730) The deflection is by formula **118**, $s = a\dfrac{Wl^3}{EI} = \dfrac{1}{192}\dfrac{Wl^3}{EI}$, the coefficient being found from the table of Bending Moments.

Transposing, $W = \dfrac{192\,s\,E\,I}{l^3}$; $l = 120$; $E = 30{,}000{,}000$; $I = .7854$; $s = \dfrac{1}{8}$.

Then, $W = \dfrac{192 \times 30{,}000{,}000 \times .7854}{8 \times 120^3} = 327.25$ lb. Ans.

(731) (*a*) The maximum bending moment is, according to the table of Bending Moments, $\dfrac{Wl}{4} = \dfrac{6{,}000 \times 60}{4} = 90{,}000$ inch-pounds.

By formula **117**,

$$M = 90{,}000 = \frac{S_t}{f}\frac{I}{c}.$$

$S_t = 120{,}000$; $f = 10$. $\quad \dfrac{I}{c} = \dfrac{\frac{\pi d^4}{64}}{\frac{1}{2}d} = \dfrac{\pi d^3}{32}$.

$$\text{Hence, } 90{,}000 = \frac{120{,}000}{10}\,\frac{\pi d^3}{32},$$

$$\text{or } d = \sqrt[3]{\frac{90{,}000 \times 10 \times 32}{120{,}000 \times 3.1416}} = 4.244'' = 4\tfrac{1}{4}'', \text{ nearly.}$$

(*b*) Using formula **123**,

$$d = c_1\sqrt[4]{\frac{H}{N}} = 4.7\sqrt[4]{\frac{75}{80}} = 4\tfrac{5}{8}'', \text{ nearly.} \quad \text{Ans.}$$

(732) (*a*) The graphic solution is shown in Fig. 63. On the vertical through the support *O 1* is laid off equal to the uniform load between the support and F_1; *1-2* is laid off

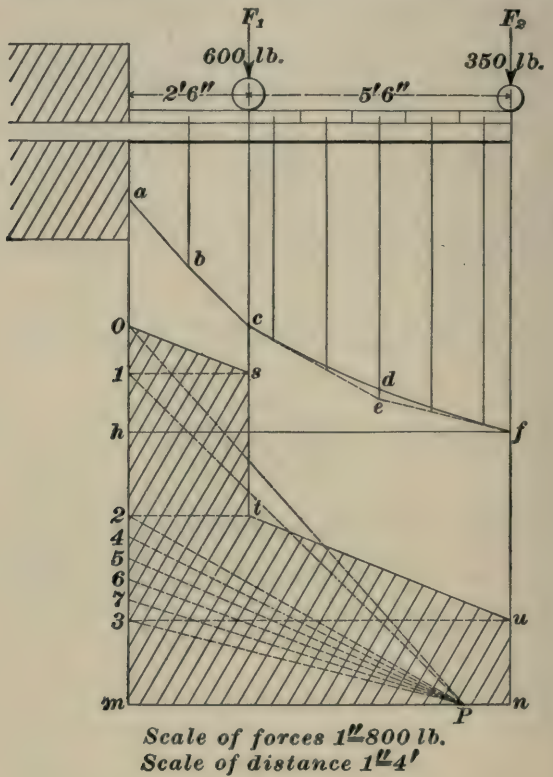


FIG. 63.

to represent F_1. *2-3* represents to the same scale the remainder of the uniform load, and *3 m* represents F_2. The pole *P* is chosen and the rays drawn. The polygon *a b c e f h* is then drawn, the sides being parallel, respectively, to the corresponding rays. If the uniform load between F_1 and F_2 be considered as concentrated at its center of gravity, the polygon will follow the broken line *c e f*. It will be better in this case to divide the uniform load into several parts, *2-4*, *4-5*, *5-6*, etc., thus obtaining the line of the polygon

c d f. To draw the shear diagram, project the point *1* across the vertical through F_1, and draw $O\,s$. Next project the point *2* across to *t*, and *3* across to *u*, and draw *t u*. *O s t u n m* is the shear diagram. The maximum moment is seen to be at the support, and is equal to $a\,h \times P\,m$. To the scale of distances, $a\,h = 58.8$ in., while $P\,m = H =$ 1,400 lb. to the scale of forces. Hence, the maximum bending moment is $58.8 \times 1{,}400 = 82{,}320$ in.-lb. Ans.

(*b*) From formula **117**,

$$M = \frac{S_1}{f}\frac{I}{c} = 82{,}320. \quad S_1 = 12{,}500;\ f = 8.$$

$$\text{Therefore, } \frac{I}{c} = \frac{82{,}320 \times 8}{12{,}500} = 52.68.$$

$$\text{But, } \frac{I}{c} = \frac{\frac{1}{12}\,b\,d^3}{\frac{1}{2}\,d} = \frac{b\,d^2}{6}. \text{ and } d = 2\tfrac{1}{2}\,b, \text{ or } b = \frac{2\,d}{5}.$$

$$\text{Hence, } \frac{I}{c} = \frac{b\,d^2}{6} = \frac{d^3}{15} = 52.68. \quad d^3 = 52.68 \times 15 = 790.2.$$

$$d = \sqrt[3]{790.2} = 9.245''. \quad b = \frac{2\,d}{5} = 3.7'', \text{ nearly.} \quad \text{Ans.}$$

(733) Referring to the table of Moments of Inertia,

$$I = \frac{(b\,d^2 - b_1\,d_1^2)^2 - 4\,b\,d\,b_1\,d_1(d - d_1)^2}{12\,(b\,d - b_1\,d_1)} =$$

$$\frac{[8 \times 10^2 - 6 \times (8\tfrac{1}{2})^2]^2 - 4 \times 8 \times 10 \times 6 \times 8\tfrac{1}{2}\,(10 - 8\tfrac{1}{2})^2}{12\,(8 \times 10 - 6 \times 8\tfrac{1}{2})} =$$

$$280.466.$$

$$c = \frac{d}{2} + \frac{b_1\,d_1}{2}\left(\frac{d - d_1}{b\,d - b_1\,d_1}\right) =$$

$$\frac{10}{2} + \frac{6 \times 8\tfrac{1}{2}}{2}\left(\frac{10 - 8\tfrac{1}{2}}{8 \times 10 - 6 \times 8\tfrac{1}{2}}\right) = 6.319.$$

(*a*) From the table of Bending Moments, the maximum bending moment is $\frac{W\,l}{4}$.

$$S_1 = 120{,}000;\ f = 7;\ l = 35 \times 12 = 420 \text{ in.}$$

Using formula **117**, $M = \frac{W\,l}{4} = \frac{S_1\,I}{f\,c}$, or

$$W = \frac{4\,S_1\,I}{l\,f\,c} = \frac{4 \times 120{,}000 \times 280.466}{420 \times 7 \times 6.319} = 7{,}246 \text{ lb.} \quad \text{Ans.}$$

(*b*) In this case $f = 5$, and the maximum bending moment is $\frac{w\,l^2}{8}$. Hence, from formula **117,**

$$M = \frac{w\,l^2}{8} = \frac{S_1\,I}{f\,c}, \text{ or } w = \frac{8\,S_1\,I}{l^2\,f\,c}.$$

Therefore, $W = w\,l = \frac{8\,S_1\,I}{l\,f\,c} = \frac{8 \times 120{,}000 \times 280.466}{420 \times 5 \times 6.319} =$ 20,290 lb. Ans.

(734) (*a*) According to formula **115,**

$$I = A\,r^2, \text{ or } r = \sqrt{\frac{I}{A}} = \sqrt{\frac{72}{24}} = \sqrt{3} = 1.732. \quad \text{Ans.}$$

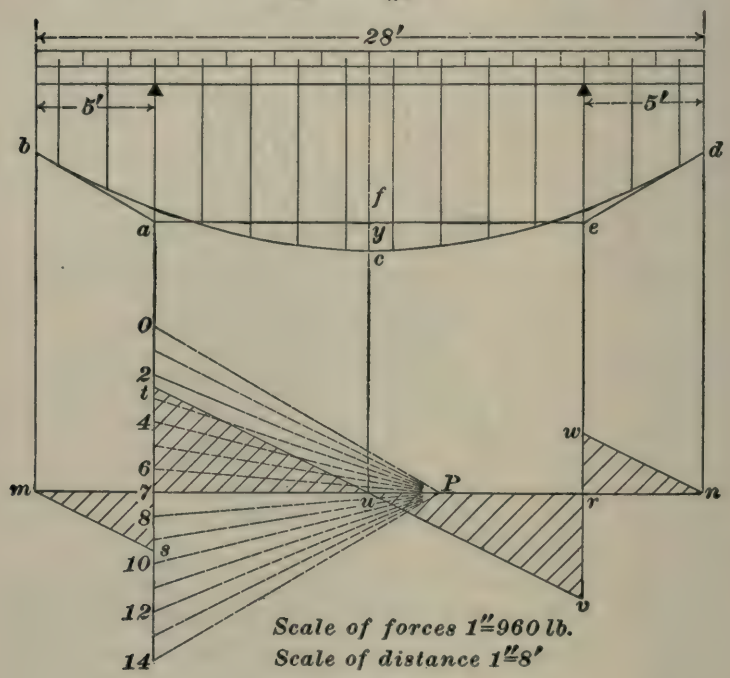


FIG. 64.

(*b*) From the table of Moments of Inertia, $I = \frac{1}{12}\,b\,d^3 =$ 72; $A = b\,d = 24$. Dividing, $\frac{\frac{1}{12}\,b\,d^3}{b\,d} = \frac{72}{24}$, or $\frac{1}{12}\,d^2 = 3$. $d^2 =$ 36; $d = 6''$ and $b = 4''$. Ans.

(*c*) As above, $r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi d^2}{4}}} = \sqrt{\frac{d^2}{16}} = \frac{d}{4}$. Ans.

(735) Using formula **112**, $p\,d = 4\,t\,S$, we have $t = \frac{p\,d}{4\,S}$.

Using a factor of safety of 6,

$$p\,d = \frac{4\,t\,S}{6}, \text{ or } t = \frac{6\,p\,d}{4\,S} = \frac{6 \times 100 \times 8}{4 \times 20{,}000} = .06'' \quad \text{Ans.}$$

(736) The graphic solution is shown in Fig. 64. The uniform load is divided into 14 equal parts, and lines drawn through the center of gravity of each part. These loads are laid off on the line through the left reaction, the pole P chosen, and the rays drawn. The polygon $b\,c\,d\,e\,f\,a$ is then drawn in the usual manner. The shear diagram is drawn as shown. The maximum shear is either $t\,7$ or $r\,v = 540$ lb. The maximum moment is shown by the polygon to be at $f\,c$ vertically above the point u, where the shear line crosses the shear axis. The pole distance $P\,7$ is 1,440 lb. to the scale of forces, and the intercept $f\,c$ is 14 inches to the scale of distances. Hence, the bending moment is 20,160 in.-lb.

(737) From formula **117**,

$$M = \frac{S_t}{f}\frac{I}{c} = 20{,}160. \quad S_t = 9{,}000;\ f = 8.$$

$$\text{Then, } \frac{I}{c} = \frac{20{,}160 \times 8}{9{,}000} = 17.92.$$

$$\text{But, } \frac{I}{c} = \frac{\frac{1}{12}\,b\,d^3}{\frac{1}{2}\,d} = \frac{1}{6}\,b\,d^2 \text{ for a rectangle.}$$

$$\text{Hence, } \frac{1}{6}\,b\,d^2 = 17.92, \text{ or } b\,d^2 = 107.52.$$

Any number of beams will fulfil this condition.

$$\text{Assuming } d = 6'',\ b = \frac{107.52}{36} = 3'', \text{ nearly.}$$

$$\text{Assuming } d = 5'',\ b = \frac{107.52}{25} = 4.3''.$$

(738) Using the factor of safety 10, in formula **114**,

$$p = \frac{9{,}600{,}000}{10}\,\frac{t^{2.18}}{l\,d} = \frac{960{,}000 \times .2^{2.18}}{108 \times 2.5} = 106.43 \text{ lb.} \quad \text{Ans}$$

(739) Using formula **130,**

$$P = 12{,}000\, d^2, \text{ or } d = \sqrt{\frac{P}{12{,}000}} = \sqrt{\frac{5 \times 2{,}000}{12{,}000}} = .913''. \text{ Ans.}$$

(740) The radius r of the gear-wheel is 24″. Using formula **123,** $d = c\sqrt[3]{Pr} = .297\ \sqrt[3]{350 \times 24} = 2.84''$. Ans.

(741) Area of cylinder $= .7854 \times 12^2 = 113.1$ sq. in.
Total pressure on the head $= 113.1 \times 90 = 10{,}179$ lb.

Pressures on each bolt $= \frac{10{,}179}{10} = 1{,}017.9$ lb.

Using formula **108,**

$$P = A\,S, \text{ or } A = \frac{P}{S} = \frac{1{,}017.9}{2{,}000} = .5089 \text{ sq. in., area of bolt.}$$

$$\text{Diameter of bolt} = \sqrt{\frac{.5089}{.7854}} = .8'', \text{ nearly.} \quad \text{Ans.}$$

(742) (*a*) The graphic solution is clearly shown in Fig. 65. On the vertical through F_1, the equal loads F_1 and F_2

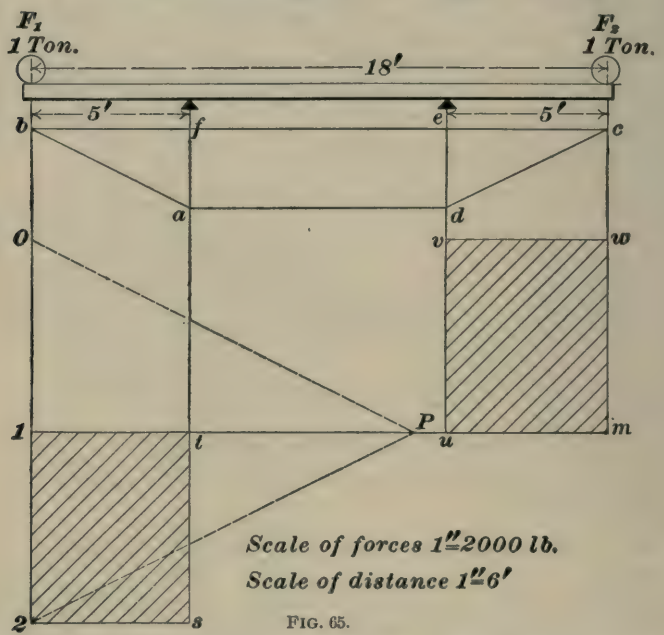


FIG. 65.

are laid off to scale, *O 1* representing F_1 and *1-2* representing

F_2. Choose the pole P, and draw the rays $P\,O$, $P\,1$, $P\,2$. Draw $a\,b$ between the left support and F_1 parallel to $P\,O$; $b\,c$ between F_1 and F_2 parallel to $P\,1$, and $c\,d$ parallel to $P\,2$, between F_2 and the right support. Through P draw a line parallel to the closing line $a\,d$. $O\,1 = 1\text{-}2$; hence, the reactions of the supports are equal, and are each equal to 1 ton. The shear between the left reaction and F_1 is negative, and equal to $F_1 = 1$ ton. Between the left and the right support it is 0, and between the latter and F_2 it is positive and equal to 1 ton. The bending moment is constant and a maximum between the supports. To the scale of forces $P\,1 = 2$ tons $= 4{,}000$ lb., and to the scale of distances $a\,f = 30$ in. Hence, the maximum bending is $4{,}000 \times 30 = 120{,}000$ in.-lb. Ans.

(*b*) Using formula **117**,

$$M = \frac{S_t}{f}\frac{I}{c} = 120{,}000. \quad S_t = 38{,}000;\ f = 6.$$

$$\text{Then, } \frac{I}{c} = \frac{120{,}000 \times 6}{38{,}000} = \frac{360}{19} = 19, \text{ nearly.}$$

$$\text{But, } \frac{I}{c} = \frac{\frac{\pi\, d^4}{64}}{\frac{d}{2}} = \frac{\pi\, d^3}{32}.$$

$$\text{Hence, } \frac{\pi\, d^3}{32} = 19, \text{ or } d^3 = \frac{32 \times 19}{3.1416}.$$

$$d = \sqrt[3]{\frac{32 \times 19}{3.1416}} = 5.784''. \quad \text{Ans.}$$

(743) Since the deflections are directly as the cubes of the lengths, and inversely as the breadths and the cubes of the depths, their ratio in this case is

$$\frac{18^3}{2 \times 6^3} : \frac{12^3}{3 \times 8^3}, \text{ or } \frac{27}{2} : \frac{9}{8} = 12.$$

That is, the first beam deflects 12 times as much as the second. Hence, the required deflection of the second beam is $.3 \div 12 = .025''$. Ans.

(744) The key has a shearing stress exerted on two sections; hence, each section must withstand a stress of $\frac{20,000}{2} = 10,000$ pounds.

Using formula **108,** with a factor of safety of 10,

$$P = \frac{A S_s}{10}, \text{ or } A = \frac{10 P}{S_s} = \frac{10 \times 10,000}{50,000} = 2 \text{ sq. in.}$$

Let b = width of key;

t = thickness.

Then, $b t = A = 2$ sq. in. But, from the conditions of the problem,

$$t = \frac{1}{4} b.$$

$$\left.\begin{aligned} \text{Hence, } b t = \frac{1}{4} b^2 = 2;\ b^2 = 8;\ b = 2.828''. \\ t = \frac{2.828}{4} = .707''. \end{aligned}\right\} \text{ Ans.}$$

(745) From formula **118,** the deflection $s = a \frac{W l^3}{E I}$, and, from the table of Bending Moments, the coefficient a for the beam in question is $\frac{1}{48}$.

$W = 30$ tons $= 60,000$ lb.; $l = 54$ inches; $E = 30,000,000$; $I = \frac{\pi d^4}{64}$.

$$\text{Hence, } s = \frac{1 \times 60,000 \times 54^3}{48 \times 30,000,000 \times \frac{3.1416 \times 12^4}{64}} = .0064 \text{ in.} \quad \text{Ans.}$$

(746) (*a*) The circumference of a 7-strand rope is 3 times the diameter; hence, $C = 1\frac{1}{4} \times 3 = 3\frac{3}{4}''$.

Using formula **129,** $P = 1,000\ C^2 = 1,000 \times (3\frac{3}{4})^2 = 14,062.5$ lb. Ans.

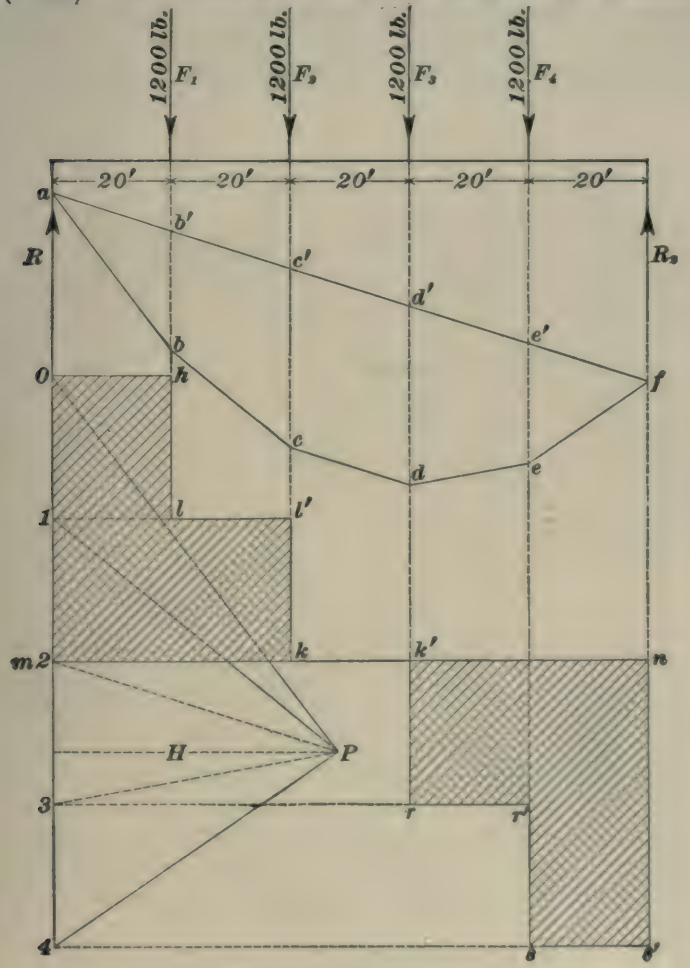
(*b*) Using formula **127,**

$$P = 100\ C^2, \text{ or } C = \sqrt{\frac{P}{100}} = \sqrt{\frac{1\frac{3}{4} \times 2,000}{100}} = 5.92''. \quad \textbf{Ans.}$$

(747) Using formula **113**,

$$p = \frac{S_1 t}{r + t} = \frac{120{,}000}{\frac{8}{2} + 6} = 12{,}000 \text{ lb.} \quad \text{Ans.}$$

(748) The construction of the diagram of bending



Scale of forces 1″=1600 lb.
Scale of distance 1″=32′

FIG. 66.

moments and shear diagram is clearly shown in Fig. 66. It is so nearly like that of Fig. 58 that a detailed description is unnecessary. It will be noticed that between k and k' the shear is zero, and that since the reactions are equal the shear at either support $= \frac{1}{2}$ of the load $= 2{,}400$ lb. The greatest intercept is $c\,c' = d\,d' = 30$ ft. The pole distance $H = 2{,}400$ lb. Hence, the bending moment $= 2{,}400 \times 30 =$ 72,000 ft.-lb. $= 72{,}000 \times 12 = 864{,}000$ in.-lb.

APPLIED MECHANICS.

(QUESTIONS 749–808.)

(749) See Fig. 341 in Art. **1434.** The construction is shown in Fig. 67.

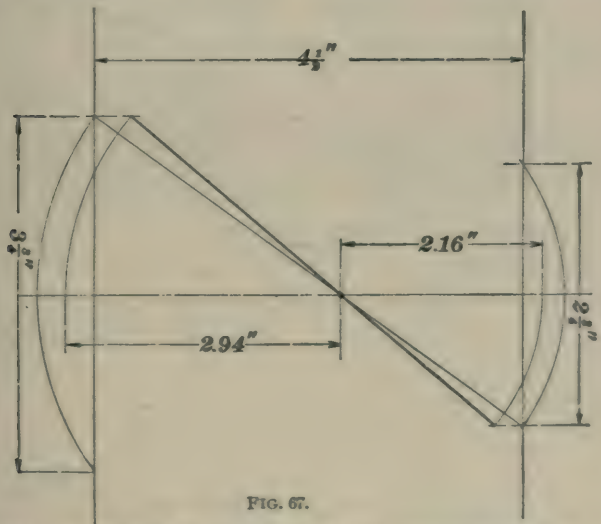


FIG. 67.

(750) The arms should be in the proportion 3 : 4. See Fig. 345 of Art. **1438.**

(751) In Fig. 68, let $A\ B$ and $C\ D$ be the two center lines of motion, making an angle of 10° with each other. Draw $a\ b$ parallel to $A\ B$, and at a distance of 3″ from it; also, $c\ d$ parallel to $C\ D$, and at a distance of 4″ from it. O, the point of intersection, will be the center of the lever, and $O\ E$ and $O\ H$ will be the center lines of the arms when in mid-position.

(752) The lengths of the lever arms will be as $3 \times 12 : 3.5 :: 36 : 3.5$. Either the motion shown in Fig. 343 or one of those shown in Fig. 344 may be used.

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(753) Circumference of pinion $= 12 \times 3.1416 = 37.699''$. Distance each moves $= 37.699 \times \frac{6.5}{360} = .68''$. Ans.

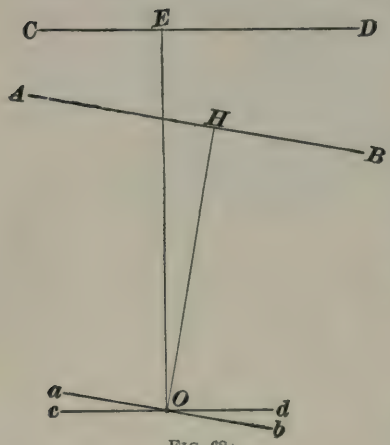

FIG. 68.

(754) The construction is shown in Fig. 69. This, in connection with the principles explained in Arts. **1432** to **1438**, should enable the student to lay out the motion.

(755) See Art. **1441**.

(756) The forward stroke, in this case, is the stroke from left to right, while the crank-pin moves through the lower part of its circle. Cross-head A leads during the forward stroke, and

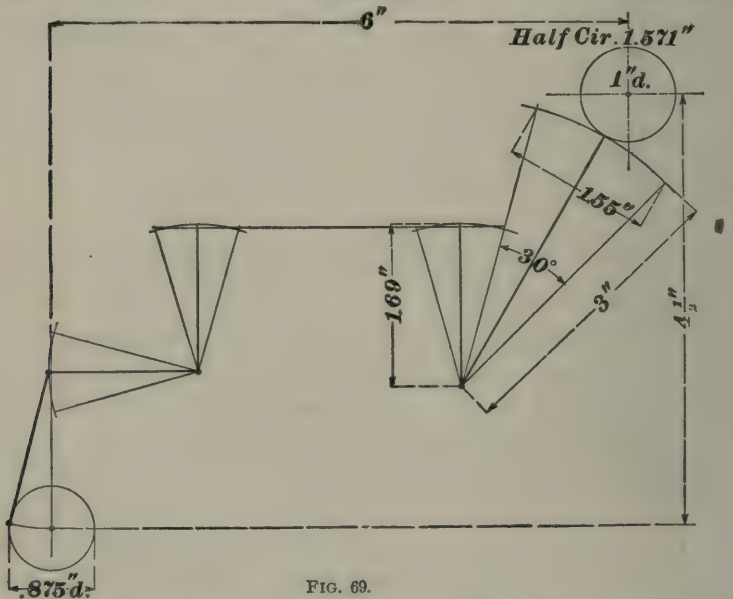

FIG. 69.

cross-head B during the return stroke. The construction to be used in determining this is shown in Fig. 349.

(757) The effect is the same as though one toggle-joint were used instead of two. First, find the pressure exerted by the screw. The distance that the power moves in one turn of the lever $= 20 \times 3.1416 = 62.832''$; distance that the nuts move $= \frac{1}{4}''$; velocity ratio $= \frac{62.832}{\frac{1}{4}} = 251.328$; force exerted $= 75 \times 251.328 = 18{,}849.6$ lb.

When distance $B = 16''$, the distance corresponding to h, in Fig. 352, $= \frac{16 - 12}{2} = 2''$. The distance corresponding to $O\,r$, therefore, equals $\sqrt{18^2 - 2^2} = \sqrt{320} = 17.888''$. Whence, applying formula **135,**

$$P = \frac{18{,}849.6 \times 17.888}{2 \times 2} = 84{,}295.4 \text{ lb.} \quad \text{Ans.}$$

(758) In the diagram in Fig. 70, $H\,E$, $C\,E$, $E\,R$, and

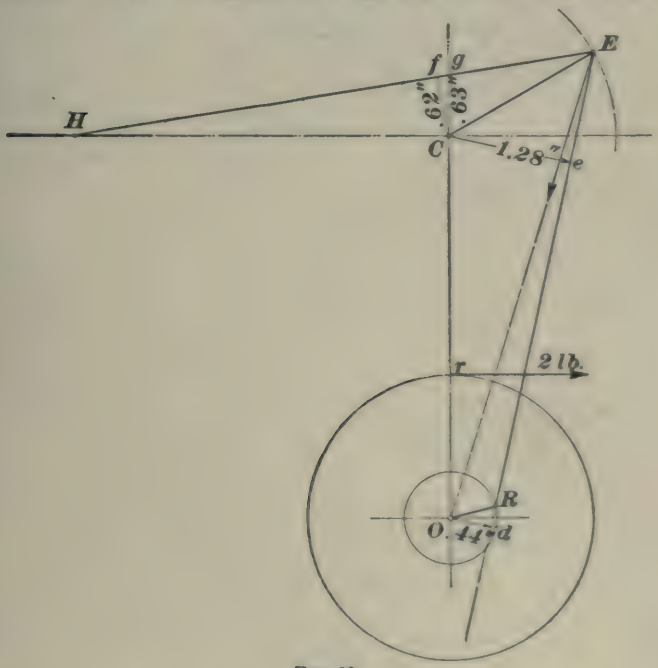

FIG. 70.

$O\,R$ are center lines of the parts which have the same letters in Fig. 22, question No. 758.

Draw $O\,d$ perpendicular to $E\,R$ produced; then by the principle of moments (see Art. **1447**), the thrust in the direction of $E\,R = \frac{2 \times O\,r}{O\,d} = \frac{2 \times 1.5}{.44} = 6.82$ lb. Erect the perpendicular $C\,e$, and draw $C\,f$ perpendicular to $H\,E$ and $C\,g$ perpendicular to $H\,C$. Then, the pull on the pin at H will equal $\frac{6.82 \times C\,e}{C\,f} = \frac{6.82 \times 1.28}{.62} = 14.08$ lb. Ans. Horizontal thrust of cross-head $= \frac{6.82 \times C\,e}{C\,g} = \frac{6.82 \times 1.28}{.63} = 13.86$ lb. Ans.

(759) See Art. **1457.**

(760) The construction is shown in Fig. 71. The various points are lettered in the same manner as in Fig. 358, and the description in Art. **1452** will apply to Fig. 71 in all respects, except that the circle must be divided into five equal parts instead of three, and the arc $d\,c\,b$ must contain three of these parts in order to have the ratio 3:2.

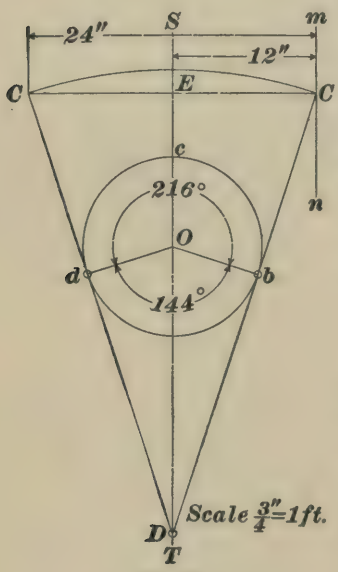

FIG. 71.

(761) (*a*) Fig. 72 (*a*) shows a diagram giving the proportions of the various parts; the arc $a\,e\,g$ is one-half as long as the arc $a\,c\,g$. See Arts. **1454** and **1455.**

(*b*) $360 \div 20 = 18$; hence, the crank-pin circle $d\,c\,b\,e$ must be divided into 18 equal parts. The motion is shown in Fig. 72 (*b*). See Art. **1455.**

(762) See Art. **1459.** (*a*) Greatest rate $= 50 \times \frac{1}{\cos 28°} = \frac{50}{.88295} = 56.63$ rev. Least rate $= 50 \times .88295 = 44.15$ rev.

(*b*) Connecting as described would double the variation

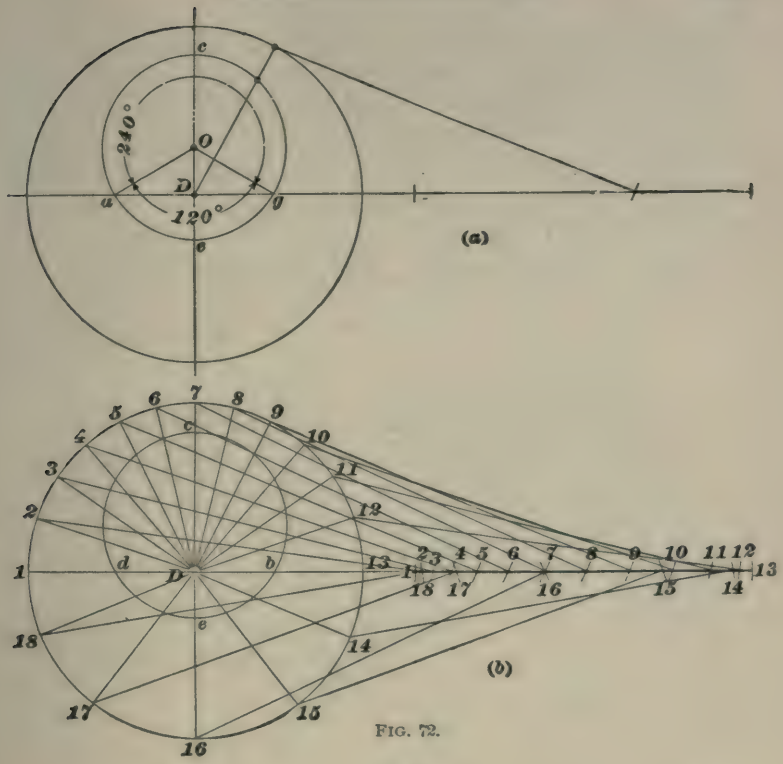
c
240°
O
D
a
120°
g
e
(a)
5 6 7 8 9 10 11 12 13
4 c
3
2
1 d D b
18 17 16 15 14
e
(b)

FIG. 72.

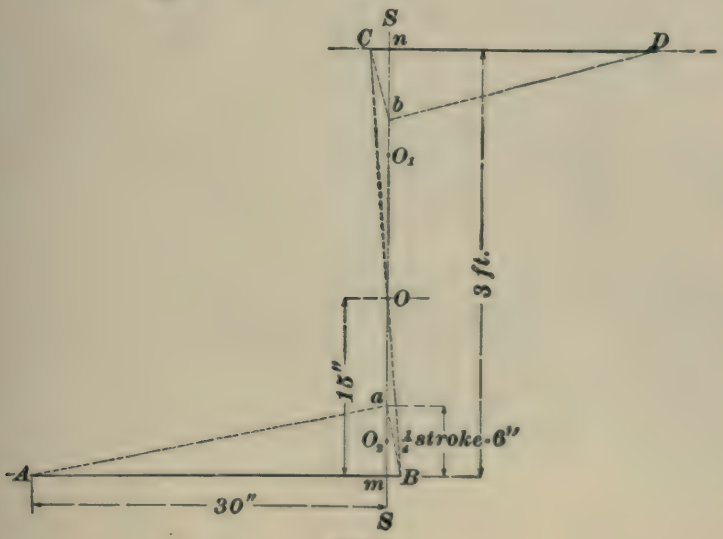
S
C n D
b
O1
3 ft.
O
15"
a
O2
¼ stroke-6"
A
m
B
30"
S

FIG. 73.

(763) In Fig. 73, which is lettered the same as Fig. 364, draw the given center lines and locate points n and A from the data of the problem. Lay off $m\,a = \frac{1}{4}$ stroke; also $n\,b = \frac{1}{4}$ stroke. Connect A and a and draw $a\,B$ perpendicular to $A\,a$. Through O, draw $B\,O\,C$; connect C and b, and through b draw $b\,D$ perpendicular to $C\,b$. Measure the lengths $C\,D$ and $A\,B$.

(764) The construction of the cam is shown in Fig. 74.

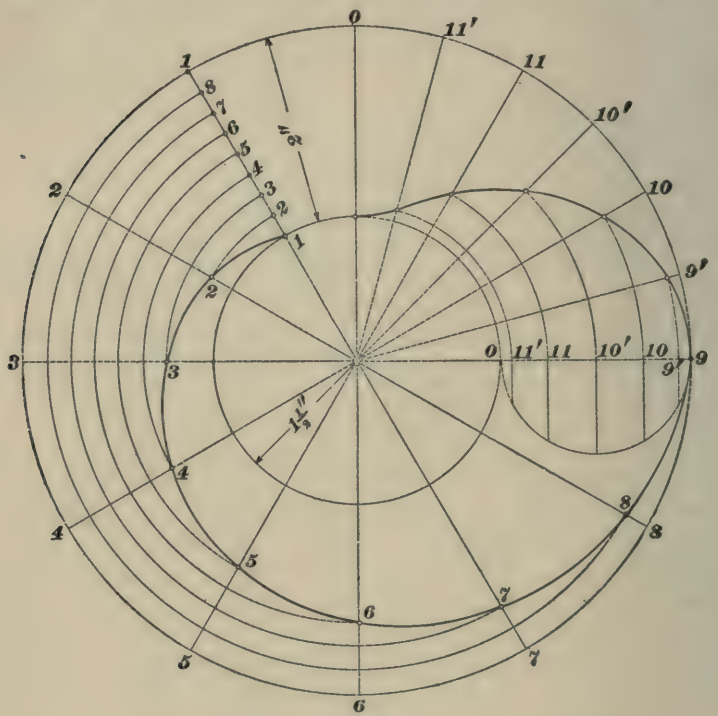

FIG. 74.

(765) The construction of the cam is shown in Fig. 75. It is similar in principle to the one shown in Fig. 366.

(766) The shape of the development of the groove is shown in Fig. 76. The method of laying it out is also indicated.

(767) This cam is shown in Fig. 77, and is like the one shown in Fig. 369 in all respects except that the dimensions are different. The description given in Art. **1474** will apply to this case.

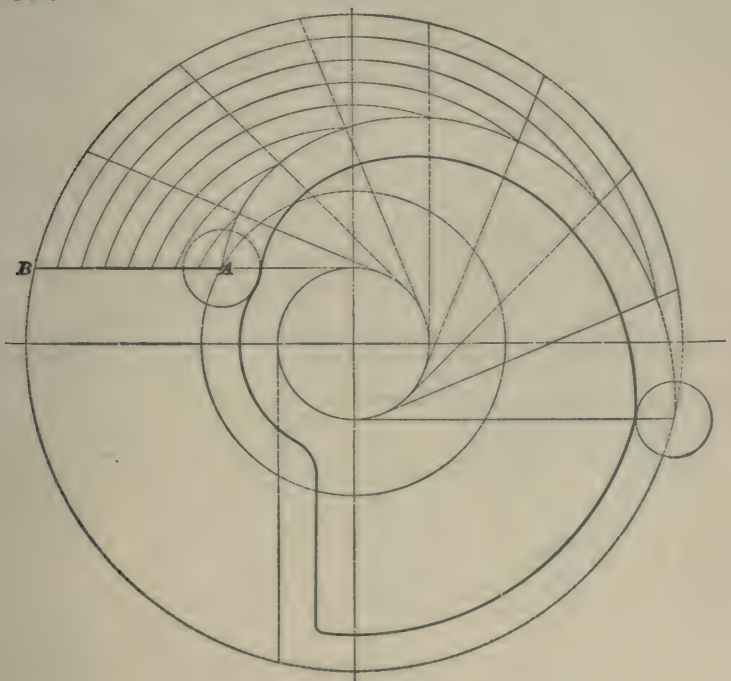

FIG. 75.

(768) See Art. **1478.** $\frac{52 \times 320}{48} = 346\frac{2}{3}$ rev. per min. Ans.

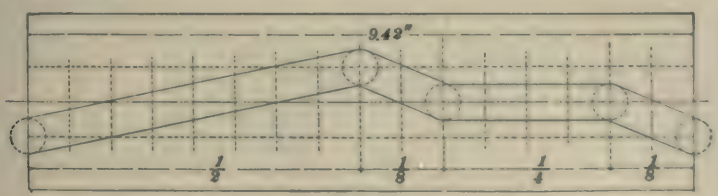

FIG. 76.

(769) $\frac{500 \times 10}{210} = 23.8'' = 23\frac{4}{5}''$. Ans.

(770) Substituting in formula **137**, $189 \times D_1 \times 20 = 1{,}800 \times 6 \times 3.5$, or

$$D_1 = \frac{1{,}800 \times 6 \times 3.5}{189 \times 20} = 10''. \quad \text{Ans.}$$

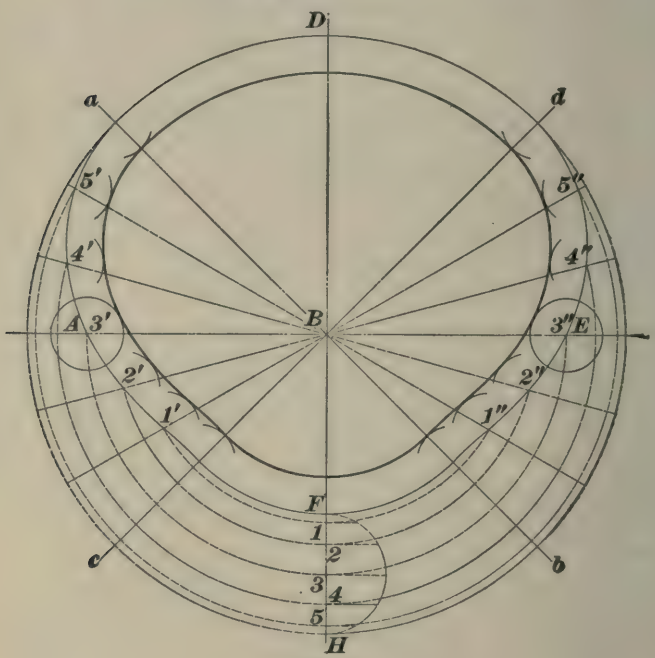

FIG. 77.

(771) See Art. **1481.** $900 \div 150 = 6$; $2 \times 3 = 6$, and $4 \times 1\frac{1}{2} = 6$. The ratios would then be 2 : 1 and 3 : 1; also 4 : 1 and $1\frac{1}{2}$: 1. Other ratios could be used.

(772) See Art. **1483.** (*a*) Effective pull in foot-pounds $= \dfrac{5 \times 33{,}000}{200 \times 3.1416 \times 3} = 87.54$ lb., nearly. Ans.

(*b*) Diameter in inches $= \dfrac{5 \times 33{,}000 \times 12}{50 \times 200 \times 3.1416} = 63$ inches. Ans.

(773) By laying out the two pulleys, the arc of contact is found to be about 150°. The allowable effective pull per inch in width, taken from the table is, therefore, 33.8 lb. Ans.

The effective pull of the belt $= \frac{6 \times 33{,}000 \times 12}{4 \times 3.1416 \times 1{,}500} = 126.05$ lb. $126.05 \div 33.8 = 3.7$ inches. A 4-inch belt would be used. Ans.

(774) The speed of the belt in feet per minute $=$ $14 \times 3.1416 \times 90$. Substituting in formula **142,**

$$W = \frac{630 \times 250}{14 \times 3.1416 \times 90} = 40'', \text{ nearly.}$$

The wheel should be 41 inches wide. Ans.

(775) Speed of belt $= 2 \times 205 \times 3.1416$. Substituting in formula **138,** $H = \frac{6 \times 2 \times 205 \times 3.1416}{900} = 8.6$ H. P., nearly. Ans.

(776) Calculating first for a double belt, we find that

$$H = \frac{6 \times 2 \times 205 \times 3.1416}{630} = 12.3, \text{ nearly.}$$

Next, calculating for a single belt with 30-inch pulley, we find that

$$H = \frac{6 \times 2.5 \times 205 \times 3.1416}{900} = 10.7, \text{ nearly.}$$

The double belt would, therefore, be the more effectual remedy. Ans.

(777) (*a*) Substituting in formula **147,**

$18 = d\sqrt{\frac{400}{70}}$; or $d = 18 \div \sqrt{5.7143} = 7.53$ inches. Ans.

(*b*) Substituting in formula **146,** $N = \sqrt{70 \times 400} = 167\frac{1}{3}$ rev. per min. Ans.

(778) (*a*) Substituting in formula **144,**

$$D = \frac{n_1 d}{N} = \frac{400 \times d}{225} = 1.78\,d.$$

Ratio, therefore, $= 1.78 : 1$. Ans.

(*b*) Substituting in formula **146,**

$225 = \sqrt{400 \times n_2}$, or $n_2 = \frac{225^2}{400} = 126.56$ rev. per min Ans

(779) First calculate the middle diameter, using formula **148.**

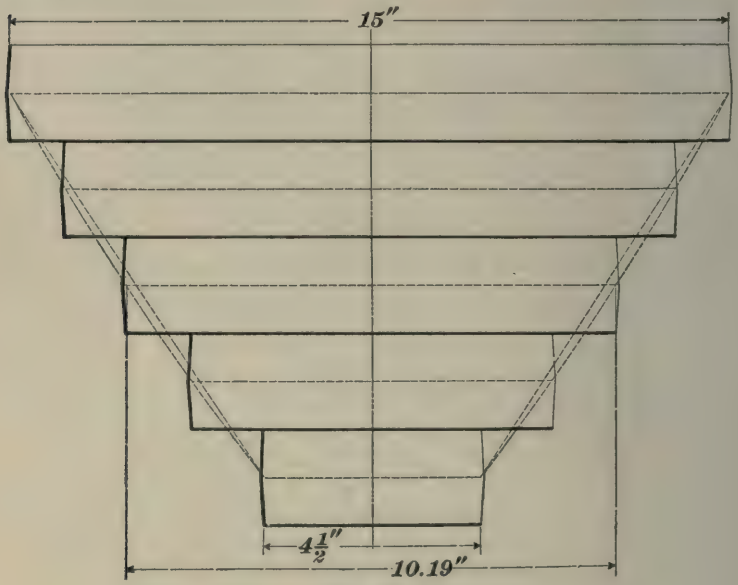

FIG. 78.

$$M = \frac{15 + 4.5}{2} + \frac{.08\,(15 - 4.5)^2}{20} = 9.75 + .44 = 10.19 \text{ inches.}$$

One of the pulleys is shown in Fig. 78.

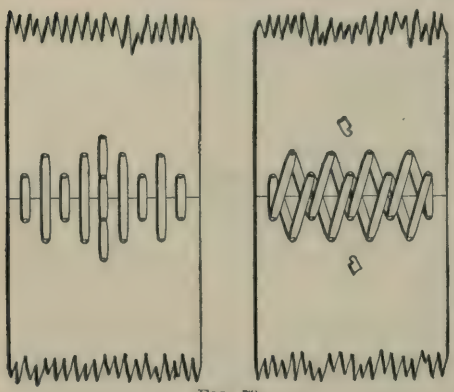

FIG. 79.

(780) See Fig. 79. An 8-inch belt should have a double row of holes, five in the row nearest the end. Space the holes equally.

(781) The arrangement of the pulleys is shown in Fig. 80. The belt leads from point *a*, on pulley *A*, into the plane

of pulley B; and from point b, on pulley B, into the plane of pulley A.

(782) Fig. 81 shows how the two shafts may be connected. The arrangement is entirely similar to that shown in Fig. 392.

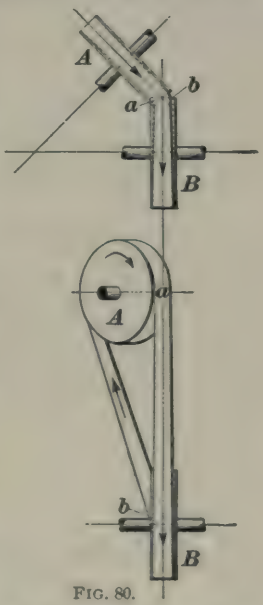

FIG. 80.

(783) $\frac{60 \times 30 \times 60 \times 30}{15 \times 15 \times 10} =$ number of revolutions of last shaft $= 1{,}440$ rev. per min. Ans.

The crossed belt and gears each reverse the direction of rotation; the open belt does not change the direction. Hence, the first and last shafts turn in the same direction.

(784) Gears B and C, being idlers, do not affect the number of rotations made by D. Hence, D makes $\frac{90}{90} = 1$ turn for every turn of A. As the number of axes is even, the first and last turn in opposite directions.

(785) Referring to Fig. 398, and applying formula **137**, we have $N \times d_1 \times d_2 = n \times f_1 \times f_2$, N and n being the number of revolutions of the spindle and lead screw, respectively. Solving for d_2,

$$d_2 = \frac{n \times f_1 \times f_2}{N \times d_1}.$$

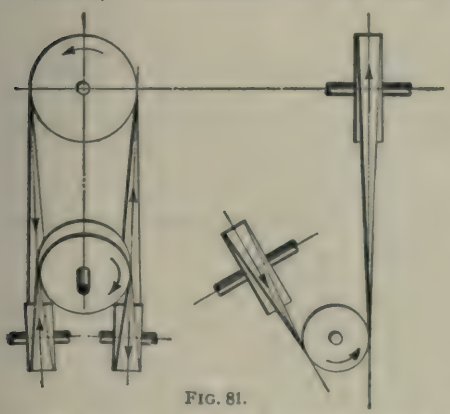
FIG. 81.

Now, to cut 4 threads per inch, the spindle must turn 4 times while the lead screw turns 8 times. Hence, sub-

stituting in the formula, we have $n = 8, f_1 = 2, N = 4, d_1 = 1$, and $d_2 = \frac{8 \times 2 \times f_2}{4 \times 1} = \frac{16 f_2}{4} = 4 f_2$.

Since $16 f_2$ remains constant for this lathe, we may hereafter write the formula as $d_2 = \frac{16 f_2}{N}$.

For 5 threads, $d_2 = \frac{16 f_2}{5} = \frac{16}{5} f_2$.

For 6 threads, $d_2 = \frac{16 f_2}{6} = \frac{8}{3} f_2$.

For 7 threads, $d_2 = \frac{16 f_2}{7} = \frac{16}{7} f_2$.

For 8 threads, $d_2 = \frac{16 f_2}{8} = 2 f_2$, etc.

To make out the table, we should aim to use the same gears as many times as possible. Start with 96 teeth for the gear on stud. Then, $f_2 = \frac{96}{4} = 24$ teeth. For 5 threads, $f_2 = \frac{5}{16} \times 96 = 30$, when $d_2 = 96$; for 6 threads, $f_2 = \frac{3}{8} \times 96 = 36$, when $d_2 = 96$; for 7 threads, $f_2 = \frac{7}{16} \times 96 = 42$; for 8 threads, $f_2 = \frac{1}{2} \times 96 = 48$. The other gears are found in the same manner.

(786) The gear makes $\frac{1}{2}$ a turn in passing the center at each end of the rack, two turns on top of the rack, and two turns while traveling over the under side.

(*a*) The circumference of the gear $= 10 \times 3.1416 = 31.416''$. Distance that the rack travels uniformly $= 2 \times 31.416 = 62.832'' = 5$ ft. $2.832''$. Ans.

(*b*) The motion is harmonic as the gear passes over the ends, and the horizontal distance traveled by the rack at each end = the radius of the gear; distance traveled at both ends $= 2 \times$ radius, or 10 inches. Hence, 5 ft. $2.832'' + 10'' = 6$ ft. $.832''$, the total travel. Ans.

(787) When driving through the worm and worm-wheel, 40 turns of the worm produce one turn of the driven shaft T. When driving through the gearing, 40 turns of the shaft S produce $\frac{40 \times 80}{20} = 160$ turns of T. Hence, the ratio of the "quick return" is 160 : 1.

(788) Circumference of gear $D_2 = 3.5 \times 3.1416 = 10.9956 =$ 11″, nearly. Hence, for every foot travel of the table, D_2 makes $\frac{12}{11} = 1\frac{1}{11}$ revolutions. To find the number of turns made by the pulley for each foot passed over by the table, we have, therefore, $N \times 3.5 = \frac{12}{11} + 26$, or $N = \frac{12 \times 26}{11 \times 3.5} =$ 8.103 +. This, multiplied by the circumference of the pulley, $= \frac{8.103 \times 30 \times 3.1416}{12} = 63.6$. Hence, the ratio is 63.6 : 1. Ans.

(789) (*a*)

	D	F	A
Wheels locked	$+10$	$+10$	$+10$
Arm stationary	-10	$-10 \times \frac{100}{99}$	0
	0	$+10 - 10 \times \frac{100}{99}$	$+10$

Number turns of $F = 10 - \frac{1,000}{99} = -.101$ turn.

The wheel I, being an idler, it is not considered.

(*b*)

	D	F	A
Wheels locked	$+10$	$+10$	$+10$
	-10	$-10 \times \frac{100}{101}$	0
	0	$+10 - 10 \times \frac{100}{101}$	$+10$

Number turns of $F = 10 - \frac{1,000}{101} = +.099$. Ans.

(790) (*a*)

	D	F	A
Wheels locked	$+4$	$+4$	$+4$
Arm stationary	-4	$-\frac{5}{4}\times 4$	0
Arm stationary	$+1$	$+1\frac{1}{4}$	0
	$+1$	$+\frac{1}{4}$	$+4$

$F = +4 - 5 + 1\frac{1}{4} = +\frac{1}{4}$. Ans.

(*b*)

	D	F	A
Wheels locked	$+5$	$+5$	$+5$
Arm stationary	-5	$-\frac{5}{4}\times 5$	0
Arm stationary	-1	$-1\frac{1}{4}$	0
	-1	$-2\frac{1}{2}$	$+5$

$F = +5 - 6\frac{1}{4} - 1\frac{1}{4} = -2\frac{1}{2}$. Ans.

(791)

	D	F, D_1	F_1	A
Train locked	$+6$	$+6$	$+6$	$+6$
Arm fixed	-6	-60	$+\frac{40}{20}\times 60$	
	0	-54	$+126$	$+6$

(792) 20 turns of shaft S will produce $\frac{20\times 12\times 10}{3\times 100} =$ 8 turns of D, L. H. Hence,

	E	H, K	D
Wheels locked	-8	-8	-8
Arm fixed	$+8$	-8	0
Arm fixed	$+20$	-20	0
	$+20$	-36	-8

Pulley K makes -36 revolutions. Ans.

(793) See Arts. **1550** and **1558.**

(794) (*a*) See Art. **1552.**

(*b*) Substituting in formula **150,**

$$C = \frac{3.1416}{4\frac{1}{2}} = \frac{3.1416\times 2}{9} = .698 \text{ inch.} \quad \text{Ans.}$$

(*c*) Substituting in formula **149,** $P = \frac{3.1416}{1.1424} = 2\frac{3}{4}$. Ans.

(795) (*a*) From formula **151**,

$$D = \frac{N}{P} = \frac{30}{3} = 10 \text{ inches.} \quad \text{Ans.}$$

(*b*) Substituting in formula **152**,

$$OD = \frac{30 + 2}{3} = 10.667 \text{ inches.} \quad \text{Ans.}$$

(*c*) Addendum $= \frac{1}{3} = .3333'$; working depth $= 2 \times .3333 = .667$ inch.

(*d*) Clearance $= .3333 \times \frac{1}{8} = .042'$; whole depth $= .667 + .042 = .709'$. Ans.

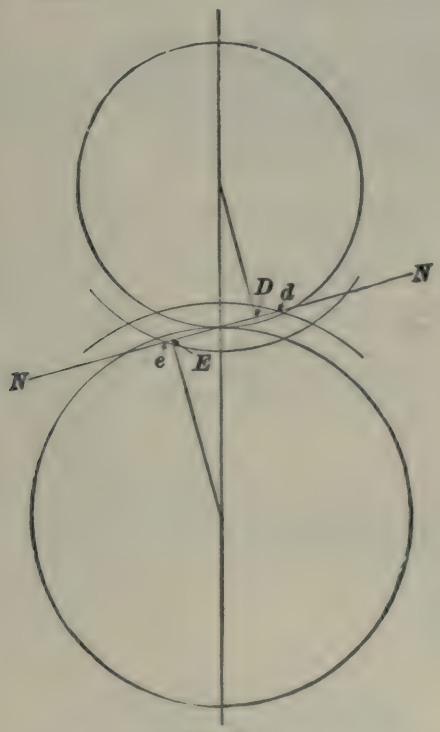

FIG. 82.

(796) See Fig. 414.

(797) (*a*) Number of teeth $= 48 \times 10 = 480$. Ans

(*b*) Number of teeth $= (14 - 2 \times \frac{1}{8}) \times 6 = 82$. Ans.

(798) From formula **153,** $D = \frac{1.25 \times 70}{3.1416} = 27.852$ inches. Ans.

(799) Substituting in formula **154,**

$$d = \frac{2\,A\,V}{V+v} = \frac{2 \times 15.5 \times 2}{2+3} = \frac{62}{5} = 12\tfrac{2}{5} \text{ inches.} \quad \text{Ans.}$$

Substituting in formula **155,** $D = \frac{2 \times 15.5 \times 3}{3+2} = \frac{93}{5} = 18\tfrac{3}{5}$ inches. Ans.

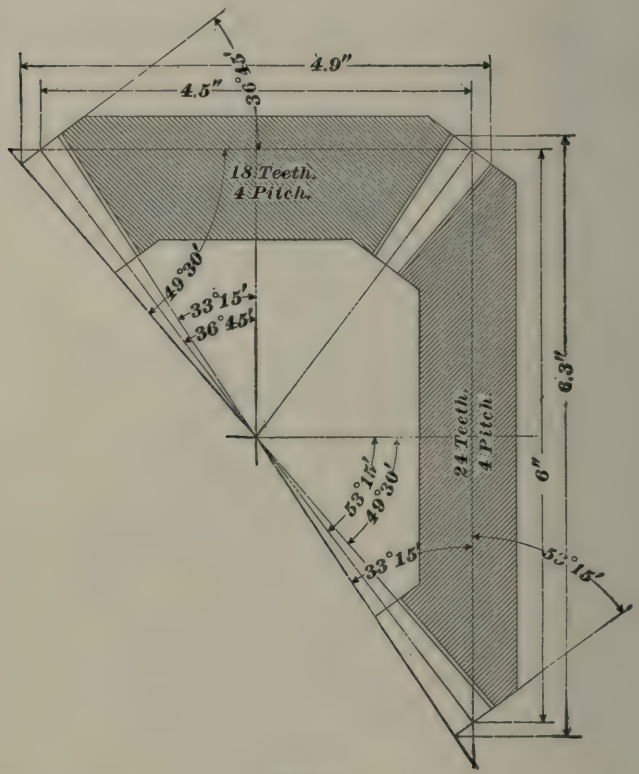

FIG. 83.

(800) (*a*) See Art. **1560.**

(*b*) See Art. **1563.**

(801) See Art. **1573.**

(802) Diameter of internal gear $= 30 \div 3 = 10$ inches.
Diameter of pinion $= 21 \div 3 = 7$ inches.

Diameter of describing circle $= \frac{10 - 7}{2} = 1\frac{1}{2}$ inches.

See Art. **1565.**

(803) To determine whether two involute gears will interfere, use the construction shown in Fig. 425. In Fig. 82, it will be observed that the addendum circles of both gears cut the line of action $N\,N$, at points c and d, outside of points E and D. Hence, both gears will interfere.

(804) The solution to this example is shown in Fig. 83.

(805) Since the worm is triple-threaded, it will drive the wheel three teeth at every revolution; in 100 revolutions, it will drive it $100 \times 3 = 300$ teeth, or $\frac{300}{40} = 7.5$ revolutions. Ans.

(806) (*a*) The wheel must have the same circular pitch as the worm, or $\frac{3}{4}''$, and, as the worm is single-threaded, the number of teeth in the wheel is the same as the ratio of speeds, or 40. Hence, $D = \frac{\frac{3}{4} \times 40}{3.1416} = 9.55''$. From Table 34, the addendum $= .3 \times \frac{3}{4} = .23''$; outside diameter $= 9.55 + (2 \times .23) = 10.01$, or, say $10''$; whole depth, $.23 + (.35 \times \frac{3}{4}) = .23 + .26 = .49''$.

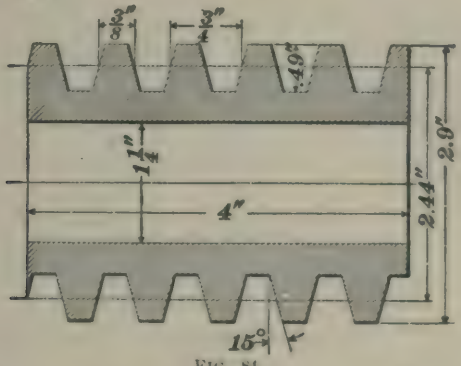

FIG. 84.

(*b*) The section of the worm is shown in Fig. 84.

(807) See Art. **1581.** Tangent of angle of thread $= \frac{\text{circum. of cylinder}}{\text{pitch}} = \frac{2 \times 3.1416}{\frac{1}{3}} = 2 \times 3 \times 3.1416 = 18.8496.$ The angle, therefore, equals 86° 58′, nearly. Ans.

(808) For every tooth moved, the screw would make $\frac{1}{72}$ of a turn. For every turn of the screw, the "feed" would be $\frac{1}{11\frac{1}{2}} = \frac{2}{23}$ inch. Hence, $\frac{1}{72} \times \frac{2}{23} = \frac{2}{1656} = \frac{1}{828}$ inch = the required "feed." Ans.

APPLIED MECHANICS.

(QUESTIONS 809–858.)

(809) Applying formula **156,** H. P. $= .0001904\,RWN$ $= .0001904 \times 4.5 \times 70 = 275 = 16.49$ H. P. Ans.

(810) From formula **156,**

$$R = \frac{\text{H. P.}}{.0001904\,WN} = \frac{6}{.0001904 \times 50 \times 200} = 3.15.$$

Might use an arm $3\frac{1}{4}$ or $3\frac{1}{2}$ feet long. Ans.

(811) Substituting in formula **157,** H. P. $= .00001586\,rWN = .00001586\,(24 + .4) \times (315 - 8) \times 160 = 19.01$ H. P. Ans.

(812) (*a*) The angle of advance for a valve with neither lap nor lead is 0°.

(*b*) Cut-off would occur at the extreme end of the stroke when the valve was in mid-position.

(*d*) Adding lap would make the cut-off earlier, since it would necessitate turning the eccentric ahead in order to bring the lead right.

(813) Admission, cut-off, compression (on other end), release admission and cut-off (on other end), compression, release (on other end). See Figs. 452 and 453.

(814) (*a*) See Art. **1606.**

(*b*) See Art. **1606.**

(815) Increasing the lap without changing the angle of advance would delay admission, hasten cut-off, and decrease the port opening.

(816) First move the eccentric ahead. This will hasten the cut-off and also increase the lead. To diminish the lead

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and make it the same as before, add lap to the valve, which will also have the effect of hastening the cut-off still more.

(817) Inside lap hastens compression and delays release.

(818) See Art. **1639.**

(819) Mark the position of the cross-head on the guides at the dead points. This gives the position of the ends of the stroke. With the engine on one center, say the forward center, set the valve with the proper lead. Now, move the crank forward until the valve cuts off, and measure the displacement of the cross-head. Move the crank forward until the valve cuts off on the return stroke, and measure the displacement of the cross-head. If the cut-off is earlier at the head than at the crank end, the valve spindle is too long, and *vice versa*. Suppose the valve spindle to be too long; then, to shorten it, turn the engine forward until it gives the head end lead, shorten the valve spindle, which will increase the lead; then, reduce the angle of advance until it gives the proper lead. Try the operation again. The lead at the other end of the cylinder will, of course, be too small, unless the engine is specially designed to give equal cut-off.

(820) Increasing the lead is accomplished by moving the eccentric ahead, which hastens the action of the valve, thus causing admission to occur earlier; when the crank reaches the center, therefore, the port will be open wider, or the lead will be greater. The only effect upon the port opening will be to make it occur earlier. It will not increase the port opening.

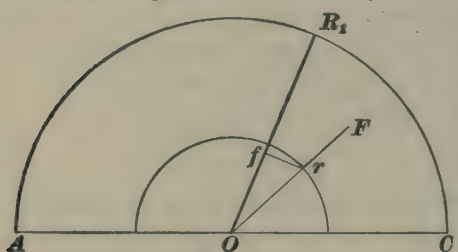

FIG. 85.

(821) (*a*) At mid-position, where the displacement is zero. The displacement of a valve is measured from mid-position.

(*b*) The diagram is shown in Fig. 85. The displacement of the valve for the crank position is equal to $r\ f$.

(822) (*a*) The port opening is always equal to the displacement minus the lap. See Art. **1611.**

(*b*) See Arts. **1613** and **1614** and Figs. 446, 447, and 448, etc.

(823) (*a*) See Art. **1622.** Since the steam should not flow over 6,000 feet per minute, we must have the area of the port $\times$ 6,000 = area of piston $\times$ piston speed in feet per minute.

$$\text{Area of piston} = 18^2 \times .7854 = 254.47 \text{ sq. in.}$$

$$\text{Piston speed} = \frac{40}{12} \times 2 \times 90 = 600.$$

Hence, area of port $\times$ 6,000 = 254.47 $\times$ 600, or

$$A = \frac{254.47 \times 600}{6{,}000} = 25.45 \text{ sq. in.}$$

$25.45 \div 18 = 1.41''$, or $1\frac{7}{16}''$, nearly. Ans.

(*b*) Assume a port opening of .8 of this, or $1.41 \times .8 = 1.128''$, say $1\frac{1}{8}''$. Ans.

(*c*) See Art. **1623.** The width of the exhaust port $= 1\frac{7}{16} + 3 + \frac{1}{8} - \frac{1}{16} = 3\frac{5}{8}''$. Ans.

(824) The diagram is shown in Fig. 86. Draw the valve circle with a radius of $2\frac{1}{2}''$, and the lap circle with a radius of $\frac{5}{8}''$, allowing $\frac{1}{16}''$ lead as shown. Maximum port

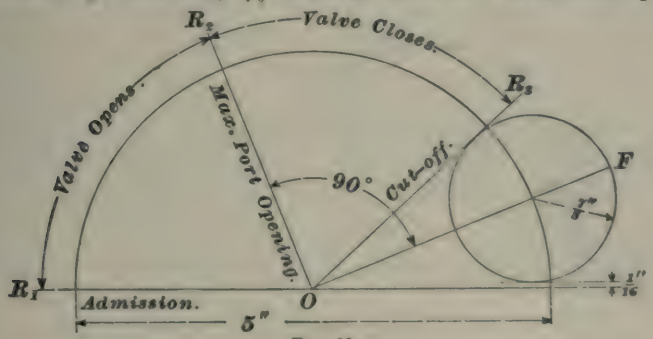

FIG. 86.

opening occurs at crank position $O\,R_2$, at right angles to the angle of advance line $O\,F$. The port, therefore, is open from crank position $O\,R_1$, at admission to $O\,R_2$, and is being closed from $O\,R_2$ to crank position at cut-off at $O\,R_3$.

(825) The diagrams are shown in Figs. 87 and 88. The crank positions at cut-off and release must be found by the

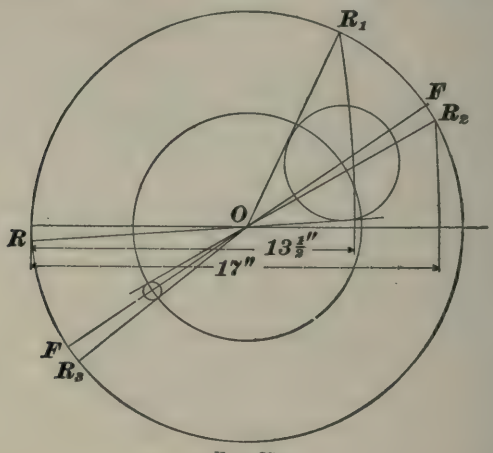

FIG. 87.

method explained in connection with Figs. 445 and 455. In Fig. 87, $O\,R$ is the point of admission; $O\,R_1$, of cut-off; $O\,R_2$, of release, and $O\,R_3$, of compression for the head end.

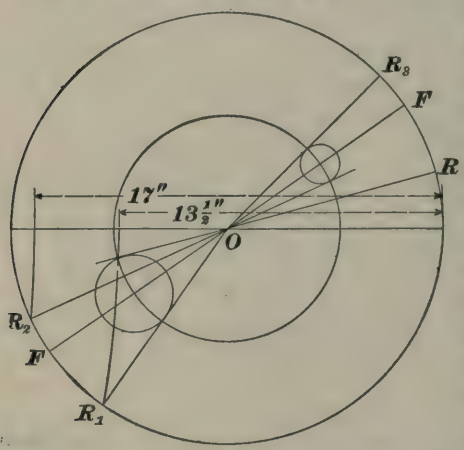

FIG. 88.

In Fig. 88, the same letters refer to corresponding positions of the crank end. $F\,F$ is the angle of advance line in each case, and the diameters of the outside and inside lap circles are determined by drawing these circles tangent to the given crank positions for cut-off and release, respectively.

(826) The length of stroke of the piston must be known and the travel of the valve, both of which can be determined

by turning the engine through a revolution. The other dimensions easiest obtained would be the lead and laps, the former by measuring when the valve-chest cover is off and the engine is on a center, and the latter by removing the valve, taking its dimensions, and also the dimensions of the valve seat. Having these data, the diagram is easily drawn.

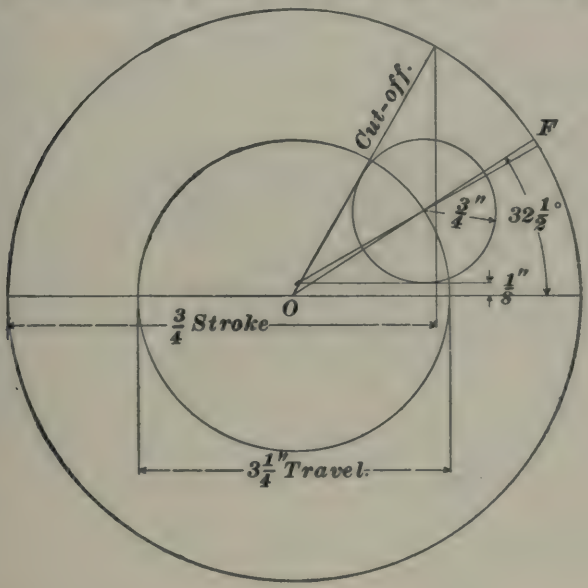

FIG. 89.

(827) The method of solving this problem is entirely similar to the method used in the general problem, Arts. **1627** to **1631.** The diagram is shown in Fig. 89; *O F* is the angle of advance line.

(828) First calculate the width of steam ports and maximum port opening.

$$\text{Area of piston} = 17^2 \times .7854 = 226.98 \text{ sq. in.}$$

$$\text{Area of port} \times 6{,}000 = 226.98 \times \frac{26 \times 2 \times 150}{12} = 24.59 \text{ sq. in.}$$

This, divided by 17, which we will assume to be the length of port, gives $1.44 = 1\frac{7}{16}$", nearly, as the width.

Assume the port opening to be .8 of this, or $1.44 \times .8 = 1\frac{3}{16}''$, nearly. To draw the diagram, draw the stroke circle to a scale of $3'' = 1$ ft., and the valve circle full size. First, draw the stroke circle, and lay off the points of cut-off and release. Then, draw the inner circle with a radius equal to the maximum port opening, or $1\frac{3}{16}''$. Draw the lead line *a b*

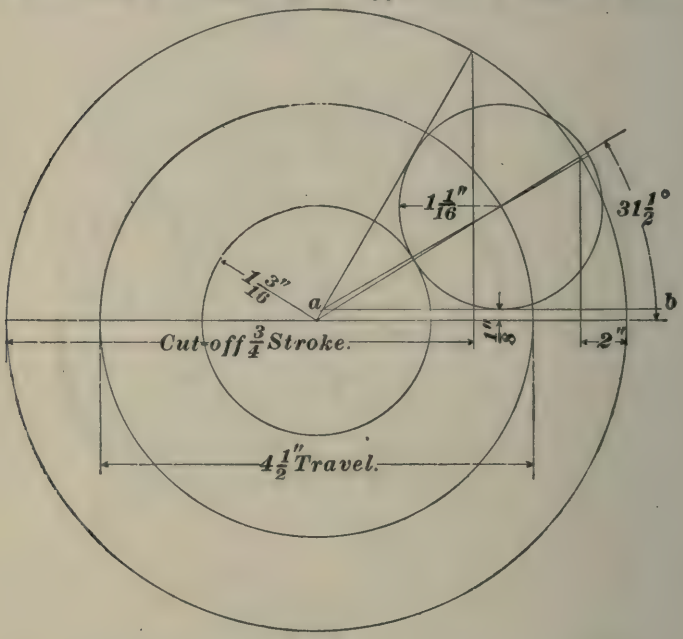

FIG. 90.

to give a lead $= \frac{1}{8}''$ and describe the lap circle tangent to this line, the port opening circle, and the line representing the crank position at cut-off. Measuring the diagram, it will be found that the travel equals $4\frac{1}{2}'' +$, lap $= 1\frac{1}{16}''$, angle of advance $= 31\frac{1}{2}°$.

The inside lap is so small that it may be taken equal to 0.

Before the section of the valve and ports can be drawn, the width of the exhaust port must be calculated. It equals $1\frac{7}{16}'' + 2\frac{1}{4}'' - 1'' = 2\frac{11}{16}''$.

Since a reversing rocker is used the eccentric will be behind the crank an angle equal to $90° - 31\frac{1}{2}° = 58\frac{1}{2}°$. Ans.

The throw of the eccentric should be equal to $\frac{4}{5} \times 4\frac{1}{2} = \frac{4}{5} \times \frac{9}{2} = 3.6''$. Ans.

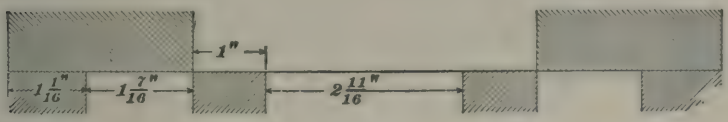

FIG. 91.

The diagram is shown in Fig. 90, and a section of the valve and ports in Fig. 91.

(829) (*a*) Piston valves are used because the steam pressure upon them is perfectly balanced.

(*b*) This type of valve makes it necessary to have a large clearance space in the passages leading to it, which is sometimes a disadvantage. They also offer no relief to any pressure that may arise, owing to water becoming encased in the end of the cylinder near the end of the stroke.

(830) See Art. **1636.**

(831) By means of the equations, Art. **1638,** we have

(*a*) width of passage $= \frac{1}{2}\ (1\frac{1}{2} - \frac{5}{8}) = {}_1{}^7{}_6''$;

(*b*) valve travel $= 2 \times (1\frac{1}{2} + \frac{1}{8}) = 3\frac{1}{4}''$;

(*c*) width of face $= 2 \times (\frac{5}{8} + {}_1{}^7{}_6 + \frac{1}{8}) - \frac{5}{8} = 1\frac{3}{4}''$;

(*d*) width of exhaust port $= 1\frac{1}{2} + \frac{5}{8} + {}_1{}^7{}_6 + \frac{1}{8} + \frac{1}{8} + {}_1{}^7{}_6 - \frac{1}{4} = 2\frac{3}{8}''$.

(832) See Arts. **1637** and **1638.**

(833) Draw the crank circle of some convenient diameter upon the stroke line $A\ C$ (Fig. 92). Lay off $A\ d$ equal to $\frac{1}{8}$ stroke and describe arc $d\ K$, with a radius equal to the length of the connecting-rod. Draw a line through $K\ O$, showing the crank position at cut-off. The diametrically opposite point at K' will be the crank position at cut-off on the forward stroke. Draw lines $l\ l$ and $l'\ l'$ for equal lead at each end. In the figure, the crank circle is drawn to a diameter of 3", so that it serves also for the valve circle. Therefore, with a center upon this circle, describe the lap

circle tangent to $O\ R$ and $l\ l$, thus locating the angle of advance line $F\ F'$. Now, it is evident that the other lap circle

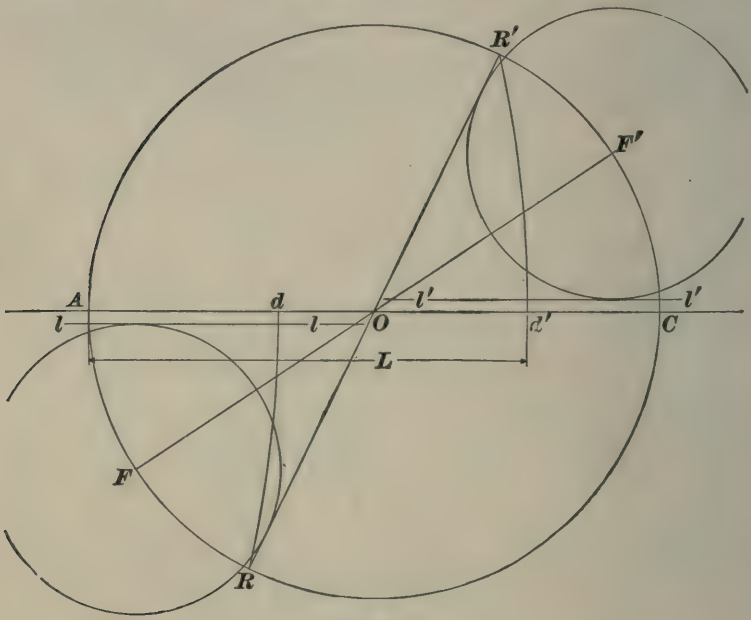

FIG. 92.

which must be drawn about F' will be tangent to $O\ R'$, if it is drawn tangent to $l'\ l'$. Therefore, $O\ R'$ is the required crank position for cut-off, and L is the distance the piston has moved at this point $= \frac{37}{48}$ stroke.

(834) See Art. **1648.**

(835) (*a*) and (*b*). See Art. **1649.**

(836) Its parts are few and simple, and it effects close regulation for high rotative speeds.

(837) An inspection of Fig. 463 will show that screwing the two parts of the cut-off valve together will make the cut-off later, and *vice versa*.

(838) Using formula **156,**

$$\text{H. P.} = .0001904 \times \frac{40}{12} \times 2\tfrac{5}{8} \times 263 = .438158 \text{ H. P.} \quad \text{Ans.}$$

(839) See Arts. **1652** and **1653.**

(840) See Art. **1657.**

(841) For (*a*), (*b*), and (*c*) see Art. **1654.** For (*d*) and (*e*) see Arts. **1655** and **1656.**

(842) See Art. **1657.**

(843) See Art. **1658.**

(844) See Art. **1661.**

(845) See Art. **1660.**

(846) See Art. **1651.**

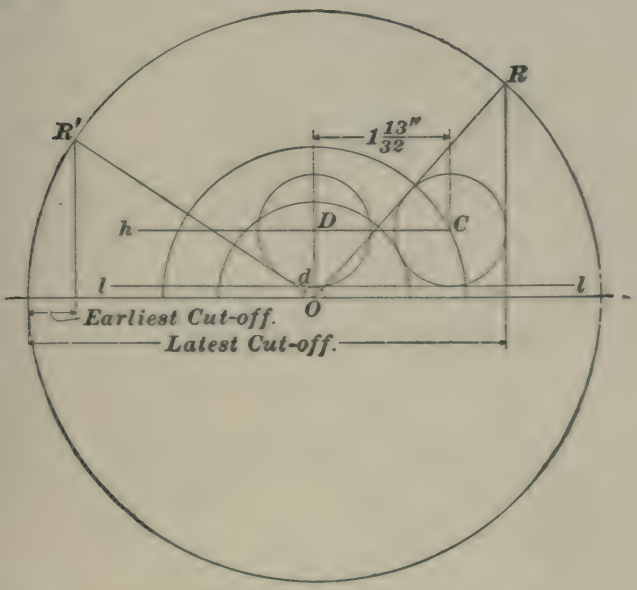

FIG. 93.

(847) The student should answer this question briefly in his own words. Art. **1651.**

(848) (*a*) See Art. **1651.**

(*b*) The same effect could not be produced by this method. Giving the valve lead in this way would hasten the release, it is true, but it would delay the compression by an equal amount.

(849) See Art. **1650.**

(850) No. Changing the angle of advance varies the lead.

(851) See Fig. 93. First find the crank positions $O\ R$ and $O\ R'$ for the latest and earliest points of cut-off. Draw the lap circle about C tangent to $O\ R$ and draw the lead line $l\ l$. Through C draw the horizontal line $C\ h$, and with the compass point upon this line draw a lap circle of the same size as the former one, tangent to $O\ R'$. Its center will fall at D, and $C\ D$ is the distance that the eccentric must shift. The port opening for shortest cut-off is equal to $O\ d$, or $\frac{1}{8}$ inch.

(852) The valve diagram is laid out for full gear posi-

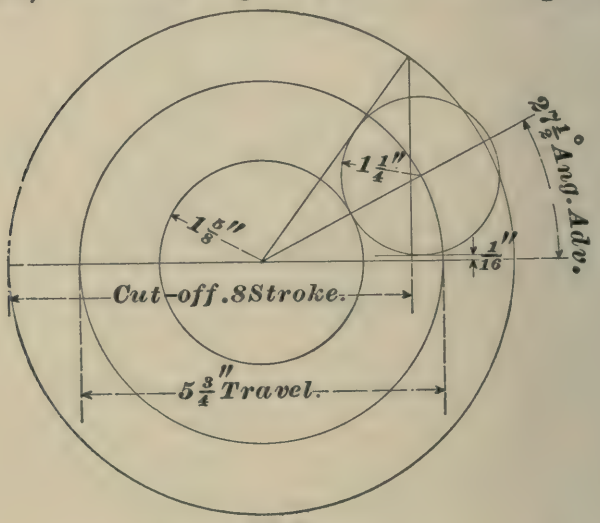

FIG. 94.

tion as shown in Fig. 94. This part of the process is like designing a plain slide valve, remembering that the valve is to over-travel $\frac{3}{8}$ of an inch.

In Fig. 95 are drawn the two link diagrams. In each, the eccentrics are drawn with the angle of advance found above. These diagrams are similar to and are lettered the same as the diagrams shown in Figs. 473 and 474, and further explanation seems to be unnecessary.

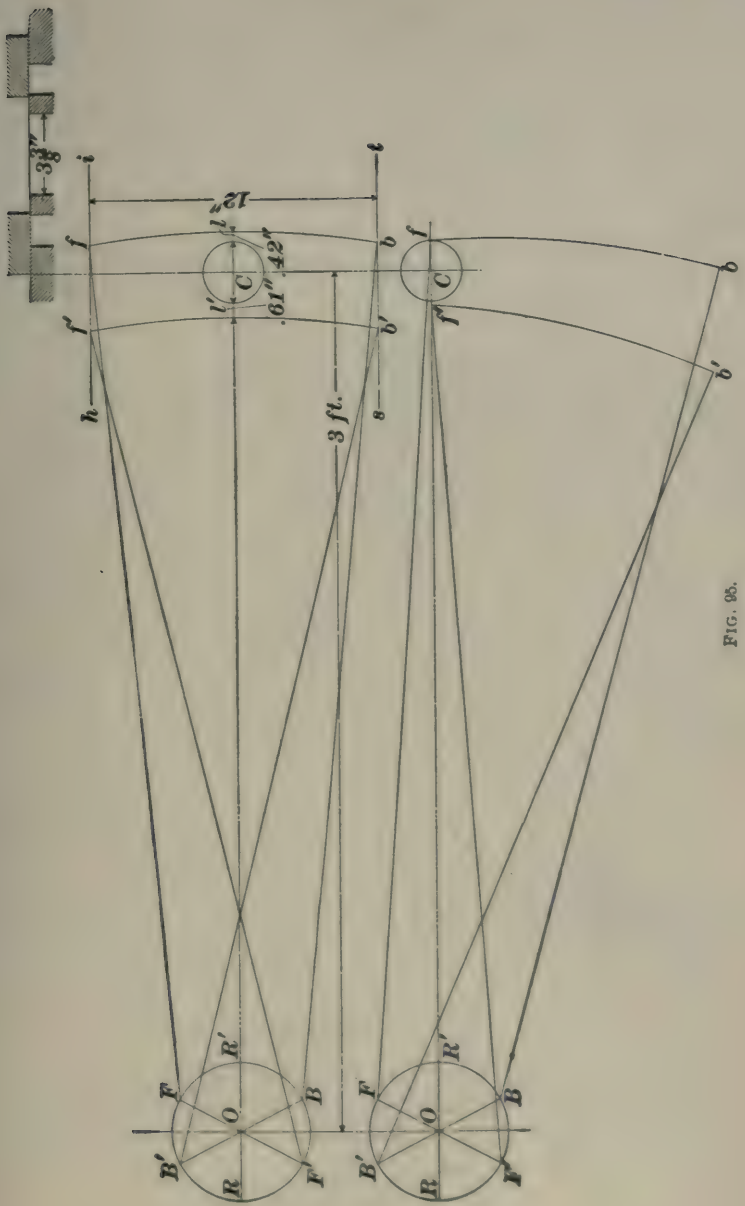

FIG. 95.

(853) (*a*) See Art. **1668.**

(*b*) See Art. **1675.**

(854) Use formula **158,** $N^2 = \frac{35{,}294}{h}$. For a height of 20″ we have $N^2 = 35{,}294 \div 20$, or $N = 42.008$ rev.; for a height of 18″, $N = 42.28$; for a height of 22″, $N = 40.053$ rev.

(*a*) $44.28 - 42.008 = 2.272$ rev. increase. Ans.

(*b*) $42.008 - 40.053 = 1.955$ rev. decrease. Ans.

(855) (*a*) See Art. **1674.**

(*b*) Compare formula **159** with formula **158,** Art. **1668.** Since W equals five times the weight of both balls together, it is equal to $10 \times B$ and

$$H = \left(1 + \frac{W}{B}\right) h = \left(1 + \frac{10}{1}\right) \times 1 = 1 + 10 = 11''. \quad \text{Ans.}$$

(856) See Art. **1676.**

(857) Substituting in formula **163,**

$$B = \frac{5}{.00000114 \times 10 \times 240^2} - \frac{4 \times 5.85 \times 6.5 + (3 \times 5.4 \times 14)}{9 \times 10} =$$

3.4045 lb.

Substituting in formula **162,**

$$W = \tfrac{1}{5}\,[.000028 \times 240^2\,(4 \times 5.85 \times 6.5 + 3.4045 \times 9 \times 10 + 3 \times 5.4 \times 14) - (3.4045 \times 9 + 4 \times 5.85 + 3 \times 5.4)]$$
$$= 115 \text{ lb.} \quad \text{Ans.}$$

(858) See Art. **1591.**

STEAM BOILERS.

(QUESTIONS 859–958.)

(859) See Arts. **1698** and **1702.**

(860) See Arts. **1764** to **1791.**

(861) Applying formula **168,**

$$t = \sqrt{\frac{p\,l\,d}{1{,}600{,}000}} = \sqrt{\frac{80 \times 15 \times 12 \times 3}{1{,}600{,}000}} = \frac{3}{16}'', \text{ nearly.} \quad \text{Ans.}$$

(862) Applying formula **177,**

$$d = 1.13\sqrt{\frac{A\,p}{T}} = 1.13\sqrt{\frac{12 \times 13\frac{1}{2} \times 125}{6{,}000}} = 2\frac{1}{16}'', \text{ nearly.} \quad \text{Ans.}$$

(863) Applying formula **184,**

$$t = h\sqrt{\frac{p}{k}} = 8\sqrt{\frac{150}{40{,}000}} = .49'', \text{ say } \frac{1}{2}''. \quad \text{Ans.}$$

(864) (*a*) Using formula **190,**

$$h = 14{,}500 \times .80 + 62{,}032\left(.08 - \frac{.06}{8}\right) = 16{,}097.32 \text{ B.T.U.} \quad \text{Ans.}$$

(*b*) Using formula **189,**

$$W = 11.6 \times .80 + 34.8\left(.08 - \frac{.06}{8}\right) = 11.8 \text{ lb.} \quad \text{Ans.}$$

(865) Since the number of B. T. U. contained in the gases is directly proportional to the temperature, 600° − 250° = 350° may be considered to represent the amount of heat saved. Then,

$$350 \div 2{,}800 = .125 = 12.5\%. \quad \text{Ans.}$$

(866) From Table 41, the factor of evaporation for 120° feed and 80 lb. pressure = 1.131; for 120° feed and 90 lb. pressure = 1.133; hence, the factor for 120° feed and 85 lb. pressure is evidently 1.132. In a similar manner it is found

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that for 130° feed and 85 lb. pressure, the factor is 1.121. Consequently, for a difference of 10° in the feed, difference in the factors is $1.132 - 1.121 = .011 = .0011$ for a difference of 1°. $127° - 120° = 7°$, and $.0011 \times 7 = .0077 =$ difference for a difference of 7°. Hence, the factor for 127° feed and 85 lb. pressure $= 1.132 - .0077 = 1.1243$, and $\frac{28,930 \times 1.1243}{34.5 \times 10}$ $= 94.3$ H. P. Ans.

(867) From the table of the Properties of Saturated Steam, the latent heat of steam at 80 lb. gauge pressure is 886.59, and the temperature is 323.71. Applying formula **204,** and substituting,

$$Q = \frac{1}{886.59}\left[\frac{360}{22}(110 - 44) - (323.71 - 110)\right] = 97.7\%. \text{ Ans.}$$

(868) From the steam table, the volume of a pound of steam at 75 lb. gauge pressure is 4.811 cu. ft. Then, the area of the cross-section of the pipe must be

$$\frac{7,200 \times 4.811 \times 144}{4,000 \times 60} = 20.78 \text{ sq. in.},$$

and the diameter not less than $5\frac{3}{16}''$. Ans.

(869) This example might be worked by applying formula **186** for each pressure; but since there is a constant difference of 5 lb. between any two consecutive pressures, there will also be a constant difference between any two consecutive graduations, and the example may be worked as follows: Applying formula **186** for a pressure of 40 lb.,

$$d = \frac{4.5\,(40 \times 12.5664 - 8.5) - 21 \times 19}{107} = 17.05''. \text{ Ans.}$$

Applying again for 60 lb.,

$$d = \frac{4.5\,(60 \times 12.5664 - 8.5) - 21 \times 19}{107} = 27.62''. \text{ Ans.}$$

Then, $\frac{27.62 - 17.05}{4} = 2.64\frac{1}{4}'' =$ distance which weight must be moved in order to produce a difference of 5 lb. per

sq. in. in the blow-off pressure. Hence, for a blow-off pressure of 45 lb.,

$$d = 17.05 + 2.64\frac{1}{4} = 19.69''. \quad \text{Ans.}$$

For 50 lb., $d = 17.05 + 2.64\frac{1}{4} \times 2 = 22.34''$. Ans.

And for 55 lb., $d = 17.05 + 2.64\frac{1}{4} \times 3 = 24.98''$. Ans.

(870) See Art. **1695,**

(871) (*a*) 4 ft. 3 in. = 4.25 ft. Total force exerted along one seam $= \frac{12 \times 4.25 \times 144 \times 80}{2} = 293{,}760$ lb. Ans.

(*b*) Total force along a girth seam $= 4.25^2 \times .7854 \times 80 \times 144 = 163{,}426$ lb. Ans.

(872) (*a*) to (*d*) See Arts. **1736** to **1739.**

(*e*) The double-riveted butt joint. (See formula **171.**)

(873) Applying formula **179,**

$$a = .885\, d\sqrt{\frac{T}{p}} = .885 \times 1\frac{3}{4}\sqrt{\frac{9{,}000}{140}} = 12.42'', \text{ say } 12\frac{1}{2}''. \text{ Ans.}$$

(874) (*a*) and (*b*) See Arts. **1764** and **1765.**

(*c*) See Art. **1770.**

(*d*) See Art. **1772.**

(875) (*a*) See Art. **1804.**

(*b*) See Art. **1805.** (*c*) $400 \times .4 = 160$ lb. Ans.

(*d*) See Art. **1805.**

(876) Using formula **193,**

$$p = H\left(\frac{7.6}{T_a} - \frac{7.9}{T_c}\right),$$

$$\text{or } H = \frac{p}{\frac{7.6}{T_a} - \frac{7.9}{T_c}} = \frac{.77}{\frac{7.6}{460+60} - \frac{7.9}{460+580}} = 110 \text{ ft.} \quad \text{Ans.}$$

(877) See Art. **1833.**

(878) Using formula **205,**

$$Q = \frac{W+R}{W+R+w} = \frac{20.5+\frac{2}{16}}{20.5+\frac{2}{16}+\frac{14}{16}} = 95.9\%. \quad \text{Ans.}$$

(879) See Art. **1894.**

(880) See Arts. **1698** to **1703.**

(881) (*a*) Force $= \frac{65 \times 4\frac{1}{2} \times 12}{2} = 1{,}755$ lb. Ans.

(*b*) Force $= 1{,}755 \div 2 = 877.5$ lb. Ans.

(882) See Art. **1740.**

(883) Applying formula **178,**

$$d = 1.13\, a\sqrt{\frac{p}{T}} = 1.13 \times 3.75\sqrt{\frac{128}{4{,}000}} = .758'', \text{ say } \frac{3}{4}''. \quad \text{Ans.}$$

(884) From Table 41, the factor of evaporation for a feed temperature of 65° and a pressure of 85 lb. is 1.189; for a feed temperature of 200° and the same steam pressure, it is 1.049. Hence,

$$\text{gain} = \frac{1.189 - 1.049}{1.189} = .118 = 11.8\%. \quad \text{Ans.}$$

(885) Applying formula **190,**

$$h = 14{,}500 \times .94 + 62{,}032\left(.01 - \frac{.02}{8}\right) = 14{,}095.24 \text{ B. T. U.}$$

$$14{,}095.24 \div 966.1 = 14.59 \text{ lb.} \quad \text{Ans.}$$

(886) Applying formula **192,**

$$p = H\left(\frac{7.6}{T_a} - \frac{7.9}{T_c}\right) = 135\left(\frac{7.6}{460+65} - \frac{7.9}{460+420}\right) = \frac{3}{4}''$$ of water, nearly. Ans.

(887) See Arts. **1832** and **1813.**

(888) See Arts. **1859** to **1867.**

(889) See Arts. **1897** and **1898.**

(890) See Arts. **1707** to **1709** and Art. **1879.**

(891) Applying formula **165** and taking $\frac{S_1}{f} = 9{,}500$,

$$t = \frac{f p d}{2 S_1 y} = \frac{1 \times 110 \times 38}{2 \times 9{,}500 \times .60} = .37'' = \frac{3}{8}''. \quad \text{Ans.}$$

(892) See Arts. **1740** and **1741.**

(893) Using formula **178,**

$$t = 1.13 a \sqrt{\frac{p}{T}} = 1.13 \times 4.5 \sqrt{\frac{160}{8{,}000}} = .719'', \text{ say } \frac{3}{4}''. \quad \text{Ans.}$$

(894) (*a*) $120 \times 30 = 3{,}600$ lb. of steam per hour. Applying formula **188,**

$$a = \frac{.5\, w}{p + 10} = \frac{.5 \times 3{,}600}{70 + 10} = 22.5 \text{ sq. in.} \quad \text{Ans.}$$

(*b*) $22.5 \times 70 = 1{,}575$ lb. Ans.

(895) See Art. **1802.**

(896) (*a*) Applying formulas **197** and **194,**

$$d = 13.54 \sqrt{E} + 4 = 13.54 \sqrt{\frac{.3 \text{ H. P.}}{\sqrt{H}}} + 4 =$$

$$13.54 \sqrt{\frac{.3 \times 700}{\sqrt{105}}} + 4 = 65.3''. \quad \text{Ans.}$$

(*b*) Applying formulas **196** and **194,**

$$S = 12 \sqrt{\frac{.3 \times 700}{\sqrt{105}}} + 4 = 58.3''. \quad \text{Ans.}$$

(897) Using formula **202,** in which $W = 150 \times 34.5$,

$$G = \frac{W}{Fc} = \frac{150 \times 34.5}{16 \times 10.25} = 31.55 \text{ sq. ft.} \quad 31.55 \times 36 = 1{,}136 \text{ sq. ft.} \quad \text{Ans.}$$

(898) See Arts. **1868** to **1873.**

(899) Applying formula **200,**

$F = 2.25 \sqrt{H} = 2.25 \sqrt{135} = 26.14$ lb. of coal per sq. ft. of grate per hour. Then, $26.14 \times 3.5 \times 5 \times 10 = 4{,}575$ lb. Ans.

(900) See Arts. **1717** to **1721.**

(901) Applying formula **165,**

$$t = \frac{f p d}{2 S_1 y} = \frac{4 \times 140 \times 13 \times 12}{2 \times 64{,}000 \times .72} = \frac{15}{16}''. \quad \text{Ans.}$$

(902) Applying formula **170,**

$$h = \left(1 + \frac{7\,d}{4\,t}\right) d = \left(1 + \frac{7}{4} \times \frac{.75}{.375}\right) \times .75 = 3\frac{3}{8}''. \quad \text{Ans.}$$

(*b*) Applying formula **172,**

$$d = t + \frac{3}{8} = \frac{3}{8} + \frac{3}{8} = \frac{3}{4}''. \quad \text{Ans.}$$

(*c*) Applying formula **173,**

$$y = \frac{h - d}{h} = \frac{3\frac{3}{8} - \frac{3}{4}}{3\frac{3}{8}} = .78 = 78\%. \quad \text{Ans.}$$

(903) Using formula **179,**

$$a = .885\, d \sqrt{\frac{T}{p}} = .885 \times 1 \sqrt{\frac{6{,}000}{150}} = 5.6''. \quad \text{Ans.}$$

(904) See Arts. **1773** to **1782.**

(905) $11.6 \times .95 = 11.02$ lb. of air required per pound of coal. Then, since 1 lb. of air at 62° has a volume of 13.14 cu. ft., the total number of cubic feet to be furnished per minute =

$$11.02 \times 1\frac{1}{2} \times 13.14 \times \frac{4 \times 6 \times 12}{60} = 1{,}042.6 \text{ cu. ft.} \quad \text{Ans.}$$

(906) (*a*) From Table 38, 632 H. P. Ans.

(*b*) Applying formulas **195** and **194,** we have, since 3 ft. $8'' = 44''$,

$$\text{H. P.} = 3.33\, E \sqrt{H} = 3.33\,(A - .6\sqrt{A})\sqrt{H} =$$

$$3.33 \left(\frac{44^2}{144} - .6\sqrt{\frac{44^2}{144}}\right)\sqrt{80} = 336 \text{ H. P.} \quad \text{Ans.}$$

(907) From Table 41, the factor of evaporation for a feed temperature of 120° and a steam pressure of 105 lb. gauge = 1.137; and for 200° feed and 105 lb. gauge = 1.054. Then,

$$\text{gain} = \frac{1.137 - 1.054}{1.137} = .073 = 7.3\%. \quad \text{Ans.}$$

(908) See Arts. **1876** and **1877.**

(909) (See Art. **1900.**) $300 \times 30 = 9{,}000$ lb. of water required per hour. Adding 15% for emergencies and 20% for leakage of water past the piston, etc., $9{,}000 + 9{,}000\,(.15 + .20) = 12{,}150$ lb. per hour $= \frac{12{,}150}{62.5} = 194.4$ cu. ft. per hour. Assuming the pump to make 40 strokes per minute, the displacement per stroke $= \frac{194.4 \times 1{,}728}{60 \times 40} = 140$ cu. in. Taking the stroke as twice the diameter,

$d^3 \times .7854 \times 2\,d = 140$, or $d = \sqrt[3]{\frac{140}{2 \times .7854}} = 4.47''$, say $4\frac{1}{2}''$. Ans.

Then, stroke $= 4\frac{1}{2} \times 2 = 9''$. Ans.

(910) See Art. **1716.**

(911) In Art. **1366** it is stated that a cylinder is twice as strong against transverse rupture as it is against longitudinal rupture. Since, in example 901, the efficiency of the joint is 72%, the relative strengths of the two seams are $\frac{2 \times .72}{1 \times .72} = \frac{2}{1}$, i.e., the girth seam is twice as strong as the longitudinal seam.

(912) See Arts. **1744** and **1745.**

(913) Applying formula **180,**

$d = 1.13\sqrt{\frac{A\,p}{T \cos B}} = 1.13\sqrt{\frac{35 \times 80}{6{,}000 \cos 18^\circ}} = .791''$, say $\frac{13}{16}''$. Ans.

(914) Applying formula **185,**

$W = \frac{a\,(p\,A - W_1) - W_2\,c}{d} = \frac{4\,(110 \times 5^2 \times .7854 - 8) - 12 \times \frac{40}{2}}{40} =$ 209.2 lb. Ans.

(915) See Arts. **1806** and **1807.**

(916) The easiest and most accurate way to solve this example is as follows: Let H, T_a, and T_e represent the height

and absolute temperatures before the economizer is introduced, and H', T_a, and T_c' the same quantities after the economizer has been introduced. Then, since p is the same in both cases,

$$H\left(\frac{7.6}{T_a}-\frac{7.9}{T_c}\right)=H'\left(\frac{7.6}{T_a}-\frac{7.9}{T'_c}\right).$$

Solving this equation for H',

$$H'=H\left[\frac{(7.6\,T_c-7.9\,T_a)\,T_a\,T_c'}{(7.6\,T_c'-7.9\,T_a)\,T_a\,T_c}\right].$$

Substituting for the letters their values and canceling the two factors T_a,

$$H'=90\left[\frac{(7.6\times 1{,}080-7.9\times 520)\,730}{(7.6\times 730-7.9\times 520)\,1{,}080}\right]=173.2 \text{ ft.}$$

Hence, the additional height $= 173.2 - 90 = 83.2$ ft. Ans.

(917) See Art. **1839.**

Applying formula **203,** we have

(*a*) $\dfrac{(H-t+32)}{966.1}=\dfrac{1{,}182.233-113+32}{966.1}=1.1399.$ Ans.

(*b*) $\dfrac{1{,}188.110-73+32}{966.1}=1.1874.$ Ans.

(*c*) $\dfrac{1{,}190.989-87+32}{966.1}=1.1758.$ Ans.

(*d*) $\dfrac{1{,}168.626-158+32}{966.1}=1.0792.$ Ans.

(918) See Arts. **1878** and **1879.**

(919) See Art. **1896.**

(920) See Arts. **1722** to **1728.**

(921) (*a*) and (*b*) See Art. **1750.**

(*c*) The spherical vessel. Art. **1366.**

(922) See Arts. **1748** and **1749.**

(923) Using formula **181,**

$$d=\sqrt[3]{\frac{p\,h\,l^2}{6{,}000}}=\sqrt[3]{\frac{120\times 8\times 33^2}{6{,}000}}=5.585'',\text{ say } 5\tfrac{5}{8}''.\quad\text{Ans.}$$

(924) Applying formula **186,**

$$d = \frac{a(pA - W_1) - W_2 c}{W} = \frac{4(110 \times 5^2 \times .7854 - 8) - 12 \times 20}{265}$$

$= 31.6''$. Ans.

(925) See Art. **1808.**

(926) Applying formula **200,**

(*a*) $2.25\sqrt{H} = 2.25\sqrt{80} = 20\frac{1}{8}$ lb. per sq. ft. of grate per hour. Ans.

(*b*) $2.25\sqrt{115} = 24\frac{1}{8}$ lb. per sq. ft. of grate per hour. Ans.

(927) See Arts. **1840** to **1852.**

(928) See Art. **1880.**

(929) (*a*) At 65 lb. gauge pressure the latent heat of steam is (see steam table) 895.296 B. T. U., and the temperature is 311.605°. Applying formula **204,**

$$Q = \frac{1}{l}\left[\frac{W}{w}(t_2 - t_1) - (t - t_2)\right] =$$

$$\frac{1}{895.296}\left[\frac{327}{24}(120 - 40) - (311.605 - 120)\right] = 1.00346.$$ Ans.

(*b*) The answer to (*a*) shows that the steam is superheated, and the number of degrees of superheat can be determined by means of the expression given in Art. **1853.** Thus,

$$\frac{(Q - 1)l}{.48} = \frac{(1.00346 - 1)\,895.296}{.48} = 6.5°,$$ nearly. Ans.

(930) See Arts. **1725, 1726,** and **1714.**

(931) Applying formula **167,** and substituting 12,000 for $\frac{S_1}{f}$,

$$p = \frac{2tS_1y}{fd} = \frac{2 \times \frac{5}{16} \times 12{,}000 \times .66}{42} = 118$$ lb. per sq. in. Ans.

(932) Solving formula **174** for p, and substituting,

$$p = \frac{3\,t^2\,S}{2\,r^2} = \frac{3 \times \left(\frac{1}{2}\right)^2 \times 10{,}000}{2 \times 24^2} = 6.5 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(933) Applying formula **182,**

$$p = \frac{6{,}000\,d^3}{h\,l^2} = \frac{6{,}000 \times 7.5^3}{10 \times 42^2} = 143.5 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(934) See Arts. **1783** to **1791.**

(935) See Arts. **1820, 1812,** and **1823.**

(936) Applying formula **199,**

$$F = 1.5\sqrt{H} - 1, \text{ or } H = \left(\frac{F+1}{1.5}\right)^2 = \left(\frac{14+1}{1.5}\right)^2 = 100 \text{ ft.}$$

Ans.

(937) (*a*) See Art. **1841.**

(*b*) Total heat of 1 lb. of steam at 60 lb. gauge pressure = 1,175.625 B. T. U. Heat units required to vaporize the water at 52° into steam at 60 lb. gauge = 1,175.625 − 52 + 32 = 1,155.625 B. T. U.

$$\text{Efficiency} = \frac{1{,}155.625 \times 175{,}000}{19{,}000 \times 13{,}500} = 78.8\,\%. \quad \text{Ans.}$$

(938) See Art. **1866.**

(939) See Art. **1861.**

(940) See Art. **1712.**

(941) See solution to (*b*), example 929.

$$\frac{(Q-1)\,l}{.48} = \frac{(1.0148-1)884.98}{.48} = 27.29°. \quad \text{Ans.}$$

(942) Solving formula **174** for t and applying,

$$t = \sqrt{\frac{2\,r^2\,p}{3\,S}} = \sqrt{\frac{2 \times 17^2 \times 75}{3 \times 5{,}000}} = 1.7''. \quad \text{Ans.}$$

(943) Applying formula **183,**

$$p = \frac{k\,t^2}{h^2} = \frac{28{,}000 \times \left(\frac{5}{16}\right)^2}{4.5^2} = 135 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(944) See Arts. **1794** and **1800.**

(945) $\frac{175 \times 40}{60} = 116\frac{2}{3}$ lb. of coal per minute.

Applying formula **191,**

$$\text{H. P.} = \frac{p\,H\,V}{33{,}000\,y} = \frac{12 \times 116\frac{2}{3} \times 230}{33{,}000 \times .625} = 15.61 \text{ H.P.} \quad \text{Ans.}$$

(946) $\frac{3{,}000}{9\frac{1}{2} \times 15} = 21$ sq. ft. **Ans.**

(947) (*a*) $\frac{175{,}000 \times 1{,}155.625}{966.1} = 209{,}331$ lb. **Ans.**

(*b*) $209{,}331 \div 19{,}000 = 11.02$ lb. per pound of coal. **Ans.**

(*c*) $209{,}331 \div (19{,}000 \times .93) = 11.85$ lb. per pound of combustible. **Ans.**

(*d*) Horsepower $= \frac{209{,}331}{34.5 \times 22} = 275.8$ H.P. **Ans.**

(948) See Arts. **1881** to **1883** and **1698** to **1703.**

(949) Applying formula **190** to find the heat of combustion, $h = 14{,}500 \times .643 + 62{,}032\left(.042 - \frac{.100}{8}\right) = 11{,}153.5$ B. T. U. Heat required to change 1 lb. of water at 100° into steam having a temperature of 240° is found from the steam table to be 1,087.14 B. T. U. Hence, $\frac{11{,}153.5}{1{,}087.14} = 10.26$ lb. **Ans.**

(950) From Table 41, the factor of evaporation for 55° feed and 40 lb. gauge pressure is 1.187. Consequently, $\frac{935 \times 1.187}{34.5} = 32.2$ H. P. nearly. **Ans.**

(951) Applying formula **166,**

$$p = \frac{2\,t\,S_1}{f\,d} = \frac{2 \times \frac{3}{8} \times 12{,}000}{30} = 300 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(952) Applying formula **176,**

$$t = \frac{a}{2}\sqrt{\frac{p}{S}} = \frac{20}{2}\sqrt{\frac{30}{12{,}000}} = \frac{1}{2}''. \quad \text{Ans.}$$

(953) Applying formula **184,**

$$t = h\sqrt{\frac{p}{k}} = 4\sqrt{\frac{110}{24{,}000}} = .2712'', \text{ say } \frac{9}{32}''. \quad \text{Ans.}$$

(954) $18 - 11.8 = 6.2$ lb. free air. $8 + 6 = 14\%$ of hydrogen and oxygen. Then, $H - \frac{O}{8} = .08 - \frac{.06}{8} = .0725 =$ 7.25% free hydrogen. Therefore, percentage of water $=$ $14 - 7.25 = 6.75\%$, and the composition of the fuel may be written $C = 80\%$, $H = 7.25\%$, water 6.75%, and ash 6%. The products of combustion are :

Carbonic acid	$80 \times 3.67 =$	2.936 lb.
Water (steam)..	$.0675 + .0725 \times 9 =$	0.72 lb.
Nitrogen	$11.8 \times .77 =$	9.086 lb.
Free air.....	$=$	6.2 lb.
Ash	$=$	0.06 lb.
Total	$=$	19.00 lb.

Then, as in Art. **1802,**

$$t = \frac{16{,}097.32 - .72 \times 966.1}{2.936 \times .217 + .72 \times .4805 + 9.086 \times .2438 + 6.2 \times .2375} =$$

3,297°. Ans.

(955) See Arts. **1816** and **1814.**

(956) $\frac{28{,}930 \times 1.1243}{3{,}120} = 10.43$ lb. of water per pound of coal from and at 212°. Ans.

(957) See Art. **1853.**

(958) See Arts. **1887** to **1893.**

Applying formula **206,**

$$t = 0.0001\,p\,d + \frac{1}{8} = 0.0001 \times 105 \times 4\frac{1}{2} + \frac{1}{8}'' = .172'',$$

$$\text{say } \frac{11}{64}''. \quad \text{Ans.}$$

MACHINE DESIGN.

(QUESTIONS 959–1023.)

(959) See Art. **1910.**

(960) See Art. **1911.**

(961) (*a*) See Art. **1915.**

(*b*) See Art. **1912.**

(962) See Arts. **1924, 1925, 1927,** and **1928.**

(963) From Table 43 the number of threads per inch corresponding to the diameters given are 13, 9, 7, and $4\frac{1}{2}$, respectively. Applying formula **211:**

$$d_1 = d - \frac{1.3''}{n} = \frac{1}{2} - \frac{1.3''}{13} = .4''. \quad \text{Ans.}$$

$$d_1 = d - \frac{1.3''}{n} = \frac{7}{8} - \frac{1.3''}{9} = .731''. \quad \text{Ans.}$$

$$d_1 = d - \frac{1.3''}{n} = 1\frac{1}{4} - \frac{1.3''}{7} = 1.064''. \quad \text{Ans.}$$

$$d_1 = d - \frac{1.3''}{n} = 2 - \frac{1.3''}{4\frac{1}{2}} = 1.711''. \quad \text{Ans.}$$

(964) (*a*) Apply formula **208.**

For $\frac{1}{4}''$ diameter, $p = .24\sqrt{.25 + .625} - .175'' = .0495''$

For $\frac{5}{8}''$ diameter, $p = .24\sqrt{.625 + .625} - .175'' = .0933''$

For $1''$ diameter, $p = .24\sqrt{1 + .625} - .175'' = .131''.$

For $1\frac{1}{2}''$ diameter, $p = .24\sqrt{1.5 + .625} - .175'' = .175''.$

Now, applying formula **210,**

$$n = \frac{1}{.0495} = 20.2, \text{ say 20 threads per inch.} \quad \text{Ans.}$$

$$n = \frac{1}{.0933} = 10.7, \text{ say 11 threads per inch.} \quad \text{Ans.}$$

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$$n = \frac{1}{.131} = 7.6, \text{ say } 8 \text{ threads per inch.} \quad \text{Ans.}$$

$$n = \frac{1}{.175} = 5.7, \text{ say } 6 \text{ threads per inch.} \quad \text{Ans.}$$

(b) By formula **210**, $n = \frac{1}{p}$; or, $p = \frac{1}{n}$. Hence,

$$p = \frac{1}{20}'' = \text{pitch for } \tfrac{1}{4}'' \text{ screw.} \quad \text{Ans.}$$

$$p = \frac{1}{11}'' = \text{pitch for } \tfrac{5}{8}'' \text{ screw.} \quad \text{Ans.}$$

$$p = \frac{1}{8}'' = \text{pitch for } 1'' \text{ screw.} \quad \text{Ans.}$$

$$p = \frac{1}{6}'' = \text{pitch for } 1\tfrac{1}{2}'' \text{ screw.} \quad \text{Ans.}$$

(965) (a) See Art. **1931.**

(b) $\frac{1}{2}'' \div 3 = \frac{1}{6}''$. Ans.

(966) See Art. **1930.**

(967) From Table 25, the ultimate tensile strength of wrought iron is 55,000 lb. per sq. in. From Table 28, the factor of safety for a varying stress is 6. Hence, the safe stress per square inch is $\frac{55{,}000}{6}$, and $11{,}800 \div \frac{55{,}000}{6} = \frac{11{,}800 \times 6}{55{,}000} = 1.2873$ sq. in. = area at bottom of thread. From Table 43, the nearest area in the last column is 1.2928 sq. in., corresponding to a nominal diameter of $1\frac{1}{2}''$. Hence, the bolt should have a diameter of $1\frac{1}{2}''$. Ans.

(968) Using formula **217,**

$$a = \frac{11{,}800}{6{,}000} = 1.967 \text{ sq. in.}$$

From Table 43, the nearest diameter corresponding to this area is $1\frac{7}{8}''$. Ans.

The reason that this diameter should be used instead of that calculated in the last example is that the bolt is greatly weakened by the sharp corners left by the thread cutting tool,

and the strength is not equal to that of a round bar whose diameter is the same as the diameter at the bottom of the thread.

(969) Using formula **218,**

$$d = .0228\sqrt{12{,}000} = 2\tfrac{1}{2}''. \quad \text{Ans.}$$

$$d_1 = 2.5 \times \frac{4}{5} = 2''. \quad \text{Ans.}$$

Using formula **219,**

$$n_1 = \frac{12{,}000}{300 \times 2^2} = 10 \text{ threads.} \quad \text{Ans.}$$

(970) The dimensions of both head and nut are the same. Hence, apply formulas **220-226.**

Height of head or nut =

$$h = \frac{7}{8}'' - \frac{1}{16}'' = \frac{13}{16}''. \quad \text{Ans.}$$

Diameter of nut or head across flats =

$$D = 1\frac{1}{2} \times \frac{7}{8}'' + \frac{1}{16}'' = 1\frac{3}{8}''. \quad \text{Ans.}$$

Diameter of nut or head across corners =

$$D_1 = 1.73 \times \frac{7}{8}'' + .07'' = 1.58''. \quad \text{Ans.}$$

Diameter of washer =

$$D_2 = 1\frac{1}{8} \times 1.58'' = 1.78'', \text{ say } 1\frac{25}{32}''. \quad \text{Ans.}$$

Thickness of washer =

$$t = .15 \times \frac{7}{8}'' = .13'', \text{ say } \frac{1}{8}''. \quad \text{Ans.}$$

Fig. 605 shows the manner of representing the bolt, except that a hexagonal head is to be shown instead of a square one, and the letters are to be replaced by the sizes calculated.

(971) In the last example, D was found to be $1\frac{3}{8}''$. The other dimensions are readily obtained from Fig. 606. The student should give the handle of the wrench a slight taper to improve the looks.

(972) From Table 28, Art. **1362,** the factor of safety for a varying stress is 6 for wrought iron. Hence, area of cross-section of eye bolt $= 2{,}500 \div \frac{55{,}000}{6} = .273$ sq. in. The corresponding diameter $= d = \sqrt{\frac{.273}{.7854}} = .59''$, say $\frac{5}{8}''$. Ans.

Applying formula **227,**

$$d_1 = .8 \times \frac{5}{8} = \frac{1}{2}''. \quad \text{Ans.}$$

Taking $a\,b$ equal to $\frac{1}{2}d^2$, $a\,b = \frac{1}{2} \times \left(\frac{5}{8}\right)^2 = .195$ sq. in.

If we assume that $a = \frac{1}{2}b$, $a\,b = \frac{1}{2}b^2$, and $b = \sqrt{.195 \times 2} = .624''$, say $\frac{5}{8}''$. Ans.

Whence, $$a = \frac{1}{2} \times \frac{5}{8} = \frac{5}{16}''. \quad \text{Ans.}$$

(973) See Art. **1944.**

(974) See Fig. 623. Take height of thin nut as one-half that of outer or standard size nut.

(975) Both devices increase the friction between the threads of the screw and the threads in the nut; in this respect the principle is the same in both devices.

(976) Taking d as $1\frac{1}{4}''$, the various dimensions are readily obtained from Fig. 642. The student may supply any dimension that is omitted, taking such dimensions as he deems best.

(977) Applying formula **233,**

$$b = \frac{1}{4} \times 4 = 1''. \quad \text{Ans.}$$

$$t = \frac{1}{6} \times 4 = \frac{2}{3}'', \text{ say } \frac{11}{16}''. \quad \text{Ans.}$$

(978) Applying formula **233,**

$$b = \frac{1}{4} \times 12 = 3''. \quad \text{Ans.}$$

$$t = \frac{1}{6} \times 12 = 2''. \quad \text{Ans.}$$

(979) Applying formula **235**,

$$b = .2 \times 2\frac{7}{16} + .16 = .65'', \text{ say } \frac{11}{16}''. \quad \text{Ans.}$$

$$t = .1 \times 2\frac{7}{16} + .16 = .4'', \text{ say } \frac{7}{16}''. \quad \text{Ans.}$$

(980) Applying formula **237**,

$$b = \frac{1.5''}{8} + \frac{1}{16}'' = \frac{1}{4}''. \quad \text{Ans.}$$

$$t = \frac{3 \times 1.5''}{16} + \frac{1}{16}'' = \frac{5.5''}{16}, \text{ say } \frac{3}{8}''. \quad \text{Ans.}$$

(981) Applying formula **238**,

$$d = 5\sqrt[3]{\frac{1.85}{110}} = 1.26''.$$

Using formulas **236**,

$$b = \frac{1.26''}{3} = .42'', \text{ say } \frac{7}{16}''. \quad \text{Ans.}$$

$$t = \frac{1.26''}{5} = \frac{1}{4}''. \quad \text{Ans.}$$

(982) Area of rod corresponding to diameter $d_1 = \frac{7,000}{6,000} = 1.167$ sq. in.

Hence, $d_1 = \sqrt{\frac{1.167}{.7854}} = 1.22''$, say $1\frac{1}{4}''$. Ans.

From formula **239**,

$$d_1 = .816\,d, \text{ or } d = \frac{d_1}{.816} = \frac{1.25}{.816} = 1.53'' = 1\frac{17}{32}''. \quad \text{Ans.}$$

The remaining dimensions are readily calculated by using formula **239.** The dimension b in Figs. 656 and 658 should extend to the top of the cotter instead of to the top of the slot, as shown. The student may take b as the depth of the cotter.

(983) Applying formula **241**,

$$d = 1.5\sqrt{\frac{10,000}{\sqrt{14,000 \times 400}}} = 3'', \text{ nearly.} \quad \text{Ans.}$$

Hence, $l = \frac{10,000}{3 \times 400} = 8.33'', \text{ say } 8\frac{3}{8}''$. Ans.

(984) Applying formula **244,**

$$d = 1.06\sqrt{\frac{15{,}000}{\sqrt{14{,}000 \times 1{,}200}}} = 2'', \text{ nearly.} \quad \text{Ans.}$$

Using formula **245,**

$$l = \frac{15{,}000}{2 \times 1{,}200} = 6\tfrac{1}{4}''. \quad \text{Ans.}$$

(985) From formula **246,**

$$d_1 = d\sqrt[3]{\frac{l_1}{l}} = 1.5\sqrt[3]{\frac{2.5}{2.125}} = 1.583'', \text{ say } 1\tfrac{19}{32}''. \quad \text{Ans.}$$

(986) Applying formula **251,**

$$d_1 = \sqrt{8^2 + \frac{15{,}000}{15 \times \pi \times 6}} = 10.9'', \text{ say } 10\tfrac{7}{8}''. \quad \text{Ans.}$$

Since $t = .8\,e$, and $e = \dfrac{d_1 - d}{2}$, $t = \dfrac{.8(d_1 - d)}{2} = .4\,(10\tfrac{7}{8} - 8)$ $= 1.15''$, say $1\tfrac{3}{16}''$. Ans.

(987) Applying formula **249,** second column of Table 47,

$$d = .07\sqrt{1{,}200} = 2.425'', \text{ say } 2\tfrac{7}{16}''. \quad \text{Ans.}$$

(988) (*a*) Apply formula **256.** Here, $m = \dfrac{4}{7}$; hence,

$$d_1 = 10\sqrt[3]{\frac{1}{1 - (\frac{4}{7})^4}} = 10.38'', \text{ say } 10\tfrac{3}{8}''. \quad \text{Ans.}$$

(*b*) Diameter of hole $= 10.38'' \times \dfrac{4}{7} = 6''$, nearly. Ans.

(*c*) Weight of solid shaft per foot of length $=$

$$\frac{.7854 \times 10^2}{144} \times 490 = 267 \text{ lb., nearly.}$$

Weight of hollow shaft per foot of length $=$

$$\frac{.7854\,[(10\tfrac{3}{8})^2 - 6^2]}{144} \times 490 = 191 \text{ lb., nearly.}$$

Difference $= 267 - 191 = 76$ lb. per foot. Ans.

(989) (*a*) Using formula **208,**

$$p = .24\sqrt{3.25 + .625} - .175 = .3'', \text{ nearly.}$$

By formula **210,**

$$n = \frac{1}{.3} = 3.333, \text{ say } 3\frac{1}{2}.$$

Then, $p = \frac{1}{n} = \frac{1}{3.5} = \frac{2''}{7}$. Ans.

(*b*) Using formula **212,**

$$p = \frac{7}{5} = 1.4'', \text{ say } 1\frac{3}{8}''. \quad \text{Ans.}$$

(*c*) Using formula **215,**

$$p = \frac{2 \times 12}{15} = 1.6'', \text{ say } 1\frac{5}{8}''. \quad \text{Ans.}$$

(990) (*a*) Using formula **210** for each case,

$n = 1 \div \frac{2}{7} = 3\frac{1}{2}$; $n = 1 \div 1\frac{3}{8} = \frac{8}{11}$, and $n = 1 \div 1\frac{5}{8} = \frac{8}{13}$. Ans.

(*b*) Using formula **211** for the first case,

$$d_1 = d - \frac{1.3}{n}'' = 3.25'' - \frac{1.3}{3.5}'' = 2.879''. \quad \text{Ans.}$$

For the second case, it will be remembered that $t = \frac{1}{2}p$; hence,

$$d_1 = d - 2\left(\frac{1}{2}p\right) = d - p = 7'' - 1\frac{3}{8}'' = 5\frac{5}{8}''. \quad \text{Ans.}$$

For the third case, $t_1 = \frac{3}{4}t = \frac{3}{4}p$; hence,

$$d = d - 2\left(\frac{3}{4}p\right) = d - 1.5p = 12 - 1.5 \times 1.625 = 9\frac{9}{16}''. \quad \text{Ans.}$$

(991) See Art. **1930.**

(992) See Art. **1935.** Using formula **217,**

$$a = \frac{15,600}{6,000} = 2.6 \text{ sq. in.}$$

Referring to Table 43, this area lies between the areas corresponding to diameters of 2″ and 2¼″; choosing the larger, the outside diameter should be 2¼″. Ans.

(993) From Table 43, the effective area of a 4-inch bolt is 9.993 sq. in. Referring to Art. **1935,** and using formula **217,**

$W = a\,S_t = 2 \times 9.993 \times 8{,}000 = 159{,}888$ lb., say 80 tons. Ans.

(994) (*a*) Using formula **218,**

$$d = .0228\sqrt{3{,}600} = 1.368'', \text{ say } 1\tfrac{3}{8}''. \quad \text{Ans.}$$

(*b*) Using formula **219,**

$$n_1 = .0052 \times \frac{3{,}600}{1.375^3} = 9.89, \text{ say 10 threads.} \quad \text{Ans.}$$

(995) Applying formula **217,** and taking S_t as 6,000 lb. per sq. in.,

$$a = \frac{3{,}200}{6{,}000} = .533 \text{ sq. in.}$$

From Table 43, the nearest diameter is 1″.

Applying formulas **220, 221, 224,** and **226,**

$$D = 1\tfrac{1}{2} \times 1'' + \tfrac{1}{16}'' = 1\tfrac{9}{16}''. \quad \text{Ans.}$$

$$h = h' = 1'' - \tfrac{1}{16}'' = \tfrac{15}{16}''. \quad \text{Ans.}$$

$$D_1 = 2.12 \times 1'' + .09'' = 2.21''. \quad \text{Ans.}$$

The student can readily make the drawing by referring to Fig. 605. Omit the washer.

(996) The dimensions are readily obtained by aid of formula **229.**

(997) (*a*) Using formula **233** for all three cases,

$$b = \tfrac{1}{4}d = \tfrac{1}{4} \times 3\tfrac{1}{2} = \tfrac{7}{8}''. \quad \text{Ans.}$$

$$t = \tfrac{1}{6}d = \tfrac{1}{6} \times 3\tfrac{1}{2} = \tfrac{7}{12}'', \text{ say } \tfrac{9}{16}''. \quad \text{Ans.}$$

(*b*) $b = \tfrac{1}{4} \times 5 = 1\tfrac{1}{4}''$. Ans.

$t = \tfrac{1}{6} \times 5 = \tfrac{5}{6}''$, say $\tfrac{13}{16}''$. Ans.

(*c*) $b = \frac{1}{4} \times 1\frac{1}{2} = \frac{3}{8}''$. Ans.

$t = \frac{1}{6} \times 1\frac{1}{2} = \frac{1}{4}''$. Ans.

(998) Each key takes up $1{,}000 \div 2 = 500$ H. P. Hence, applying formula **232,**

$$b = \frac{18\,H}{l\,d\,N} = \frac{18 \times 500}{10 \times 10 \times 50} = 1.8'', \text{ say } 2''. \quad \text{Ans}$$

$$t = \frac{2}{3}\,b = \frac{2}{3} \times 2 = 1.333'', \text{ say } 1\frac{3}{8}''. \quad \text{Ans.}$$

(999) For the driven pulley, apply formula **234.**

$$b = \frac{2.75''}{7} + \frac{1}{4}'' = .64'', \text{ say } \frac{5}{8}''. \quad \text{Ans.}$$

$$t = \frac{2.75''}{12} + \frac{3}{16}'' = .42'', \text{ say } \frac{7}{16}''. \quad \text{Ans.}$$

For the driving pulley, apply formula **235.**

$$b = .2 \times 4.5'' + .16'' = 1.06'', \text{ say } 1\frac{1}{16}''. \quad \text{Ans.}$$

$$t = .1 \times 4.5'' + .16'' = .61'', \text{ say } \frac{5}{8}''. \quad \text{Ans.}$$

(1000) Using formulas **237,**

$$b = \frac{1.75}{8} + \frac{1}{16} = .28'', \text{ say } \frac{5}{16}''. \quad \text{Ans.}$$

$$t = \frac{3 \times 1.75}{16} + \frac{1}{16} = .39'', \text{ say } \frac{3}{8}''. \quad \text{Ans.}$$

(1001) It is evident that a $3\frac{3}{4}''$ shaft will transmit a great deal more than 6 H. P. at 135 R. P. M. Hence, using formulas **238** and **236,**

$$d = 5\sqrt[3]{\frac{6}{135}} = 1.77''. \quad \text{Therefore,}$$

$$b = \frac{1.77}{3} = .59'', \text{ say } \frac{5}{8}''. \quad \text{Ans.}$$

$$t = \frac{1.77}{5} = .354'', \text{ say } \frac{3}{8}''. \quad \text{Ans.}$$

(1002) The calculation of d, d_1, and the other dimensions is in all respects similar to that required in example 982. The various dimensions in the figure may be obtained by using formula **239.**

(1003) According to Art. **1969,** $b\,t$ should equal $\frac{5}{4}$ times the sectional area of the strap, and $t = \frac{1}{4}\,b$. Hence, $b \times \frac{1}{4}\,b = \frac{5}{4} \times \frac{3}{4} \times 3.5$; or, $b^2 = 13.125$ sq. in., and $b = \sqrt{13.125} = 3.623''$, say $3\frac{5}{8}''$. Ans.

$$t = \frac{1}{4}\,b = \frac{1}{4} \times 3\frac{5}{8} = \frac{29}{32}'', \text{ say } \frac{15}{16}''. \quad \text{Ans.}$$

The other dimensions are readily obtained from Fig. 661, and the drawing can be made by referring to it.

(1004) Art. **1969** states that b may be made equal to d when a steel cotter is used in a wrought-iron rod. Hence,

$$b = d = 2\frac{1}{2}''. \quad \text{Ans.}$$

Also, $$t = \frac{1}{4}\,b = \frac{1}{4} \times 2\frac{1}{2} = \frac{5}{8}''. \quad \text{Ans.}$$

(1005) From Table 43, the area at the bottom of the thread for a $3\frac{1}{2}''$ standard screw is 7.55 sq. in. Referring to Art. **1935,** it would not be advisable to take S_t greater than 4,000 lb. per sq. in. in this case. Hence, $W = a\,S_t = 7.55 \times 4{,}000 = 30{,}200$ lb. = load which rod may safely carry.

When held by a cotter, the rod is weakened owing to the cutting out of the slot, the equivalent diameter being (see formula **239**) $d_1 = .816\,d = .816 \times 3.5 = 2.856''$; that is, a rod having a diameter of $2.856''$ would be, theoretically, of the same strength as a rod having a slot in it for a cotter and $3\frac{1}{2}''$ in diameter. Hence, the equivalent area is $2.856^2 \times .7854 = 6.41$ sq. in. The area is less than in the first case, but the value of S_t may now be taken as $\frac{55{,}000}{10} = 5{,}500$ lb. per sq.

in., and the safe load is $6.41 \times 5{,}500 = 35{,}255$ lb. Hence, the rod and cotter are, apparently, slightly stronger than the rod and nut.

(1006) (*a*) Applying formula **240,**

$$d = 2.26\sqrt{\frac{5{,}000}{4{,}250} \times 1.5} = 3''. \quad \text{Ans.}$$

$$l = 1\tfrac{1}{2}d = 1\tfrac{1}{2} \times 3 = 4\tfrac{1}{2}''. \quad \text{Ans.}$$

(*b*) Pressure per square inch of projected area =

$$p = \frac{5{,}000}{3 \times 4.5} = 370 \text{ lb.} \quad \text{Ans.}$$

(1007) Applying formula **240,**

$$d = l = 2.26\sqrt{\frac{10{,}000}{8{,}500} \times 1} = 2.45'', \text{ say } 2\tfrac{1}{2}''. \quad \text{Ans.}$$

(1008) See example in Art. **1975,** and Table 46, Art. **1978.**

$$d = 1.5\sqrt{\frac{17{,}800}{\sqrt{12{,}000} \times 750}} = 3.65'', \text{ say } 3\tfrac{5}{8}''. \quad \text{Ans.}$$

Whence, $l = \dfrac{17{,}800}{3\frac{5}{8} \times 750} = 6.55'', \text{ say } 6\tfrac{1}{2}''.$ Ans.

(1009) As in example 1008,

$$d = 1.5\sqrt{\frac{1{,}750}{\sqrt{8{,}500} \times 275}} = 1.6'', \text{ say } 1\tfrac{5}{8}''. \quad \text{Ans.}$$

Whence, $l = \dfrac{1{,}750}{1\frac{5}{8} \times 275} = 3.92'', \text{ say } 4''.$ Ans.

(1010) Here, $l = 2d$. From formula **242,** $l\,d = \dfrac{P}{p}$.

Hence, $2d \times d = 2d^2 = \dfrac{3{,}750}{550}$; or, $d = 1.846'', \text{ say } 1\tfrac{7}{8}''.$ Ans

Also, $l = 2d = 2 \times 1\tfrac{7}{8} = 3\tfrac{3}{4}''.$ Ans.

(1011) Circumference of wheel $= 2.75\,\pi$.

$$\text{R. P. M.} = N = \frac{5{,}280 \times 40}{2.75\,\pi \times 60} = 407.$$

Then, $p = \frac{220{,}000}{407} = 540$ lb. per sq. in.

Applying formula **241**, and getting S_f from Table 46,

$$d = 1.5\sqrt{\frac{10{,}000}{\sqrt{12{,}000 \times 540}}} = 2.97'', \text{ say } 3''. \quad \text{Ans.}$$

Hence, $l = \frac{10{,}000}{3 \times 540} = 6.17'', \text{ say } 6\frac{1}{4}''.$ Ans.

The other dimensions may be obtained from Fig. 665 and formula **247.**

(1012) (*a*) Using formula **243** and Table 46,

$$d = 1.13\sqrt{\frac{9{,}600}{7{,}000} \times 2.5} = 2.09'', \text{ say } 2\tfrac{1}{8}''. \quad \text{Ans.}$$

$$l = 2.5\,d = 2.5 \times 2\tfrac{1}{8} = 5\tfrac{5}{16}'', \text{ say } 5\tfrac{1}{4}''. \quad \text{Ans.}$$

(*b*) From formula **242,**

$$p = \frac{P}{l\,d} = \frac{9{,}600}{2\frac{1}{8} \times 5\frac{1}{4}} = 860 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(1013) Using formula **250,**

$$d = .004\sqrt{PN} = .004\sqrt{25 \times 3{,}000} = 1.1'', \text{ say } 1\tfrac{1}{8}''. \quad \text{Ans.}$$

(1014) Using formula **249,**

$$d = .05\sqrt{P} = .05\sqrt{9{,}000} = 4.74'', \text{ say } 4\tfrac{3}{4}''. \quad \text{Ans.}$$

(1015) Applying formula **251,**

$$d_1 = \sqrt{6^2 + \frac{7{,}800}{15\pi \times 4}} = 8.8'', \text{ say } 8\tfrac{3}{4}''. \quad \text{Ans.}$$

$$e = \frac{1}{2}(d_1 - d) = \frac{1}{2}(8\tfrac{3}{4} - 6) = 1\tfrac{3}{8}''. \quad \text{Ans.}$$

$$t = .8\,e = .8 \times 1\tfrac{3}{8}'' = 1.1'', \text{ say } 1\tfrac{1}{8}''. \quad \text{Ans.}$$

Taking s as $2\,t$, $s = 2 \times 1\frac{1}{8}'' = 2\frac{1}{4}''.$ Ans.

(See Fig. 668 for manner of representing the journal.)

(1016) This shaft should be calculated with the help of Table 49. To determine Z, we have, from the table of Bending Moments,

$$B = W\frac{l_1 l_2}{l} = 10 \times \frac{3 \times 5}{8} = 18\frac{3}{4} \text{ tons.}$$

The twisting moment $T = PR = 63{,}025 \times \frac{H}{N}$ in.-lb. (see formula **231**) =

$$\frac{63{,}025}{2{,}000 \times 12} \times \frac{H}{N} \text{ ft.-tons} = \frac{63{,}025}{2{,}000 \times 12} \times \frac{300}{60} = 13.13 \text{ ft.-tons.}$$

Then, $$Z = \frac{B}{T} = \frac{18.75}{13.13} = 1.4, \text{ nearly.}$$

From Table 49, $\sqrt[3]{k} = 1.461$ for $Z = 1.4$. Hence (see Table 32, Art. **1416**),

$$d_1 = d\sqrt[3]{k} = 1.461 \times 3.3\sqrt[3]{\frac{H}{N}} = 1.461 \times 3.3\sqrt[3]{\frac{300}{60}} = 8.25''.$$

Ans.

(1017) Applying formula **253**,

$$d = 68.5\sqrt[3]{\frac{90}{110 \times 7{,}000}} = 3.35''.$$

$$Z = \frac{.75\ T}{T} = .75.$$

From Table 49, $\sqrt[3]{k} = 1.209$ for $Z = .6$, and 1.277 for $Z = .8$. Hence, $1.277 - 1.209 : .8 - .6 = x : .75 - .6$; or, $x = .051$.

Therefore, for $Z = .75$, $\sqrt[3]{k} = 1.209 + .051 = 1.26$, and $d_1 = 1.26 \times 3.35 = 4.22''$, say $4\frac{1}{4}''$. Ans.

(1018) Use formula **256.** In this case $m = \frac{6}{13}$.

Hence, $d = d_1\sqrt[3]{1 - m^3} = 13\sqrt[3]{1 - \left(\frac{6}{13}\right)^3} = 12.8''$, say $12\frac{13}{16}''$.

Ans.

(1019) Use formula **256.** Here $m = \frac{1}{2} = .5$. Hence,

$$d_1 = 16\frac{3}{4}\sqrt[3]{\frac{1}{1 - .5^3}} = 17.11'', \text{ say } 17\frac{1}{8}''. \quad \text{Ans.}$$

$$d_2 = 17\frac{1}{8} \div 2 = 8\frac{9}{16}''. \quad \text{Ans.}$$

(1020) The student should be able to draw the joint by referring to Art. **2000** and Fig. 676. The keys may be proportioned by formula **233.** The student should use his own judgment regarding any dimensions not given.

(1021) Fig. 673 and formula **258** are a sufficient guide for the drawing of this coupling.

(1022) (*a*) From formula **260,**

$$n = \frac{1}{3}D + 2 = \frac{1}{3} \times 11 + 2 = 5\frac{2}{3}, \text{ say } 6. \quad \text{Ans.}$$

(*b*) From formula **259** and Table 50,

$$d = k\,D = .239 \times 11 = 2.629'', \text{ say } 2\frac{5}{8}''. \quad \text{Ans.}$$

(*c*) From formula **260,**

$$B = D + 2\,d = 11 + 2 \times 2.625 = 16\frac{1}{4}''. \quad \text{Ans.}$$

(*d*) $C = D + 4.25\,d = 11 + 4.25 \times 2.625 = 22.16$, say $22\frac{3}{16}''$. Ans.

(*e*) $T = \frac{2\,D + 1}{7} = \frac{2 \times 11 + 1}{7} = 3\frac{2}{7}''$, say $3\frac{1}{4}''$. Ans.

(1023) Using formula **250,**

$d = .004\sqrt{PN} = .004\sqrt{100 \times 600} = .98''$, say $1''$. Ans.

MACHINE DESIGN.

(QUESTIONS 1024–1063.)

(1024) See Art. **2004.**

(1025) See Art. **2016.**

(1026) Radius of gear $= 15'' = 1\frac{1}{4}$ ft. $= R$. From formula **231,**

$$p = \frac{63{,}025\,H}{n\,R} = \frac{63{,}025 \times 20}{120 \times 15} = 700 \text{ lb., nearly.}$$

From formula **267,**

$$S = \frac{9{,}600{,}000}{v + 2{,}160} = \frac{9{,}600{,}000}{2.5 \times \pi \times 120 + 2{,}160} = 3{,}094 \text{ lb. per sq. in., nearly.}$$

Hence, from formula **266,**

$$b\,C = 16.8\,\frac{p}{S} = \frac{16.8 \times 700}{3{,}094} = 3.8 \text{ sq. in.}$$

Assuming that $b = 2\frac{1}{2}C$, $b\,C = 2.5\,C^2 = 3.8$ sq. in., or $C = \sqrt{\frac{3.8}{2.5}} = 1.2329''$. Hence, number of teeth $=$

$$\frac{\pi \times 30}{1.2329} = 76 +, \text{ say } 76. \quad \text{Ans.}$$

$$\text{Circular pitch} = \frac{\pi \times 30}{76} = 1.2401''. \quad \text{Ans.}$$

(1027) The velocity of a point on the pitch circle is $\pi \times 4\frac{2}{3} \times 60 = 880$ ft. per min., nearly. Hence, from formula **267,**

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$$S = \frac{9{,}600{,}000}{880 + 2{,}160} = 3.158 \text{ lb. per sq. in., nearly.}$$

From formula **231**,

$$p = \frac{63{,}025 \times 300}{60 \times 28} = 11{,}254.5 \text{ lb.}$$

From formula **266**,

$$b\,C = \frac{16.8 \times 11{,}254.5}{3{,}158} = 59.87 \text{ sq. in.}$$

Taking b equal to $3\,C$, $3\,C^2 = 59.87$ sq. in.; whence, $C = 4.4673''$.

$$\text{Number of teeth} = \frac{\pi \times 56}{4.4673} = 39.4, \text{ say } 39 \text{ teeth.}$$

Then, $$C = \frac{3.1416 \times 56}{39} = 4.511''. \quad \text{Ans.}$$

Breadth of face =

$$b = 3\,C = 3 \times 4.511 = 13.533, \text{ say } 13\tfrac{1}{2}''. \quad \text{Ans.}$$

Thickness of tooth $= .475\,C = .475 \times 4.511 = 2.143''$. Ans.

Height above pitch line $= .3\,C = .3 \times 4.511 = 1.353''$. Ans.

Depth below pitch line $= .4\,C = .4 \times 4.511 = 1.804''$. Ans.

Using formula **123**, Art. **1415**,

$$\text{diameter of shaft} = d = 4.92\sqrt[4]{\frac{300}{60}} = 7.36'', \text{ say } 7\tfrac{3}{8}''. \quad \text{Ans.}$$

The enlargement for the wheel seat will have a diameter of $1.2\,d = 1.2 \times 7\frac{3}{8} = 8.85''$, say $8\frac{7}{8}''$. Ans.

By formula **273**, thickness of hub =

$$w = \tfrac{1}{3}\sqrt[3]{b\,C\,R} = \tfrac{1}{3}\sqrt[3]{13.5 \times 4.511 \times 28} = 3.983'', \text{ say } 4''. \quad \text{Ans.}$$

The length of the hub may be

$$1.4\,b = 1.4 \times 13\tfrac{1}{2} = 18.9'', \text{ say } 19''. \quad \text{Ans.}$$

Formula **272** gives for the number of arms,

$$z = .55\sqrt[3]{N^2\,C} = .55\sqrt[3]{39^2 \times 4.511} = 5 \text{ arms.} \quad \text{Ans.}$$

By formula **270**, width of arm at center of wheel =

$$a = \sqrt{\frac{R\,b}{z}} = \sqrt{\frac{28 \times 13\frac{1}{2}}{5}} = 8.6948'', \text{ say } 8\frac{11}{16}''. \quad \text{Ans.}$$

Assuming the taper to be $\frac{1}{48}$ on each side, the width of arm at pitch line is

$$8\frac{11}{16} - (2 \times \frac{1}{48} \times 28) = 7\frac{1}{2}'', \text{ nearly.} \quad \text{Ans.}$$

Thickness of arm $= \frac{1}{2}\,C = \frac{1}{2} \times 4.511 = 2.255'' = 2\frac{1}{4}''$. Ans.

Thickness of stiffening rib $= .4\,C = .4 \times 4.511 = 1.804''$. Ans.

(1028) By formula **275**,

$$P = Q f = 280 \times \frac{1}{3} = \frac{280}{3}.$$

$$\text{Radius of wheel} = R = \frac{6 \times 12}{2} = 36''.$$

Then, by formula **231**,

$$H = \frac{P R N}{63{,}025} = \frac{280 \times 36 \times 110}{3 \times 63{,}025} = 5.864 \text{ H. P.} \quad \text{Ans.}$$

(1029) See Art. **2049.**

(1030) (*a*) By formula **282**,

$$d = \frac{D}{9} = \frac{1}{9}''. \quad \text{Ans.}$$

(*b*) By formula **283**, $w = 1.43\,D^2 = 1.43 \times 1^2 = 1.43$ lb. Ans.

(1031) By formula **291**,

$$P = 6{,}600\,d^3 = 6{,}600 \times 1.25^3 = 10{,}312.5 \text{ lb.} \quad \text{Ans.}$$

(1032) See Art. **2005.**

(1033) The values of the different dimensions, as determined from the table of proportions in Art. **2014**, are as follows:

$$d = 14''.$$

$a = 14 + 1 = 15''$.
$b = .5 \times 14 + 1 = 8''$.
$c = .66 \times 14 = 9.24''$, say $9\frac{1}{4}''$.
$e = .825 \times 14 - .25 = 11.3''$, say $11\frac{5}{16}''$.
$f = .6 \times 14 = 8.4''$, say $8\frac{3}{8}''$.
$g = .1 \times 14 + .5625 = 1.9625''$ say $1\frac{31}{32}''$.
$h = .1 \times 14 + .25 = 1.65''$, say $1\frac{21}{32}''$.
$h' = .08 \times 14 = 1.12''$, say $1\frac{1}{8}''$.
$i = .11 \times 14 = 1.54''$, say $1\frac{17}{32}''$.
$j = \frac{5}{8}''$.
$k = .5 \times 14 + 1.25 = 8\frac{1}{4}''$.
$l = \frac{3}{8}''$.
$m = .175 \times 14 + .3125 = 2.7625''$, say $2\frac{3}{4}''$.
$n = .25 \times 14 + .25 = 3\frac{3}{4}''$.
$n' = .1 \times 14 + .375 = 1.775''$, say $1\frac{3}{4}''$.
$o = 1''$.
$p = .25 \times 14 + .625 = 4\frac{1}{8}''$
$q = 1.75 \times 14 = 24\frac{1}{2}''$.
$q' = 1.5 \times 14 = 21''$.
$r = .15 \times 14 = 2.1''$, say $2\frac{1}{8}''$
$r' = .1 \times 14 = 1.4''$, say $1\frac{3}{8}''$.
$r_1 = 14''$.
$s = .9 \times 14 = 12.6''$, say $12\frac{5}{8}''$.
$t = .15 \times 14 + .375 = 2.475''$, say $2\frac{1}{2}''$.
$t' = .9 \times 14 = 12.6''$, say $12\frac{5}{8}''$.
$u = 1.5 \times 14 = 21''$.
$v = .25 \times 14 + .375 = 3\frac{7}{8}''$.
$w = 1.45 \times 14 = 20.3''$, say $20\frac{1}{4}''$.
$w' = 1.47 \times 14 = 20.58''$, say $20\frac{5}{8}''$.
$w_1 = 1.75 \times 14 = 24\frac{1}{2}''$.
$x = .1 \times 14 = 1.4''$, say $1\frac{3}{8}''$.
$y = .3 \times 14 + .75 = 4.95''$, say $5''$.
$y' = .2 \times 14 + .5 = 3.3''$, say $3\frac{1}{4}''$.
$z = .09 \times 14 = 1.26''$, say $1\frac{1}{4}''$.
$z' = 2\frac{1}{2}''$.

The student may use his own judgment to supply any dimensions not given.

(1034) See Art. **2023.**

(1035) Number of teeth $= N = 56 \times 1\frac{1}{4} = 70.$

$$\text{Circular pitch} = C = \frac{3.1416}{1\frac{1}{4}} = 2.5133''.$$

Applying formula **272,**

$$z = .55\sqrt[4]{N^2 C} = .55\sqrt[4]{70^2 \times 2.5133} = 5.79, \text{ or } 6 \text{ arms.}$$ Ans.

(1036) See Art. **2037.**

(1037) See Art. **2057.**

(1038) See Art. **2066.**

(1039) See Art. **2074.**

(1040) See Arts. **2005** and **2006.**

(1041) See Art. **2022.**

(1042) The values of the different dimensions, as determined from the table of proportions in Art. **2011,** are:

$$d = 3\tfrac{1}{2}''.$$

$a = 3.25 \times 3.5 = 11\tfrac{3}{8}''$.
$b = 1.75 \times 3.5 = 6\tfrac{1}{8}''$.
$c = 3\tfrac{1}{2}''$.
$e = .5 \times 3.5 = 1\tfrac{3}{4}''$.
$f = .4375 \times 3.5 = 1.53125''$, say $1\tfrac{17}{32}''$.
$g = .09 \times 3.5 = .315''$, say $\tfrac{5}{16}''$.
$h = .3125 \times 3.5 = 1\tfrac{3}{32}''$.
$i = .25 \times 3.5 = \tfrac{7}{8}''$.
$j = .375 \times 3.5 = 1\tfrac{5}{16}''$.
$k = 1.0625 \times 3.5 = 3.72''$, say $3\tfrac{3}{4}''$.
$l = .875 \times 3.5 = 3\tfrac{1}{16}''$.
$m = 1.75 \times 3.5 = 6\tfrac{1}{8}''$.
$n = 1.25 \times 3.5 = 4\tfrac{3}{8}''$.
$o = \tfrac{1}{8}''$.
$p = \tfrac{7}{8}''$.
$q = .625 \times 3.5 = 2\tfrac{3}{16}''$.
$r = .25 \times 3.5 = \tfrac{7}{8}''$.
$s = .1875 \times 3.5 = \tfrac{21}{32}''$, say $\tfrac{11}{16}''$.
$t = .65 \times 3.5 = 2.275''$, say $2\tfrac{5}{16}''$.
$u = .75 \times 3.5 = 2\tfrac{3}{8}''$.
$v = 1.375 \times 3.5 = 4\tfrac{13}{16}''$, say $4\tfrac{7}{8}''$.
$x = .25 \times 3.5 = \tfrac{7}{8}''$.
$y = .5 \times 3.5 = 1\tfrac{3}{4}''$.
$z = .0625 \times 3.5 = \tfrac{7}{32}''$, say $\tfrac{1}{4}''$.

(1043) See Arts. **2020** and **2018.**

(1044) Applying formula **268,**

$$b = 4\tfrac{1}{2}\frac{H}{D} = \frac{4.5 \times 21}{44} = 2.148'', \text{ say } 2\frac{1}{8}''. \quad \text{Ans.}$$

(1045) Applying formula **274,**

$$e = \frac{Np_1}{Np_1 + K} = \frac{4 \times .5236}{4 \times .5236 + 1\tfrac{1}{4}} = 62.62\%. \quad \text{Ans.}$$

(1046) See Art. **2038.**

(1047) Applying formula **276,**

$$t = \frac{B + D}{200} + \frac{1}{16}'' = \frac{10 + 52}{200} + \frac{1}{16} = .3725'', \text{ say } \frac{3}{8}''. \quad \text{Ans.}$$

(1048) Applying formula **283,**

$$w = 1.43\,D^2 = 1.43 \times \left(\frac{7}{8}\right)^2 = 1.094 \text{ lb.}$$

$$a = 420 \div 2 = 210.$$

Applying formula **285,**

$$h = \frac{T}{2\,w} - \sqrt{\frac{T^2}{4\,w^2} - \frac{a^2}{2}} =$$

$$\frac{2{,}023}{2 \times 1.094} - \sqrt{\frac{2{,}023^2}{4 \times 1.094^2} - \frac{210^2}{2}} = 12 \text{ ft.} \quad \text{Ans.}$$

(1049) See Art. **2047.**

(1050) See Art. **2050.**

(1051) Velocity $= \dfrac{18 \times \pi \times 72}{60} = 67.86$ feet per second.

Using formula **281,**

$$H = \frac{v\,D^2}{825}\left(200 - \frac{v^2}{107.2}\right) = \frac{67.86 \times (1\frac{1}{4})^2}{825}\left(200 - \frac{67.86^2}{107.2}\right) =$$

20.18 H. P. $20.18 \times 24 = 484.32$ H. P. Ans.

(1052) By formula **283,**

$$w = 1.43\,D^2 = 1.43 \times \left(1\frac{1}{4}\right)^2 = 2.23 \text{ lb. per foot.}$$

$$a = 386 \div 2 = 193 \text{ ft.}$$

Applying formula **284,**

$$T = \frac{w\,a^2}{2\,h} + w\,h = \frac{2.23 \times 193^2}{2 \times 9.25} + 2.23 \times 9.25 = 4{,}511 \text{ lb.}$$

Ans.

(1053) See Art. **2070,**

$$n = .13\sqrt[3]{P} = .13\sqrt[3]{5{,}600} = 2.31, \text{ say } 2.$$

$$d = .0115\sqrt{\frac{P}{\sqrt{n}}} = .0115\sqrt{\frac{5{,}600}{\sqrt{2}}} = .7237'', \text{ say } \frac{3}{4}''.$$

$$d' = 1.2\,d = 1.2 \times \frac{3}{4} = .9'', \text{ say } \frac{29}{32}''.$$

Remaining dimensions are the same as those calculated in Art. **2070.**

(1054) The values of the dimensions given in the table of proportions in Art. **2019** are:

$$d = 2\tfrac{1}{2}''.$$

$a = 3 \times 2.5 = 7\tfrac{1}{2}''$. $m = .125 \times 2.5 = \tfrac{5}{16}''$.

$b = 7.5 \times 2.5 = 18\tfrac{3}{4}''$. $n = .25 \times 2.5 = \tfrac{5}{8}''$.

$c = 5.375 \times 2.5 = 13\tfrac{7}{16}''$. $o = .1875 \times 2.5 = \tfrac{15}{32}''$, say $\tfrac{1}{2}''$.

$e = 3 \times 2.5 = 7\tfrac{1}{2}''$. $p = .375 \times 2.5 = \tfrac{15}{16}''$, say $1''$.

$f = 1.75 \times 2.5 = 4\tfrac{3}{8}''$. $q = .625 \times 2.5 = 1\tfrac{9}{16}''$.

$g = 1.5 \times 2.5 = 3\tfrac{3}{4}''$. $r = 1.25 \times 2.5 = 3\tfrac{1}{8}''$.

$h = 2.125 \times 2.5 = 5\tfrac{5}{16}''$. $s = .25 \times 2.5 + .5 = 1\tfrac{1}{8}''$.

$i = .16 \times 2.5 = .4''$, say $\tfrac{7}{16}''$. $v = 1.125 \times 2.5 + .1875 = 3''$.

$j = .25 \times 2.5 = \tfrac{5}{8}''$. $w = 1.5 \times 2.5 = 3\tfrac{3}{4}''$.

$k = 1.875 \times 2.5 = 4\tfrac{11}{16}''$. $y = .0625 \times 2.5 = \tfrac{5}{32}''$.

$l = 1.125 \times 2.5 = 2\tfrac{13}{16}''$. $z = 2.5 - .0625 = 2\tfrac{7}{16}''$.

For the other dimensions necessary to draw the bracket, see Fig. 697.

(1055) Using formula **271**,

$$a = 1.75\sqrt[3]{\frac{6 \times 20}{1\frac{1}{2} \times 6}} = 4.15'', \text{ say } 4\frac{1}{8}''. \quad \text{Ans.}$$

$$\frac{1}{2}a = \frac{1}{2} \times 4\frac{1}{8} = 2\frac{1}{16}''. \quad \text{Ans.}$$

(1056) (*a*) The diameter of the wire composing the rope is, by formula **282**,

$$d = \frac{D}{9} = \frac{\frac{3}{4}}{9} = \frac{1}{12}''.$$

The weight of the rope per foot is, by formula **283**,

$$w = 1.43\ D^2 = 1.43 \times \left(\frac{3}{4}\right)^2 = .8044 \text{ lb.}$$

Stress due to bending is, by formula **287**,

$$S_b = \frac{E_1 d}{2R} = \frac{25{,}000{,}000 \times \frac{1}{12}}{13 \times 12} = 13{,}355 \text{ lb. per sq. in.}$$

Stress due to centrifugal force is, by formula **288**,

$$S_c = \frac{4\,w\,v^2}{\pi d^2 n g} = \frac{4 \times .8044 \times \left(\frac{5{,}000}{60}\right)^2}{3.1416 \times \left(\frac{1}{12}\right)^2 \times 42 \times 32.16} = 758 \text{ lb. per sq. in.}$$

Hence, $S_t = S - (S_b + S_c) = 25{,}000 - (13.355 + 758) =$ 10,887 lb. per sq. in.

$T_1 =$ maximum tension on driving side $=$

$$\frac{\pi}{4} d^2 n S_t = \frac{\pi}{4} \times \left(\frac{1}{12}\right)^2 \times 42 \times 10{,}887 = 2{,}494 \text{ lb.}$$

$T_2 = \frac{1}{2} T_1 = 2{,}494 \times .5 = 1{,}247$ lb.

$P =$ driving force $= T_1 - T_2 = 2{,}494 - 1{,}247 = 1{,}247$ lb.

$$\text{Horsepower} = H = \frac{PV}{33{,}000} = \frac{1{,}247 \times 5{,}000}{33{,}000} = 189 \text{ H. P.}$$

(*b*) The greatest deflection will, of course, be on the driven, or slack, side. Hence, applying formula **285,**

$$h = \frac{T_2}{2w} - \sqrt{\frac{T_2^2}{4w^2} - \frac{a^2}{2}} = \frac{1{,}247}{2 \times .8044} - \sqrt{\frac{1{,}247^2}{4 \times .8044^2} - \frac{212.5}{2}} =$$

14.7 ft. Ans.

(1057) Circular pitch $= C = \frac{\pi d}{84} = \frac{\pi \times 21}{84} = .7854''$.

Applying formula **272,**

$$z = .55 \sqrt[4]{84^2 \times .7854} = 4.74+, \text{ say } 5 \text{ arms.} \quad \text{Ans.}$$

(1058) Using formula **277,**

$$a = \sqrt[3]{\frac{BR}{n}} = \sqrt[3]{\frac{8 \times 11}{4}} = 2.8'', \text{ say } 2\tfrac{7}{8}''. \quad \text{Ans.}$$

Assuming the taper to be $\frac{1}{48}$ on each side, the taper for both sides will be $\frac{1}{48} \times 2 = \frac{1}{24}$. The width of the arm at rim will be

$$2\tfrac{7}{8} - \left(\frac{1}{24} \times 11\right) = 2.417'', \text{ say } 2\tfrac{3}{8}''. \quad \text{Ans.}$$

$$\text{Thickness at center} = 2\tfrac{7}{8} \times \frac{1}{2} = 1\tfrac{7}{16}''. \quad \text{Ans.}$$

$$\text{Thickness at rim} \quad = 2\tfrac{3}{8} \times \frac{1}{2} = 1\tfrac{3}{16}''. \quad \text{Ans.}$$

(1059) Using formula **279,**

$$S' = \frac{A'}{4} = \frac{7 \times 1\frac{3}{8}}{4} = 2.406 \text{ sq. in.}$$

Net section of one bolt $= 2.406 \div 6 = .401$ sq. in.

Hence, according to Table 43, $\frac{7}{8}''$ in. bolts will be used. Ans.

(1060) (1) The dimensions as taken from Table 52, are:

$A = \frac{9}{32}''$; $B = \frac{7}{16}''$; $C = 2\frac{3}{4}''$; $D = 1\frac{3}{4}''$; $E = 8\frac{3}{4}''$; $F = 1\frac{1}{4}''$; $G = \frac{7}{8}''$; $H = 1\frac{1}{2}''$, and $I = \frac{3}{8}''$.

(2) From the table of proportions in Art. **2021,**

$d = 3''$.

$a = 2 \times 3 = 6''$.
$b = 1.5 \times 3 = 4\frac{1}{2}''$.
$c = .25 \times 3 + .375 = 1\frac{1}{8}''$.
$e = 1.25 \times 3 = 3\frac{3}{4}''$.
$f = .2 \times 3 + .125 = .725$, say $\frac{3}{4}''$.
$g = 1.75 \times 3 = 5\frac{1}{4}''$.
$h = 1.4 \times 3 = 4.2''$, say $4\frac{1}{4}''$.
$j = .4 \times 3 = 1.2''$, say $1\frac{1}{4}''$.
$k = .3 \times 3 = .9''$, say $\frac{15}{16}''$ or $1''$.
$l = .15 \times 3 = .45''$, say $\frac{13}{32}''$ or $\frac{1}{2}''$.
$m = 1.5 \times 3 = 4\frac{1}{2}''$.
$n = .2 \times 3 + .25 = .85''$, say $\frac{7}{8}''$.
$o = .15 \times 3 = .45''$, say $\frac{1}{2}''$.

(3) The various dimensions are readily obtained from Fig. 735.

(4) The dimensions, as obtained from Table 54, are:

$a = 12''$; $b = .79''$; $c = \frac{7}{8}''$; $d = 3\frac{3}{4}''$; $n = 12''$; $e = 1\frac{1}{4}''$; $f = 16\frac{1}{2}''$, and $g = 19''$.

(1061) Applying formula **280,**

$$w = .3\,D^2 = .3 \times \left(1\frac{3}{4}\right)^2 = .91875 \text{ lb. per ft.}$$

Hence, total weight $= 875 \times .91875 = 803.9$ lb. **Ans.**

(1062) (*a*) Using formula **281,**

$$H = \frac{v\,D^2}{825}\left(200 - \frac{v^2}{107.2}\right) = \frac{\frac{3,800}{60} \times \left(1\frac{3}{4}\right)^2}{825}\left(200 - \frac{3,800^2}{60^2 \times 107.2}\right) =$$

38.223 H. P. Ans.

(*b*) Referring to the diagram, Fig. 723, the horsepower appears to be about 38 H. P. Ans.

(1063) Applying the method described in Art. **2063,** and assuming that $\frac{R}{d} = 850$, we proceed as follows:

The driving force $= P = \frac{33{,}000\ H}{v} = \frac{33{,}000 \times 125}{4{,}900} = 842$ lb., nearly.

Then, $T_1 = 842 \times 2 = 1{,}684$ lb.

Stress due to bending is

$$S_b = \frac{E_1\, d}{2\, R} = \frac{25{,}000{,}000}{2 \times 850} = 14{,}706 \text{ lb. per sq. in.}$$

Hence, $S_t = 25{,}000 - 14{,}706 = 10{,}294$.

Cross-section of wires $= \frac{T_1}{S_t} = \frac{1}{4}\pi d^2 n$, or

$$d = \sqrt{\frac{4\, T_1}{\pi\, n\, S_t}} = \sqrt{\frac{4 \times 1{,}684}{\pi \times 42 \times 10{,}294}} = .0704.$$

The diameter of the rope is, by formula **282,**

$$D = 9\, d = 9 \times .0704 = .6336, \text{ say } \frac{5}{8}''.$$

A recalculation taking into account the stress due to centrifugal force will not appreciably affect the result; hence, the diameter of the rope is $\frac{5}{8}''$. Ans. (*c*)

(*a*) Diameter of wires is, then,

$$\frac{5}{8} \div 9 = .625 \div 9 = .0694''. \quad \text{Ans.}$$

(*b*) Radius of pulley $= .0694 \times 850 = 58.99''$, say $59''$.

Diameter of pulley $= 59 \times 2 = 118'' = 9$ ft. $10''$, say 10 ft

Ans.

MACHINE DESIGN.

(QUESTIONS 1064–1078.)

(1064) (*a*) See Art. **2076.**

(*b*) $100 - 8 = 92\%$. See Art. **2076.**

(1065) The values of the various dimensions, as determined from the table of proportions in Art. **2111**, are as follows:

$$D = 8''.$$

$$d = 1\frac{3}{8}''.$$

$$a = 2 \times 1\frac{3}{8} = 2\frac{3}{4}''.$$

$$b = 1.5 \times 1\frac{3}{8} = 2\frac{1}{16}''.$$

$$c = 1\frac{3}{8}''.$$

$$e = .75 \times 1\frac{5}{8}'' = 1\frac{7}{32}''.$$

$$f = .2 \times 8 = 1.6'', \text{ say } 1\frac{5}{8}''.$$

$$g = \frac{7.125}{6} = 1.1875'' = 1\frac{3}{16}''.$$

$$h = \sqrt{\frac{100 \times 8^2 \times .7854 \times .6}{16.5 \times 36}} = 7.125'' = 7\frac{1}{8}''$$

$$i = .26 \times 8 + .5'' = 2.58'', \text{ say } 2\frac{9}{16}''.$$

$$k = .075 \times 8 = .6'', \text{ say } \frac{5}{8}''.$$

$$l = \frac{7.125}{18} = 4'', \text{ say } \frac{3}{8}''.$$

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$m = \frac{3}{8}''$.

$n = \frac{13}{16}''$.

o must be greater than $a + h - \frac{1}{2}d$, see wrist-pin end of connecting-rod in Fig. 760. But $a = .75 \times .2 \times 8 + .125 = 1.325''$, say $1\frac{5}{16}''$, and

$$h = \frac{.5 \times \pi \times (.155 \times 8 + .0625)^2}{4\left[1.3 \times (.155 \times 8 + .0625) - .25\sqrt{2.5 \times \frac{\pi}{4}(.155 \times 8 + .0625)^2}\right]}$$

$= .53''$, say $\frac{17}{32}''$. Then $o = 1\frac{5}{16} + \frac{17}{32} - \frac{1}{2} \times 1\frac{5}{8} + \frac{7}{32}$ (assumed) $= 1\frac{1}{4}''$.

$p = .5 \times 1\frac{3}{8} = \frac{11}{16}''$.

$q = .75 \times 1\frac{3}{8} = \frac{33}{32}''$, say $1''$.

The student may use his own judgment to supply any dimensions not given.

(1066) See Art. **2076.**

(1067) Using formula **303,** and substituting values given,

$d = .0091\sqrt{p\,L\,B} = .0091\sqrt{120 \times 9 \times 16} = 1.2''$, say $1\frac{1}{4}''$.

Ans.

(1068) The values of the various dimensions, as obtained from the table of proportions in Art. **2093,** are as follows:

$D = 20''$.

$d = 3''$.

$a = .18\sqrt{2 \times 20} - .1875'' = .9509''$, say $1''$.

$b = .45 \times 1'' = .45''$, say $\frac{7}{16}''$.

$c = .65 \times 1'' = .65''$, say $\frac{21}{32}''$.

$d' = 2 \times 3 = 6''$.

$e = \frac{.06 \times 20}{4} = .3''$, say $\frac{5}{16}''$.

$$f = 1'' - \frac{1}{8} = \frac{7}{8}''.$$

$$g = .5 \times \frac{7}{8}'' = \frac{7}{16}''.$$

$$h = .2 \times 20 + 1.5 = 5\frac{1}{2}''.$$

$$i = 1'' - \frac{1}{8} = \frac{7}{8}''.$$

$$k = \frac{1.4 \times 20}{4} = 7''.$$

$$n = .08\,(20 + 34) = 4.32, \text{ say } 4.$$

The student may use his own judgment to supply any dimensions not given.

(1069) See Art. **2083.** 160 ft. per sec. Ans.

(1070) Using formula **303,** and substituting the values given,

$$d = .01\sqrt{p\,L\,B} = .01\sqrt{100 \times 6 \times 8\tfrac{1}{4}} = .7'', \text{ say } \frac{3}{4}''. \quad \text{Ans.}$$

(1071) See Art. **2083.**

(1072) (*a*) By formula **300,**

$$d = .0118 \times 12\sqrt{100} = 1.416'', \text{ say } 1\frac{7}{16}''. \quad \text{Ans.}$$

(*b*) By formula **301,**

$$d = .038\sqrt[4]{12^2 \times 32^2 \times 100} = 2.355'', \text{ say } 2\frac{3}{8}''. \quad \text{Ans.}$$

(*c*) By Art. **2103,**

$$d = \frac{4}{25}D = \frac{4 \times 12}{25} = 1.92'', \text{ say } 1\frac{15}{16}''. \quad \text{Ans.}$$

In formula **300,** the assumption is made that the rod is subjected to tension and compression only. In deriving formula **301,** the rod is treated as though it were a long column. In the formula given in Art. **2103,** the length of the rod is assumed to be twice the cylinder diameter, and the steam pressure is assumed to be 80 lb. per sq. in., neither of which is true in this case. The value obtained by formula **301** would be used in practice.

(1073) See Art. **2081.** Take the initial pressure as .92 of the boiler pressure, or as $100 \times .92 = 92$ lb. per sq. in., gauge, and $92 + 14.7 = 106.7$ lb. per sq. in., absolute.

By formula **292,** the real cut-off $= k = \dfrac{22}{22 + 106.7} = .171''$.

By formula **293,** $p_o = \dfrac{106.7 + 16}{2} = 61.35$ lb. per sq. in. = absolute pressure at end of compression. Now draw the theoretical indicator diagram, as shown in Fig. 96. Assume

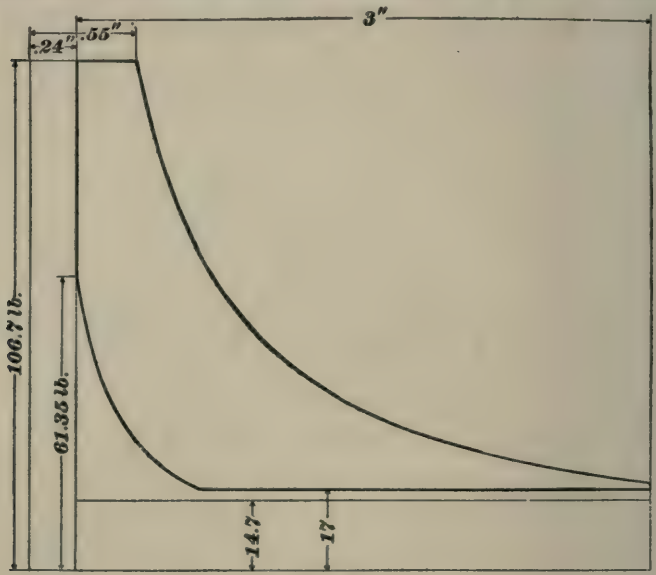

FIG. 96.

any convenient length (we have made the length 3″), also any convenient scale of pressures* (we have assumed 40 lb. = 1″), and then draw the diagram as described in Art. **2081.** Taking the clearance as 8%, the actual distance of the clearance line from the admission line $= 3 \times .08 =$ .24″, as shown. The point of cut-off will, therefore, be $3.24 \times .171 = .554''$, say 55, from the clearance line. Since

* NOTE.—The student must use a different length of diagram, and a different scale of pressures, from that used here. We suggest that he use 4″ for the length of his diagram, and 48 lb. = 1 in. or 50 lb. = 1 in., for his scale of pressures.

the engine is non-condensing, the back pressure line is taken 17 lb. above the vacuum line. The apparent cut-off $= \frac{.55 - .24}{3} = .1 = \frac{1}{10}$, very nearly.

If the diagram has been correctly drawn, the M. E. P. ought to be 26 lb. per sq. in. Assuming that the actual M. E. P. is but .85 of the theoretical M. E. P. just found, the actual M. E. P. may be taken as $26 \times .85 = 22.1$, say 22 lb. per sq. in. Then, area of cylinder

$$A = \frac{33{,}000 \times 200}{22 \times 600} = 500 \text{ sq. in.},$$

$$\text{and diameter} = \sqrt{\frac{500}{.7854}} = 25.23''.$$

To avoid fractions, take the diameter as 25″. Then, the area of the piston will be $25^2 \times .7854 = 490.875$ sq. in., and the piston speed, $\frac{33{,}000 \times 200}{22 \times 490.875} = 611$ ft. per min.

Taking the stroke as 30″, the revolutions per minute will be $\frac{611 \times 12}{2 \times 30} = 122.2$ R. P. M.

Hence, we have for our data the following:

Size of engine, 25″ × 30″.
Initial pressure, 92 lb., gauge.
Back pressure, 2.3 lb., gauge.
Pressure at end of compression, 46.65 lb., gauge.
Apparent cut-off, $\frac{1}{10}$.
Mean effective pressure, 22 lb.
Revolutions per minute, 122.2.
Piston speed, 611 ft. per min.
Clearance, 8%.

} Ans

(1074) Applying formula **296** (see Art. **2088**),

$d = .34\,D + 2\frac{1}{2}'' = .34 \times 30 + 2\frac{1}{2}'' = 12.7''$, say $12\frac{3}{4}''$. Ans.

(1075) (*a*) By formula **300**,

$d = .0167\,D\sqrt{p} = .0167 \times 38\sqrt{100} = 6.346''$, say $6\frac{3}{8}''$. Ans.

(*b*) By formula **301**,

$$d = .04\sqrt[4]{D^2 l^2 p} = .04\sqrt[4]{38^2 \times 79^2 \times 100} = 6.928'', \text{ say } 6\frac{15}{16}''.$$

Ans.

(*c*) By Art. **2103**,

$$d = \frac{D}{6} = \frac{38}{6} = 6\frac{1}{3}'', \text{ say } 6\frac{5}{16}''. \quad \text{Ans.}$$

In absence of any further data in regard to the engine, the result obtained by formula **301** ought to be taken, for the reason that more conditions are included in the derivation of the formula.

(1076) See Art. **2079.**

(1077) See Art. **2080.**

(1078) The values of the various dimensions, as obtained from the table of proportions, in Art. **2090,** are as follows:

$$D = 24''.$$

$R = 52 \div 2 = 26''.$

$d = .5 \times 24 = 12''.$

$a = 12''.$

$b = 1.75 \times 12 = 21''.$

$c = .045 \times 12 + .0625 = .6025''$, say $\frac{5}{8}''$.

$d' = .28 \times 24 = 6.72''$, say $6\frac{3}{4}''$.

$e = \frac{1}{4}''.$

$f = .375 \times 17\frac{1}{4} = 6\frac{15}{32}''$, say $6\frac{1}{2}''$.

$g =$ (by construction) $17\frac{1}{4}''$.

$h = 1.35 \times 6\frac{3}{4} = 9.1125$, say $9\frac{1}{8}''$.

$i = 1.125 \times 12 = 13\frac{1}{2}''.$

$l = .26 \times 24 + .5 = 6.74''$, say $6\frac{3}{4}''$.

PRINCIPLES OF ELECTRICITY AND MAGNETISM.

(QUESTIONS 1079–1187.)

(1079) See Art. **2240.**

(1080) One joule = .7373 foot-pound (Art. **2333**); therefore, electrical energy =

$$\frac{\text{foot-pounds}}{.7373} = \frac{123{,}750}{.7373} = 167{,}842.1266 \text{ joules.}$$

Electrical power (Art. **2349**) =

$$\frac{\text{energy}}{\text{time in seconds}} = \frac{167{,}842.1266}{60 \times 43.5} = 64.3073 \text{ watts.} \quad \text{Ans.}$$

(1081) Let C_1 = current in A and C_2 = current in B. According to Art. **2322,** the conductivity of $A = \frac{1}{16.2}$ and of $B = \frac{1}{14.1}$. Therefore (Art. **2323**), $C_1 : C_2 :: \frac{1}{16.2} : \frac{1}{14.1}$, or $\frac{C_1}{14.1} = \frac{C_2}{16.2}$; hence, $C_1 = \frac{14.1\, C_2}{16.2}$.

Now, $C_1 + C_2 = 6.37$, or $C_1 = 6.37 - C_2$. Substituting, $6.37 - C_2 = \frac{14.1\, C_2}{16.2}$, or $103.194 - 16.2\, C_2 = 14.1\, C_2$; hence, $30.3\, C_2 = 103.194$, and $C_2 = 3.4057$ amperes.

$$C_1 = 6.37 - 3.4057 = 2.9643 \text{ amperes.}$$

Therefore, the current in branch $A = 2.9643$ amperes. Ans.
Therefore, the current in branch $B = 3.4057$ amperes. Ans.

(1082) (*a*) Power, or rate of expending energy = $\frac{\text{energy}}{\text{time}} = \frac{99{,}370{,}000}{2.5} = 39{,}748{,}000$ ergs per second. From

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Art. **2264,** 1 watt = 10,000,000 ergs per second; therefore,

$$\frac{\text{ergs per second}}{10{,}000{,}000} = \frac{39{,}748{,}000}{10{,}000{,}000} = 3.9748 \text{ watts.} \quad \text{Ans.}$$

(*b*) $\text{Horsepower} = \frac{\text{watts}}{746} = \frac{3.9748}{746} = .005328$ H. P. Ans.

(**1083**) Using formula **433,**

$$\text{Ampere-turns} = \frac{N}{3.192} \times \left(\frac{l_1}{A_1 \times \mu_1} + \frac{l_2}{A_2 \times \mu_2} + \frac{2\, l_3}{A_3 \times \mu_3} \right).$$

$l_1 = 2.5 \times 2 \times 3.1416 = 15.708''$. $A_1 = 2 \times 1 = 2$ sq. in.

$B_1 = \frac{N}{A_1} = \frac{160{,}000}{2} = 80{,}000$ lines of force per sq. in.; from Table 82, $\mu = 769.2$ in wrought iron at that density. *The reluctance of the magnetic circuit in the wrought iron* =

$$\frac{l_1}{A_1 \times \mu_1} = \frac{15.708}{2 \times 769.2} = .01021.$$

$l_2 = 4 \times 1 = 4''$. $A_2 = 2 \times 2 = 4$ sq. in.

$B_2 = \frac{160{,}000}{4} = 40{,}000$ lines of force per sq. in.; from Table 79, $\mu = 152.9$ in cast iron at that density. *The reluctance of the magnetic circuit in the cast iron* =

$$\frac{l_2}{A_2 \times \mu_2} = \frac{4}{4 \times 152.9} = \frac{1}{152.9} = .00654.$$

$l_3 = .125''$. $A_3 = 2 \times 2 = 4$ sq. in. $\mu_3 = 1$. *The reluctance of the magnetic circuit in two air-gaps* =

$$\frac{.2\, l_3}{A_3 \times \mu_3} = \frac{2 \times .125}{4 \times 1} = .0625.$$

The total reluctance of the magnetic circuit = .01021 + .00654 + .0625 = .07925.

$$\text{Ampere-turns} = \frac{160{,}000}{3.192} \times .07925 = \frac{12{,}680}{3.192} = 3{,}972.431.$$

Ans.

(**1084**) (*a*) Use formula **438,** and let P = the tractive force in pounds exerted by one contact surface; $A = 2 \times 2 = 4$ sq. in., the area of one contact surface; and $\mathbf{B} = \frac{N}{A} = \frac{240{,}000}{4} = 60{,}000$ lines of force per sq. in., the magnetic density of contact surface. Substituting,

$$P = \frac{60{,}000 \times 60{,}000 \times 4}{72{,}134{,}000} = 199.6284 \text{ lb.},$$

tractive force of one polar surface; total tractive force = $199.6284 \times 2 = 399.2568$ lb. Ans.

(*b*) Use formula **442** given in Art. **2429**, $p = \frac{\mathbf{B}^2}{72{,}134{,}000}$, and let p = pounds per sq. in. of contact surface and $\mathbf{B}$ = 60,000 lines of force per sq. in., the density at contact surface. Substituting, $p = \frac{60{,}000 \times 60{,}000}{72{,}134{,}000} = 49.9071$ lb. per sq. in. Ans.

(1085) From the Greek word *Elektron*, meaning *amber*.

(1086) (Formula **409.**)

$$C = \frac{E}{R} = \frac{20}{30 + 80} = .1818 \text{ ampere.} \quad \text{Ans.}$$

(1087) One joule = .7373 foot-pound; therefore, electrical energy = $\frac{\text{foot-pounds}}{.7373} = \frac{742{,}500}{.7373} = 1{,}007{,}052.76$ joules. Power is the *rate* of expending energy, and is found by dividing energy by the time. (Art. **2349.**)

$$\text{Electrical power} = \frac{\text{joules}}{\text{seconds}} = \frac{1{,}007{,}052.76}{(4 \times 3{,}600) + (21 \times 60)} =$$

$$\frac{1{,}007{,}052.76}{15{,}660} = 64.3073 \text{ watts.} \quad \text{Ans.}$$

(1088) The sectional area of A is $.02^2 \times .7854 = .00031416$ sq. in. The sectional area of B is $.02 \times .02 = .0004$ sq. in. The conductivity of a conductor is proportional to its sectional area. Therefore, the relative conductivities of the two branches A and B are .00031416 and .0004, respectively. The current divides among the branches of a divided circuit in proportion to their conductivities. Therefore, if C_1 represents the current in branch A, and C_2 the current in branch B, then $C_1 : C_2 :: .00031416 : .0004$; or, $C_1 : C_2 :: .7854 : 1$; hence, $C_2 = \frac{C_1}{.7854}$, or $C = .7854\, C_2$. Now, $C_1 +$

$C_2 = 2.6$, or $C_1 = 2.6 - C_2$. Equating the two values of C_1, $2.6 - C_2 = .7854\, C_2$, or $C_2 = \dfrac{2.6}{1.7854} = 1.4562$ amperes in B. Ans.

$C_1 = 2.6 - 1.4562 = 1.1438$ amperes in A. Ans.

(1089) (Art. **2272.**) 1 absolute unit of current = 10 amperes; consequently, 6.74 absolute units = $6.74 \times 10 = 67.4$ amperes. Ans.

(1090) See Art. **2201.**

(1091) (Formula **409.**)

$$C = \frac{E}{R} = \frac{112}{49.2} = 2.2764 \text{ amperes.} \quad \text{Ans.}$$

(1092) (Formula **419.**)

$W = E\,C = 112.5 \times 12.2 = 1{,}372.5$ watts. Ans.

(1093) (Art. **2323.**) Let C_1 = current in A and C_2 = current in B; the conductivity of $A = \dfrac{1}{\text{resistance}} = \dfrac{1}{49.2}$; the conductivity of $B = \dfrac{1}{\text{resistance}} = \dfrac{1}{67}$. The current divides among the branches of a derived circuit in proportion to the conductivities of the branches; therefore, $C_1 : C_2 :: \dfrac{1}{49.2} : \dfrac{1}{67}$, or $\dfrac{C_1}{C_2} = \dfrac{67}{49.2}$; hence, $C_1 = \dfrac{67\, C_2}{49.2}$. But $C_1 + C_2 = .76$, or $C_1 = .76 - C_2$. Substituting, $.76 - C_2 = \dfrac{67\, C_2}{49.2}$, or $116.2\, C_2 = 37.392$; therefore, $C_2 = .3218$ ampere, the current in branch B. Ans.

$C_1 = .76 - C_2 = .76 - .3218 = .4382$ ampere, the current in A. Ans.

(1094) See Art. **2303.** 1 volt = 100,000,000 absolute units of potential; therefore, 1,764,300,000 absolute units of potential $= \dfrac{1{,}764{,}300{,}000}{100{,}000{,}000} = 17.643$ volts. Ans.

(1095) From Table 83 (Art. **2414**) the loss is .01518 watt per cubic inch for 1 cycle, when the density is 90,300

lines of force per sq. in. 1 cycle = 2 reversals; therefore, 133 reversals = 66.5 cycles.

Watts expended = number of cycles × cubic inches of iron × .01518 = 66.5 × 65 × .01518 = 65.61555 watts. Ans.

(1096) See Arts. **2205, 2275,** and **2383.**

(1097) (Art. **2310,** formula **411.**) $E = C\,R = (36.2 + 1.7) \times .127 = 37.9 \times .127 = 7.3533$ volts. Ans.

(1098) By formula **420** the power in watts is $W = C^2 R = 19.8^2 \times 11.8 = 392.04 \times 11.8 = 4{,}626.072$ watts. Ans.

(1099) (Formula **412.**) Joint resistance $= R'' = \dfrac{r_1 r_2}{r_2 + r_1} = \dfrac{7.19 \times .92}{.92 + 7.19} = \dfrac{6.6148}{8.11} = .8156$ ohm. Ans.

(1100) By formula **413** the joint resistance in parallel is

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} =$$

$$\frac{37.3 \times 49.9 \times 57.1}{(49.9 \times 57.1) + (37.3 \times 57.1) + (37.3 \times 49.9)} =$$

$$\frac{106{,}278.517}{2{,}849.29 + 2{,}129.83 + 1{,}861.27} = \frac{106{,}278.517}{6{,}840.39} = 15.537 \text{ ohms.}$$

Ans.

(1101) (Art. **2282.**) One ohm = 1,000,000,000 absolute units of resistance; therefore, 875,000,000 absolute units $= \dfrac{875{,}000{,}000}{1{,}000{,}000{,}000} = .875$ ohm. Ans.

(1102) (Art. **2419.**) 165,750 − 115,300 = 50,450 lines of force which are not useful. According to formula **436,** the per cent. leakage $= p = \dfrac{50{,}450}{165{,}750} \times 100 = 30.437\%$. Ans.

(1103) *Indestructibility* and *elasticity.*

(1104) (Formula **411.**)

$E = C\,R = 2.73 \times 49.3 = 134.589$ volts. Ans.

(1105) (Formula **421.**)

$$W = \frac{E^2}{R} = \frac{78.8^2}{2.3} = \frac{6{,}209.44}{2.3} = 2{,}699.7565 \text{ watts.}$$ Ans.

(1106) (Formula **412.**) Joint resistance in parallel is

$$R'' = \frac{r_1 r_2}{r_2 + r_1} = \frac{42.1 \times 98.3}{98.3 + 42.1} = \frac{4{,}138.43}{140.4} = 29.476 \text{ ohms.} \quad \text{Ans.}$$

(1107) See Art. **2358.**

(1108) By transposition of terms in formula **437,** the number of useful lines of force is

$$l_u = \frac{l\,(100 - p)}{100} = \frac{376{,}420 \times (100 - 33)}{100} = 252{,}201.4$$

useful lines of force. Ans.

(1109) Static charges and currents.

(1110) (Formula **410.**) $R = \frac{E}{C} = \frac{22.4}{.43} = 52.093$ ohms, total resistance of the circuit. The external resistance is the difference between the total resistance and the internal, or $52.093 - 13.4 = 38.693$ ohms. Ans.

(1111) (Formula **422.**) 1 horsepower = 746 watts; therefore, $2.33 \times 746 = 1{,}738.18$ watts. Ans.

(1112) Let r_1 = resistance of branch A and r_2 = resistance of branch B. Their joint resistance in parallel, by formula **412,** is

$$R'' = \frac{r_1 r_2}{r_2 + r_1}. \quad \text{Then, } 19.9 = \frac{r_1 r_2}{r_2 + r_1}, \text{ or } r_1 = \frac{19.9\, r_2}{r_2 - 19.9}.$$

Let $C_1 = 2.2$ amperes, the current in branch A; and $C_2 = 5.9$ amperes, the current in B. By Art. **2323,**

$$C_1 : C_2 :: \frac{1}{r_1} : \frac{1}{r_2}.$$

Substituting,

$$2.2 : 5.9 :: \frac{1}{\frac{19.9\, r_2}{r_2 - 19.9}} : \frac{1}{r_2}, \text{ or } \frac{5.9\,(r_2 - 19.9)}{19.9\, r_2} = \frac{2.2}{r_2},$$

and, $5.9\, r_2 - 117.41 = 43.78$, or $5.9\, r_2 = 161.19$, whence $r_2 = 27.3203$ ohms, the resistance of branch B. Ans.

$C_1 : C_2 :: \frac{1}{r_1} : \frac{1}{r_2}$, or $2.2 : 5.9 :: \frac{1}{r_1} : \frac{1}{27.3203}$, whence $2.2\, r_1 = 5.9 \times 27.3203$, or $r_1 = \frac{161.18977}{2.2} = 73.2681$ ohms, the resistance of branch A. Ans.

(1113) See Art. **2363.**

(1114) (Formula **437.**) The total number of lines of force is

$$l = \frac{100\, l_m}{100 - p} = \frac{100 \times 1{,}764{,}300}{100 - 61} = \frac{176{,}430{,}000}{39} = 4{,}523{,}846$$

lines of force developed by magnetizing coils. Ans.

(1115) See Art. **2204.**

(1116) (*a*) (Formula **410.**) $R = \frac{E}{C} = \frac{28.2}{5.2} = 5.423$ ohms. Ans.

(*b*) Let r = internal resistance; then, $7\, r$ = the external resistance (the external being 7 times the internal). $r + 7\, r = 5.423$, or $8\, r = 5.423$, and $r = .677875$ ohm, internal resistance. Ans. $7\, r = 7 \times .677875 = 4.745125$ ohms, external resistance. Ans.

(1117) (Formula **422.**) 746 watts = 1 horsepower; therefore, 63,426 watts $= \frac{63{,}426}{746} = 85.0214$ horsepower. Ans.

(1118) Let C_1 = current in branch A; C_2 = current in branch B; and r = resistance of branch B. Then, $4.5\, r$ = resistance of branch A, since the resistance of branch A is 4.5 times the resistance of branch B. (Art. **2323.**) $C_1 : C_2 :: \frac{1}{4.5\, r} : \frac{1}{r}$, or $C_1 = \frac{C_2}{4.5}$. (The two r's cancel.) Also, $C_1 = 23.4 - C_2$, since the sum of the currents in the two branches equals 23.4 amperes. Equating, $23.4 - C_2 = \frac{C_2}{4.5}$, or $105.3 - 4.5\, C_2 = C_2$, whence $105.3 = C_2 + 4.5\, C_2$, or $5.5\, C_2 = 105.3$, and $C_2 = 19.1454$ amperes, current in B. Ans.

$C_1 = 23.4 - C_2 = 23.4 - 19.1454 = 4.2546$ amperes, current in A. Ans.

(1119) See Art. **2366.**

(1120) See Art. **2208.**

(1121) A strength of current of 1 ampere decomposes .0014388 grain of water in 1 second (Art. **2277**); to find the total amount of water, in grains, decomposed, multiply .0014388 by the time in seconds and then by the strength of current in amperes. $.0014388 \times 60 \times 60 = 5.17968$. $5.17968 \times 10 = 51.7968$ grains of water. Ans.

(1122) (Art. **2355.**) 1 horsepower = 746 watts; therefore, 83.42 horsepower = $83.42 \times 746 = 62{,}231.32$ watts. Ans.

(1123) Let r = the resistance of branch B; then, $2.7\,r$ = the resistance of branch A, since the resistance of branch A is 2.7 times the resistance of branch B. (Art. **2325.**) Joint resistance $= \dfrac{r \times 2.7\,r}{r + 2.7\,r}$; therefore, $47.9 = \dfrac{r \times 2.7\,r}{r + 2.7\,r}$. Hence, $2.7\,r = 3.7 \times 47.9$, or $2.7\,r = 177.23$ ohms, the resistance of branch A. Ans.

$r = \dfrac{177.23}{2.7} = 65.6407$ ohms, the resistance of branch B. Ans.

(1124) See Art. **2366.**

(1125) See Art. **2438.**

(1126) (*a*) Negative. (*b*) Negative. (*c*) Positive.

(1127) (Formula **402.**) The current $C = \dfrac{w}{.0014388 \times t}$; where w = weight of water in grains, and t = time of flow in seconds. Substituting, $C = \dfrac{25}{.0014388 \times 4 \times 60 \times 60} = 1.2066$ amperes. Ans.

(1128) (Formula **423.**)

$$\text{Horsepower} = \frac{W}{746} = \frac{3{,}362}{746} = 4.5067. \quad \text{Ans.}$$

(1129) (Formula **412.**) Joint resistance in parallel $= R' = \frac{r_1 r_2}{r_2 + r_1} = \frac{2.4 \times 987.3}{987.3 + 2.4} = \frac{2,369.52}{989.7} = 2.394$ ohms. Ans.

(1130) (Formula **427.**)

$\mathbf{B} = \frac{N}{A} = \frac{103,750}{2.25 \times 1.75} = \frac{103,750}{3.9375} = 26,349.2$ lines of force per square inch. Ans.

(1131) See Art. **2441.**

(1132) To detect static charges of electricity and determine their sign, whether *positive* or *negative*.

(1133) The total resistance of the circuit $= 17.2 + 8.2 + 11.3 = 36.7$ ohms. (Formula **411.**) $E = C R = .75 \times 36.7 = 27.525$ volts, total electromotive force generated in the battery. Ans.

(Formula **411.**) $E = C R = .75 \times 11.3 = 8.475$ volts, difference of potential between a and b. Ans.

$E = C R = .75 \times 8.2 = 6.15$ volts, difference of potential between b and c. Ans.

Art. **2318.** $E' = E - C r_1 = 27.525 - (.75 \times 17.2) = 14.625$ volts, difference of potential between c and a.

(1134) (Formula **424.**) Mechanical power, or H. P. $= \frac{EC}{746} = \frac{225 \times 86.3}{746} = 26.0288$ horsepower. Ans.

(1135) The resistances may be found by Ohm's law, as follows:

The difference of potential is 11.6 volts $= E$; the current in A is 6.7 amperes $= C$; therefore, $R = \frac{E}{C} = \frac{11.6}{6.7} = 1.7313$ ohms. Likewise for branch B, the resistance $R = \frac{11.6}{4.9} = 2.3673$ ohms. Ans.

This example may also be worked out in the following manner:

The sum of the currents in the two branches is $6.7 + 4.9 = 11.6$ amperes. By formula **410,** $R = \frac{E}{C} = \frac{11.6}{11.6} = 1$ ohm, the joint resistance of the two branches in parallel. Let r_1 = the separate resistance of branch A and r_2 = the separate resistance of branch B. Then, by formula **412,** their joint resistance $= R'' = \frac{r_1 r_2}{r_2 + r_1}$. Therefore, since 1 ohm is also their joint resistance, $1 = \frac{r_1 r_2}{r_2 + r_1}$, or $r_2 = \frac{r_1}{r_1 - 1}$.

Let C_1 = the current in branch A, and C_2 the current in branch B.

Then, $C_1 : C_2 :: \frac{1}{r_1} : \frac{1}{r_2}$.

Substituting, $6.7 : 4.9 :: \frac{1}{r_1} : \frac{1}{\frac{r_1}{r_1 - 1}}$, or $\frac{4.9}{r_1} = \frac{6.7\, r_1 - 6.7}{r_1}$.

Multiplying each side by r_1, $4.9 = 6.7\ r_1 - 6.7$, or $6.7\ r_1 = 4.9 + 6.7$; whence $r_1 = 1.7313$ ohms, the resistance of branch A. Ans.

$C_1 : C_2 :: \frac{1}{r_1} : \frac{1}{r_2}$. Substituting, $6.7 : 4.9 :: \frac{1}{1.7313} : \frac{1}{r_2}$, or $4.9\ r_2 = 11.59971$, and $r_2 = 2.3673$ ohms, resistance of branch B. Ans.

(1136) The sectional area of the magnet = diameter$^2 \times .7854 = .5^2 \times .7854 = .25 \times .7854 = .19635$ sq. in. The magnetic density is found by dividing the total number of lines of force passing through a magnetic circuit or a magnet by the sectional area; therefore, by formula **427,** $\mathbf{B} = \frac{3{,}927}{.19635} = 20{,}000$ lines of force per sq. in. Ans.

(1137) (Art. **2449.**) An electromotive force of 1 volt is generated by a conductor when it is cutting lines of force at the rate of 100,000,000 per second; therefore, by formula **447,** $E = \frac{N}{100{,}000{,}000 \times t} = \frac{3{,}660{,}000{,}000}{100{,}000{,}000 \times 60} = .61$ volt. Ans

(1138) See Art. **2211.**

(1139) (Art. **2292.**) The resistance of a portion of the conductor is, by formula **406,**

$$r_2 = \frac{r_1 \times l_2}{l_1} = \frac{12.6 \times 11}{16} = 8.6625 \text{ ohms.}$$

From formula **411,** $E = C\,R = 2.3 \times 8.6625 = 19.92375$ volts, difference of potential between the end and the point. Ans.

(1140) (Art. **2356.**) Mechanical power =

$$\frac{C^2\,R}{746} = \frac{11.4^2 \times 12.3}{746} = 2.1427 \text{ horsepower.} \quad \text{Ans.}$$

(1141) First find the joint resistance between a and b, or, in other words, the joint resistance of the two branches in parallel, by formula **412** (Art. **2324**). The joint resistance is $R'' = \frac{r_1\,r_2}{r_2 + r_1} = \frac{793 \times 979.3}{979.3 + 793} = \frac{776{,}584.9}{1{,}772.3} = 438.1791$ ohms.

From formula **411,** $E = C\,R = .23 \times 438.1791 = 100.781193$ or 100.7812 volts, difference of potential between a and b. Ans.

(1142) The sectional area of the bar $= .25 \times .375 =$ .09375 sq. in. The total number of lines of force is found by multiplying the sectional area by the magnetic density; therefore, by formula **428,** $N = .09375 \times 34{,}500 = 3{,}234.375$ lines of force. Ans.

(1143) By formula **447,**

$$E = \frac{N}{100{,}000{,}000\,t} = \frac{48{,}900{,}000{,}000}{100{,}000{,}000 \times 60} = 8.15 \text{ volts.} \quad \text{Ans.}$$

(1144) If 4 units of electricity repel the lever arm of a torsion balance through an angle of 20 degrees, then it is evident that the force that would be required to repel the lever arm of the torsion balance through an angle of 1 degree would be only $\frac{1}{20^2}$ as great, that is, $\frac{1}{20^2} \times 4 = \frac{4}{400}$ $= .01$ of a unit of electricity. The quantity of electricity

required to repel the lever arm through a distance of $45° = 45^2 \times .01 = 45 \times 45 \times .01 = 20.25$ units of electricity. Ans.

(1145) Reduce the time to seconds. $60 \times 60 \times 4.5 =$ 16,200 seconds.

By transposition of formula **405,**

$$C = \frac{Q}{t} = \frac{368,422}{16,200} = 22.7421 \text{ amperes.} \quad \text{Ans.}$$

(1146) (*a*) (Formula **409.**)

$$C = \frac{E}{R} = \frac{48.4}{.23} = 210.4348 \text{ amperes.} \quad \text{Ans.}$$

(*b*) (Formula **421.**)

Electrical power $W = \frac{E^2}{R} = \frac{48.4^2}{.23} = 10,185.04$ watts. Ans.

(*c*) (Formula **423.**)

Mechanical power $= \frac{W}{746} = \frac{10,185.04}{746} = 13.6528$ horsepower. Ans.

(1147) This problem can be solved by the simple application of *Ohm's law*, and by considering each branch separately. (Art. **2310.**)

In branch *A*, the difference of potential between the two ends is 125 volts, and the resistance of the branch is 47.41 ohms; consequently, by formula **409,** the current $= \frac{E}{R} = \frac{125}{47.41} = 2.6366$ amperes. Ans.

In branch *B*, the difference of potential between the two ends is also 125 volts, and the resistance is 69.8 ohms; consequently, $C = \frac{E}{R} = \frac{125}{69.8} = 1.7908$ amperes. Ans.

(1148) The sectional area of the bar $= 2^2 \times .7854 =$ 3.1416 sq. in. Applying formula **428,** $N = A\ \mathbf{B} = 3.1416 \times 56,000 = 175,929.6$ lines of force. Ans.

(1149) (*a*) Since the force exerted between two statically charged bodies is equal to the product of their two respective charges (Art. **2214**), and a unit quantity of electricity

attracts another unit quantity of opposite sign with a force of 1 dyne at a distance of 1 centimeter, then the force of attraction exerted between the two gilt balls at a distance of 1 centimeter is $20 \times 5 = 100$ dynes. Ans.

(*b*) According to the law of attraction and repulsion of static charges (Art. **2213**), the force exerted between them varies inversely as the square of the distance; consequently, if $f =$ the force at 5 centimeters, then $100 : f :: 5^2 : 1$, or $5^2 f = 100$, or $f = 4$ dynes. Ans.

(1150) First reduce the time to seconds. $60 \times 60 \times 2.25 = 8{,}100$ seconds. From Art. **2281,** formula **405,** $Q = C\,t = 8.32 \times 8{,}100 = 67{,}392$ coulombs. Ans.

(1151) (Art. **2356.**) By formula **424,** the mechanical power =

$$\frac{E\,C}{746} = \frac{525 \times 12.5}{746} = \frac{6{,}562.5}{746} = 8.7969 \text{ horsepower.} \quad \text{Ans.}$$

(1152) (Art. **2326.**) The joint resistance of three branches in parallel is

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} =$$

$$\frac{7.9 \times 13.3 \times 25.8}{(13.3 \times 25.8) + (7.9 \times 25.8) + (7.9 \times 13.3)} =$$

$$\frac{2{,}710.806}{343.14 + 203.82 + 105.07} = \frac{2{,}710.806}{652.03} = 4.1575 \text{ ohms.} \quad \text{Ans.}$$

(1153) The end *b*; because in looking at that end the current is circulating around the solenoid in the opposite direction to the motion of the hands of a watch. (Art. **2390.**)

(1154) As soon as the two gilt balls come in contact, the $+4$ charge on ball *A* neutralizes a -4 charge on ball *B*, leaving a negative charge of -20 units, which immediately divides equally between the two balls. A charge of -10 units on each ball produces a repulsion of $10 \times 10 = 100$ dynes between the two balls at a distance of 1 centimeter; and at a distance of 2 centimeters the force will be

$\frac{100}{2^2} = 25$ dynes, since the force exerted between them varies inversely as the square of the distance. (Art. **2213.**)

(1155) Transposing formula **405,** $t = \frac{Q}{C} = \frac{86{,}700}{10} =$ 8,670 seconds, or 2 hours 24 minutes and 30 seconds. Ans.

(1156) (Art. **2356.**) Mechanical power $= \frac{C^2 R}{746} = \frac{46.8^2 \times 4.4}{746} = \frac{9{,}637.056}{746} = 12.9183$ horsepower. Ans.

(1157) Towards the East.

(1158) See Art. **2224.**

(1159) From Art. **2333,** 1 joule = .7373 foot-pound; therefore, 1 foot-pound $= \frac{1}{.7373} = 1.3563$ joules. Ans.

(1160) (Formula **426.**) Mechanical power $= \frac{E^2}{746\,R} = \frac{225.6^2}{746 \times 15.7} = \frac{50{,}895.36}{11{,}712.2} = 4.3455$ horsepower. Ans.

(1161) From formula **413,** the joint resistance of the three branches in parallel $=$

$$\frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} =$$

$$\frac{63.2 \times 76.5 \times 88.9}{(76.5 \times 88.9) + (63.2 \times 88.9) + (63.2 \times 76.5)} =$$

$$\frac{429{,}813.72}{6{,}800.85 + 5{,}618.48 + 4{,}834.8} = \frac{429{,}813.72}{17{,}254.13} = 24.9108 \text{ ohms.}$$

The difference of potential between where the three branches divide and where they unite is $C \times R = 5.7 \times 24.9108 = 141.99156$ volts.

To find the current in each branch, consider each separately and apply *Ohm's law* (formula **409**):

The current in branch A is $\frac{E}{r_1} = \frac{141.99156}{63.2} = 2.2467$ amperes. Ans.

The current in branch B is $\frac{E}{r_2} = \frac{141.99156}{76.5} = 1.8561$ amperes. Ans.

The current in branch C is $\frac{E}{r_3} = \frac{141.99156}{88.9} = 1.5972$ amperes. Ans.

(1162) The current is flowing *from* the north *towards* the south.

(1163) Positive. Since a negative charge is developed on sulphur when rubbed with silk (Art. **2209**), this negative charge will induce a *bound* positive charge on the cover towards the sulphur and a free negative charge upon the opposite side of the cover, which will neutralize with the earth and leave the positive charge alone upon the cover.

(1164) (*a*) First reduce the time to seconds. $1.25 \times 60 \times 60 = 4{,}500$ seconds. By formula **414,** electrical energy $= J = C^2 R t = 14.2 \times 14.2 \times 8 \times 4{,}500 = 7{,}259{,}040$ joules. Ans.

(*b*) By formula **417,** mechanical energy in foot-pounds F. P. $= .7373 J = .7373 \times 7{,}259{,}040 = 5{,}352{,}090.192$ foot-pounds. Ans.

(1165) Let r_1 = resistance of 1 mile of copper wire, and r_2 = resistance of 1 mile of iron wire. Since the resistances of two metals are inversely proportional to their respective conductivities, $r_1 : r_2 :: 1 : 6.08$, or $r_2 = 6.08\ r_1$. Substituting 14 for r_1, $r_2 = 6.08 \times 14 = 85.12$ ohms per mile, the resistance of the iron wire. As the resistance of the conductor is proportional to its length (Art. **2292**), then, by formula **406,** the resistance of 1,000 feet of the iron wire is

$$r_3 = \frac{85.12 \times 1{,}000}{5{,}280} = 16.1212 \text{ ohms.} \quad \text{Ans.}$$

NOTE.—In the two cases, as the diameters of the wires are equal, their sectional areas are also equal; therefore, the sectional areas can be neglected in solving the problem. See Art. **2293.**

(1166) The sum of the currents in the three branches is $7.7 + 23.9 + 15 = 46.6$ amperes. The difference of potential between the point where the branches divide and the

point where they unite is by formula **411** $= C\,R = 46.6 \times 3.2 = 149.12$ volts. To find the resistance of each branch, consider each branch separately and apply *Ohm's law;* therefore (Art. **2310**),

The separate resistance of branch A is

$$\frac{E}{C_1} = \frac{149.12}{7.7} = 19.3662 \text{ ohms.} \quad \text{Ans.}$$

The separate resistance of branch B is

$$\frac{E}{C_2} = \frac{149.12}{23.9} = 6.2393 \text{ ohms.} \quad \text{Ans.}$$

The separate resistance of branch C is

$$\frac{E}{C_3} = \frac{149.12}{15} = 9.9413 \text{ ohms.} \quad \text{Ans.}$$

(1167) From formula **430,** the intensity of magnetomotive force is $\mathsf{H} = \frac{3.192 \times a\text{-}t}{l} = \frac{3.192 \times 4{,}620}{10} = 1{,}474.704.$ Ans.

A magnetic density of 1,474.704 lines of force is produced in a magnetic circuit when $\mathsf{H} = 1{,}474.704$; therefore, the total number of lines of force in the circuit is equal to the product of the magnetic density and the sectional area, or $1{,}474.704 \times 3.2 = 4{,}719.0528$ lines of force. Ans.

(1168) The area of the surface of the sphere = diameter$^2 \times 3.1416 = 4 \times 4 \times 3.1416 = 50.2656$ sq. in. From Art. **2225,** the electric density of a charged body is found by dividing the number of units of electricity by the area of the surface upon which it resides; therefore, the electric density of the sphere is $400 \div 50.2656 = 7.9577$ units of electricity per sq. in. Ans.

(1169) From Art. **2333,** 1 joule = .7373 foot-pound; therefore, 5,326,824 foot-pounds are equivalent to $\frac{5{,}326{,}824}{.7373} = 7{,}224{,}771.463$ joules. Electrical energy in joules $= E\,Q$; therefore, $Q = \frac{\text{joules}}{E} = \frac{7{,}224{,}771.463}{220} = 32{,}839.8703$

coulombs. Transposing formula **405,**

$$C = \frac{Q}{t} = \frac{32{,}839.8703}{60 \times 60 \times 2.5} = 3.6489 \text{ amperes.} \quad \text{Ans.}$$

(1170) From Art. **2217,** the conductivity is the reciprocal of the resistance; therefore, the conductivity of copper $= \frac{1}{1} = 1$, whence the conductivity of mercury $= \frac{1}{59}$ or .01695. Ans.

(1171) (Art. **2326.**) The joint resistance of the three branches in parallel, by formula **413** $= \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \frac{.2 \times 1.9 \times 2.1}{1.9 \times 2.1 + .2 \times 2.1 + .2 \times 1.9} = \frac{.798}{3.99 + .42 + .38} = \frac{.798}{4.79} =$ .1666 ohm. The difference of potential between a and b is, therefore (formula **411**), $E = CR = 67.4 \times .1666 = 11.22884$ volts. Ans.

(1172) From formula **430,** the intensity of magnetomotive force $= \mathsf{H} = \frac{3.192 \times a\text{-}t}{l}$; transposing,

$$\textit{ampere-turns} = \frac{\mathsf{H} \times l}{3.192}$$

Substituting the values of H and l,

$$\textit{ampere-turns} = \frac{360 \times 12}{3.192} = 1{,}353.383. \quad \text{Ans.}$$

(1173) *Negative;* because the negative charge on the inside coating induces a *bound* positive charge upon the inner face of the outside coating and leaves a free negative charge upon the outer face of the outside coating.

(1174) First reduce the time to seconds; $15 \times 60 =$ 900 seconds. Then, transposing formula **417,** $J = \frac{\text{F. P.}}{.7373} = \frac{984{,}650}{.7373} = 1{,}335{,}480.808$ joules. Electrical energy in joules $= EQ$; therefore, $Q = \frac{\text{joules}}{E} = \frac{1{,}335{,}480.808}{115} = 11{,}612.8766$

coulombs of electricity. By transposition of formula **405,** $C = \frac{Q}{t} = \frac{11{,}612.8766}{900} = 12.9032$ amperes; then, applying Ohm's law, $R = \frac{E}{C} = \frac{115}{12.9032} = 8.9125$ ohms. Ans.

(1175) From Art. **2286,** the resistance of a column of mercury 41.7323 in. high and .00155 sq. in. sectional area is 1 ohm. The resistance of any conductor is directly proportional to the length of the conductor, Art. **2293;** then, if the column were 1,000 feet high, its resistance r_1, is given by the proportion $41.7323 : 12{,}000 :: 1 : r_1$. Equating, $41.7323\ r_1 = 12{,}000$, or $r_1 = 287.54705$ ohms. If the sectional area of the column were $.02^2 \times .7854 = .00031416$ sq. in., instead of .00155 sq. in., its resistance r_2 would then be represented by the proportion $287.54705 : r_2 :: .00031416 : .00155$; since the resistance of any conductor is inversely proportional to its sectional area. (Art. **2295.**) Equating, $.00031416\ r_2 = .00155 \times 287.54705$, or $r_2 = 1{,}418.69724$ ohms. If the resistance of mercury is 62.73 times the resistance of silver, then the resistance of a silver wire 1,000 feet long and .02″ in diameter would be

$$\frac{1{,}418.69724}{62.73} = 22.6159 \text{ ohms.} \quad \text{Ans.}$$

(1176) This problem is a simple application of *Ohm's law* as given in Art. **2310;** each branch can be considered as a separate conductor having a difference of potential of 225 volts between its two ends.

Let C_1 = the current in branch A,
C_2 = the current in branch B;
C_3 = the current in branch C.

The separate resistance of branch A is $\frac{E}{C_1} = \frac{225}{17} = 13.2353$ ohms. Ans.

The separate resistance of branch B is $\frac{E}{C_2} = \frac{225}{39} = 5.7692$ ohms. Ans.

The separate resistance of branch C is $\frac{E}{C_3} = \frac{225}{23} = 9.7826$ ohms. Ans.

(1177) From Art **2400,** $\mu = \frac{\mathsf{B}}{\mathsf{H}}$; where μ = the permeability of the substance, B = the magnetic density, and H = the intensity of magnetomotive force. Substituting for B and H, $\mu = \frac{54{,}300}{600} = 90.5$. Ans.

(1178) See Art. **2237.**

(1179) See Art. **2349.**

(1180) From Art. **2286,** the resistance of a column of mercury 41.7323 in. high and .00155 sq. in. sectional area is 1 ohm; if the column were 72.3 in. high, the resistance (r_1) is given by the proportion $41.7323 : 72.3 :: 1 : r_1$, since the resistance of any conductor is directly proportional to its length. (Art. **2293.**) Equating, $41.7323\, r_1 = 72.3$, or $r_1 = 1.7324$ ohms. If the sectional area be decreased to $.04^2 \times .7854 = .00125664$ sq. in., the resistance (r_2) will be given by the proportion $1.7324 : r_2 :: .00125664 : .00155$; since the resistance of any conductor is inversely proportional to its sectional area. (Art. **2295.**) Equating, $.00125664\, r_2 = 1.7324 \times .00155$, or $r_2 = 2.1368$ ohms. Ans.

(1181) Each branch can be considered as a separate conductor, with a difference of potential of 108 volts between its two ends; the current in each branch is found by the application of *Ohm's law*, as in Art. **2310.**

Let r_1 = the resistance of branch A;
r_2 = the resistance of branch B;
r_3 = the resistance of branch C.

The current in branch A is $\frac{E}{r_1} = \frac{108}{52.2} = 2.069$ amperes. Ans.

The current in branch B is $\frac{E}{r_2} = \frac{108}{73} = 1.4795$ amperes. Ans.

The current in branch C is $\frac{E}{r_3} = \frac{108}{64} = 1.6875$ amperes. Ans.

(1182) From Art. **2400,** $\mathsf{H} = \frac{\mathsf{B}}{\mu}$; where μ = the permeability, H = the intensity of the magnetomotive force, and B = the magnetic density. Substituting 850 for μ and 59,500 for B, $\mathsf{H} = \frac{59,500}{850} = 70$. Ans.

(1183) (1) Contact of two dissimilar metals. (2) Chemical action. (3) Magnetic induction.

(1184) From Art. **2357,** 746 watts = 1 horsepower; therefore, 1 watt = $\frac{1}{746}$ horsepower = .00134 horsepower. Ans.

(1185) Since the resistance (Art. **2293**) of any conductor is directly proportional to its length, the resistance of 2,000 feet of the round wire would be $1 \times \frac{2,000}{1,000} = 2$ ohms. The sectional area of the round wire is $.1^2 \times .7854 = .007854$ sq. in., and the sectional area of the square wire is $0.1 \times 0.1 = .01$ sq. in. Since the resistance of any conductor is inversely proportional to its sectional area (Art. **2295**), then the resistance (r) of 2,000 feet of square wire is given by the proportion $2 : r :: .01 : .007854$; whence, equating, $.01\ r = 2 \times .007854$, or $r = 1.5708$ ohms. Ans.

(1186) Since 1 joule = 10,000,000 ergs (Art. **2330**), then 600,000,000 ergs are equivalent to $\frac{600,000,000}{10,000,000} = 60$ joules. From Art. **2333,** 1 joule = .7373 foot-pound; then, 60 joules are equivalent to $60 \times .7373 = 44.238$ foot-pounds. Ans.

(1187) From Art. **2400,** $\mathsf{B} = \mu\ \mathsf{H}$; where B = the magnetic density, μ = the permeability of the magnetic substance, and H = the intensity of magnetomotive force. Substituting 180 for H and 450 for μ, $\mathsf{B} = 450 \times 180 = 81,000$. Ans.

ELECTRICAL MEASUREMENTS.

(QUESTIONS 1188–1229.)

(1188) The multiplying power of a shunt of 500 ohms resistance used with a galvanometer of 690 ohms resistance is (Art. **2488**)

$$n+1=\frac{R_g}{R_s}+1.$$

Substituting the values of R_s and R_g given in this example,

$$n+1=\tfrac{6}{5}\tfrac{9}{0}\tfrac{0}{0}+1=2.38.$$

The current flowing through the galvanometer is, from formula **454**,

$$C_g=\frac{C}{n+1}.$$

Substituting the values of C and $n+1$ in the above formula,

$$C_g=\frac{.06}{2.38}=.02521 \text{ ampere},$$

the current in the galvanometer when shunted.

As a current of .06 ampere gives a deflection of 68° with this galvanometer, its constant may be calculated from formula **450**, $C=K\tan m°$.

Substituting the above values,

$$.06=K\times 2.4751,$$

or

$$K=\frac{.06}{2.4751}=.024242.$$

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A current of .02521 ampere will then give

$$.02521 = .024242 \times \tan m^\circ;$$

whence, $$\tan m^\circ = \frac{.02521}{.024242} = 1.0399.$$

From the table of tangents, $m^\circ = 46^\circ\ 7'+$. Ans.

(1189) (*a*) Use formula **448,** $f = \frac{2\pi A t}{r}$.

In this example, $A = .1$, $t = 1$, $r = 12$ cm. Substituting these values,

$$f = \frac{2\pi \times .1 \times 1}{12} = \frac{.62832}{12} = .05236 \text{ dyne.} \quad \text{Ans.}$$

(*b*) In Washington, the value of the horizontal component of the earth's magnetism H is .200. (Table 84.) Then, by formula **449,** $f = H \tan m^\circ$. Substituting the above values, $f = .05236$ and $H = .200$,

$$.05236 = .200 \times \tan m^\circ,$$

or $$\tan m^\circ = \frac{.05236}{.200} = .2618.$$

From the table of tangents, $m^\circ = 14^\circ\ 40'+$. Ans.

(1190) A controlling magnet serves to vary the action of the earth's field on the magnet of the galvanometer, by increasing or decreasing the strength of the earth's field at that point. (Art. **2466.**)

(1191) The specific resistance of mica is 84 tregohms (Table 85); that is, the resistance between two opposite faces of a cube of mica, each edge being 1 cm. long, is 84 tregohms. 1 cm. = .3937 inch; therefore, the resistance of a piece of mica $\frac{1}{8}$ inch long and 1 cm. square would be, calculated from formula **406,**

$$r_2 = \frac{r_1 \times l_2}{l_1} = \frac{84 \times .125}{.3937} = 26.67 \text{ tregohms.}$$

1 sq. cm. = .3937 × .3937 = .155 sq. in. 26.67 tregohms is, then, the resistance of a piece of mica .155 sq. in. in sectional area, .125 inch long. To find the resistance of a

piece $10 \times 10 = 100$ square inches area, formula **407**, $r_2 = \frac{r_1 a_1}{a_2}$, may be used,

$$r_2 = \frac{26.67 \times .155}{100} = .04134 \text{ tregohm.}$$

From Art. **2515** the prefix *trega* implies a unit 1,000,000,000,000 times as large as the ordinary unit, so in this case, to reduce .04134 tregohm to ohms, multiply .04134 by 1,000,000,000,000, which gives 41,340,000,000 ohms.

Ans.

(1192) The diagram should be about like Fig. 97. The principle of the bridge is described in Art. **2509.**

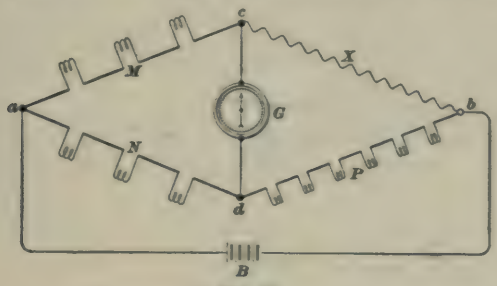

FIG. 97.

(1193) (*a*) In example 1189, a current of 1 ampere through 1 turn (section III) of the coil gave a value of $f = .05236$ dyne; a current of 4.2 amperes through 3 turns (section II) of the coil would then give a value of $f = 4.2 \times 3 \times .05236 = .65974$ dyne.

As this force gives a deflection of 40°, the value of H may be found from formula **449**, $f = H \tan m°$.

$$\text{Tan } m° = \tan 40° = .8391;$$

hence,
$$.65974 = H \times .8391,$$

or
$$H = \frac{.65974}{.8391} = .78625.$$

The value of H due to the earth's field at Philadelphia is .194. (Table 84.) Consequently, this field is increased .78625 $-.194 = .5923$ line of force per sq. cm. Ans.

(*b*) If the polarity of the controlling magnet be reversed, the earth's field will be lessened by .5923 line of force per sq. cm.; or $.194 - .5923 = -.3983$ line of force per sq. cm.; that is, the value of H acting on the galvanometer needle will be entirely due to the controlling magnet, and will be in a direction opposite to the lines of force of the earth's field. The deflection will therefore be in the opposite direction, and its amount may be found from formula **449**, $f = H \tan m°$, by substituting the values of f and H.

$f = .65974$, as before, the current being the same. $H = .3983$, as found above. (The minus sign merely indicates *direction* of the field and may be omitted.)

Hence, $.65974 = .3983 \times \tan m°$, or

$$\tan m° = \frac{.65974}{.3983} = 1.6564.$$

Then, from the table of tangents, $m° = 58° 53'$. Ans.

Being in the opposite direction, this means that the *difference* between the two deflections is $58° 53' + 40° = 98° 53'$.

(1194) (*a*) See Art. **2504.**
(*b*) See Art. **2504.**
(*c*) See Art. **2542.**

(1195) (*a*) See Art. **2475.**
(*b*) See Art. **2538.**

(1196) See Art. **2458.**

(1197) The principle of this method of measuring insulation resistance is given in Art. **2530.** The result may be found from formula **465**, $I = \frac{R\,x\,d°}{d_1°}$. As in this case $d°$ and $d_1°$ are identical, $I = R\,x$. The number of cells $x = 100$, and R is 11,800 ohms. Then $I = 11{,}800 \times 100 = 1{,}180{,}000$ ohms, the total insulation resistance of the wire. This sample of wire is 20 feet long, but 1 foot of each end projects from the liquid, so that the actual length of the piece tested is 18 feet, or 6 yards. The total

insulation resistance multiplied by the length in yards gives the insulation resistance per yard of length. (Art. **2523.**)

$$6 \times 1{,}180{,}000 = 7{,}080{,}000 \text{ ohms per yard.}$$

Dividing this result by 1,000,000 gives the megohms per yard, or

$$\frac{7{,}080{,}000}{1{,}000{,}000} = 7.08 \text{ megohms per yard.} \quad \text{Ans.}$$

(1198) See Art. **2476.**

(1199) (Art. **2513.**) Applying formula **459** for working up the readings of the slide-wire bridge,

$$X = \frac{M}{N} \times P =$$

$$\frac{n\,b}{a\,n} \times P = \frac{73.8}{26.2} \times 10 = 28.17 \text{ ohms.} \quad \text{Ans.}$$

(1200) This may be found from formula **448,**

$$f = \frac{2\,\pi\,A\,t}{r}.$$

In this example, $A = \frac{2.6}{10} = .26$; $t = 2$; $r = 11$; $2\,\pi = 6.2832$.

Substituting these values in the above formula,

$$f = \frac{6.2832 \times .26 \times 2}{11} = \frac{3.2673}{11} = .29703 \text{ dyne.} \quad \text{Ans.}$$

(1201) To find the galvanometer constant from the deflection, the strength of the current flowing must first be determined. This may be found from the increase in weight of the copper negative plate (Art. **2496**), which is 2.26 grams. As is there stated, a current of 1 ampere will deposit .0003286 gram of copper in 1 second. Then, if 2.26 grams of copper have been deposited in 1 hr. 40 min., or $(60 + 40) \times 60 = 6{,}000$ seconds, $\frac{2.26}{6{,}000} = .00037667$ gram was deposited each second. As 1 ampere would have deposited .0003286 gram in the same time, the actual current must have been $\frac{.00037667}{.0003286} = 1.1463$ amperes. Knowing the current

and the deflection it produces, the galvanometer constant may be calculated from formula **450,** $C = K \tan m°$.

Substituting the values of C and tan $m°$ in this equation,

$$1.1463 = K \times .9004, \text{ or } K = \frac{1.1463}{.9004} = 1.2731. \quad \text{Ans.}$$

(1202) By the electromagnetic effect. (Art. **2459.**)

(1203) (*a*) The specific resistance of a saturated solution of copper sulphate is 29.3 ohms. (Table 86.) The length and sectional area of the body of copper sulphate in this example being known, the actual resistance of the body may be calculated from formulas **406** and **407,** as follows:

The resistance of a body of the liquid 1 cm. long and 1 sq. cm. sectional area is 29.3 ohms, from the table.

The length of the liquid in the example is 4 inches, or $4 \times 2.54 = 10.16$ cm. Then, the resistance of a body of this liquid 1 cm. in sectional area, 10.16 cm. long, would be

$$r_2 = \frac{r_1 \times l_2}{l_1}.$$

Substituting the values of r_1, l_1, and l_2,

$$r_2 = \frac{29.3 \times 10.16}{1} = 297.7 \text{ ohms.}$$

The sectional area of the body of liquid in this example is, however, 6 in. $\times$ 8 in. $= 48$ sq. in. $= 48 \times 6.45 = 309.6$ sq. cm., and the resistance of this body, 10.16 cm. long and 309.6 sq. cm. area, may be found from the formula

$$r_2 = \frac{r_1 a_1}{a_2}.$$

Substituting the values of r_1, a_1, and a_2,

$$r_2 = \frac{297.7 \times 1}{309.6} = .9616 \text{ ohm.} \quad \text{Ans.}$$

(*b*) From the table, the specific resistance of a solution of sal ammoniac of maximum conductivity is 2.50 ohms. If the copper sulphate, specific resistance 29.3 ohms, be replaced with the sal ammoniac, specific resistance 2.50 ohms, the resistance of the body will be reduced in the

same proportion; that is, the resistance with sal ammoniac will be $\frac{2.50}{29.3} = .085324$ times that with copper sulphate.

Then, $.9616 \times .085324 = .082047$ ohm, resistance of sal ammoniac solution. Ans.

(1204) The multiplying power of a shunt is $n + 1$, where $n = \frac{R_g}{R_s}$, R_g being the resistance of the galvanometer and R_s the resistance of the shunt. (Art. **2488.**)

In this case,

(*a*) $R_g = 120$ ohms, $R_s = 20$;
then, $n = \frac{120}{20} = 6$, and $n + 1 = 6 + 1 = 7$. Ans.

(*b*) $R_g = 180$ ohms, $R_s = 20$;
then, $n = \frac{180}{20} = 9$, and $n + 1 = 9 + 1 = 10$. Ans.

(*c*) $R_g = 300$, $R_s = 20$;
then, $n = \frac{300}{20} = 15$, and $n + 1 = 15 + 1 = 16$. Ans.

(1205) See Art. **2509.**

(*a*) In this case, the various circuits of the bridge would be as represented in Fig. 98.

The balance arms are *a c* and *a d*; adjustable arm, *d b*. *G* is the galvanometer, connected to *c* and *d*, and *B* is the battery, connected to *a* and *b*. The unknown resistance is connected between *b* and *c*.

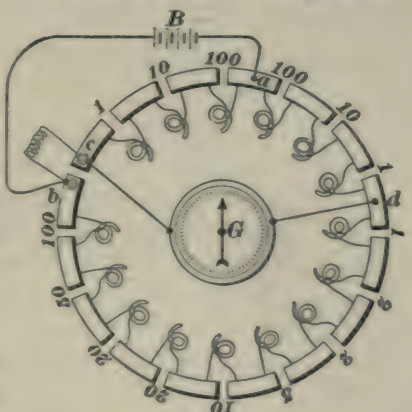

FIG. 98.

(*b*) The resistances in each balance arm are equal, being $100 + 10 + 1 = 111$ ohms each. The resistance in the adjustable arm is $1 + 2 + 2 + 5 + 10 + 20 + 20 + 50 + 100 = 210$ ohms. The unknown resistance is then found from formula **459,**

$$X = \frac{M}{N} \times P.$$

In this case, $M = 111$, $N = 111$, and $P = 210$.

Then, $X = \frac{111}{111} \times 210 = 210$ ohms. Ans.

(1206) The resistance coils of the bridge, having no temperature coefficient, would always be of the same resistance; but the resistance of the copper wire coil would vary with the temperature. (Art. **2516.**) In this example, the resistance of the copper wire coil, as measured by the bridge, is 210 ohms at 91° F. At 45° F., which is a temperature of 46° F. lower, its resistance would be found to be less. According to formula **463,**

$$r_2 = \frac{r_1}{1 + b F^\circ} = \frac{210}{1 + .802156 \times 46^\circ} = \frac{210}{1.099176} = 191.05.$$

The change in resistance is the difference between 210, its first value, and 191.05, its decreased value, or

$$210 - 191.05 = 18.95 \text{ ohms.} \quad \text{Ans.}$$

(1207) (*a*) See Art. **2520.**

(*b*) See Art. **2521.**

(1208) (*a*) A diagram similar to that shown in Fig. 991 is required.

(*b*) The calibrating coil is used to set up a field, the intensity of which can be calculated. Since this field is set up in air, wood, or other non-magnetic medium, the intensity of the field, **H**, in the coil may be calculated by formula **430,** because the number of turns on the solenoid and also its axial length are known. The kick of the galvanometer corresponding to various known values of **H** can thus be calculated, and by comparing these kicks with those obtained when the secondary coil S on the iron ring is in use, the value of **B** may be determined as explained in Art. **2486.**

(*c*) The objection to the step-by-step method is that if any mistake is made in one of the first readings, it is carried through the whole series of readings. If careful work is

done, however, this objection should not prove serious. See Art. **2486.**

(*d*) In the step-by-step method, the current in the primary is suddenly increased or decreased. In the reversal method, the current in the primary is reversed by means of the switch *H*, Fig. 991. See Arts. **2484** and **2485.**

(1209) (*a*) The heating effect of the current is made to cause the expansion of a wire. This expansion is then measured, giving a measure of the current.

(*b*) The Cardew voltmeter is one. (Art. **2541.**)

(1210) (Art. **2515.**) The specific resistance of a substance is the resistance of a cube of the substance 1 cm. on each side, measured between two parallel faces of the cube. The resistance of a piece of a substance of any length and sectional area being known, the specific resistance of that substance may be found from the formulas given, as follows:

The length of the piece in the example is 1 inch. Its resistance, if its length were 1 cm., its area being unchanged, may be found from formula **406,**

$$r_2 = \frac{r_1 l_2}{l_1}.$$

In this example $r_1 = .00000824$ ohm, $l_1 = 1$ in., $l_2 = .3937$ in. (1 cm.).

Then, $r_2 = \frac{.00000824 \times .3937}{1} = .0000032441$ ohm,

the resistance of a body of this substance 1 cm. long and 1 sq. in. sectional area. To find the resistance if the area be reduced to 1 sq. cm. (.155 sq. in.), formula **407** may be used:

$$r_2 = \frac{r_1 a_1}{a_2}.$$

In this example, as it now stands, $r_1 = .0000032441$, $a_1 =$ 1 sq. in., $a_2 = .155$ sq. in.

Then, $r_2 = \dfrac{.0000032441 \times 1}{.155} = .00002093$ ohm. Ans.

Or, multiplying by 1,000,000, gives 20.93 microhms. Ans.

(1211) Formula **451,** $f = H \sin m°$, may be used in solving this example.

The magnet being short in proportion to the diameter of the coil, the value of f may be found from formula **448,**

$$f = \frac{2\pi A t}{r}.$$

In this example, $A = \dfrac{.362}{10} = .0362$, $t = 8$, $r = 12$, $2\pi =$ 6.2832.

Then, $$f = \frac{6.2832 \times .0362 \times 8}{12} = .15163.$$

As the instrument is set up in Chicago, the value of H (Table 84) is .184;

then, $$.15163 = .184 \times \sin m°,$$

or $$\sin m° = \frac{.15163}{.184} = .82408,$$

and, from the table, $m° = 55°\ 30'$, nearly. Ans.

(1212) The fundamental equation of the Wheatstone bridge (formula **453**) is

$$X = \frac{M}{N} \times P.$$

It will be seen from the connections of the battery and galvanometer circuits in the diagram that the coils lying between c and a form the upper balance arm of the bridge, and hence, in this example, $M = 1$ ohm; the coils between a and d form the lower balance arm, and hence $N = 100$ ohms; and the coils between d and b form the adjustable arm, and hence $P = 500 + 200 + 20 + 2 + 1 = 723$ ohms. Substituting these values in the fundamental equation,

$$X = \frac{M}{N} \times P = \frac{1}{100} \times 723 = 7.23 \text{ ohms.} \quad \text{Ans}$$

(1213) (Art. **2546.**) Formula **466,** $R = r\left(\frac{d}{d_1} - 1\right)$, applies to this example.

The source of the E. M. F. in this case being a dynamo giving a constant potential of 115 volts, the deflection of the voltmeter when connected directly to the dynamo will be 115; i. e., $d = 115$.

From the example, $d_1 = 17.5$ and $R = 18{,}000$.

Then, $$R = 18{,}000 \times \left(\frac{115}{17.5} - 1\right) =$$

$$18{,}000 \times 5.5714 = 100{,}285 \text{ ohms},$$

or, practically, .1 megohm. Ans.

(1214) The ratio of the drops through two conductors in series is the same as the ratio of their resistances. If at 15° C. their resistances are equal, at any higher temperature their resistances would both be increased by an amount depending on their temperature coefficient.

The temperature coefficient of copper is .00388, while that of mercury is .00088, so that the resistance of the copper at a temperature of 62° C., which is a rise of 47° C., would be, from formula **460,**

$$r_2 = r_1 (1 + .00388\ C^\circ),$$

where r_1 = resistance at 15° C. and r_2 = resistance at 15 + C.°,

or $$r_2 = r_1 \times [1 + (.00388 \times 47)] = 1.1824\, r_1,$$

and the resistance of the mercury would be

$$r_2 = r_1 (1 + .00088\ C^\circ),$$

or $$r_2 = r_1 \times [1 + (.00088 \times 47)] = 1.0413\, r_1.$$

The value of r_1 being the same for both the mercury and the copper part of the circuit, at 15° C., since the drop is the same, at 62° C. the drops will be directly proportional to the resistances, or

drop through mercury : drop through copper :: $1.0413\, r_1$: $1.1824\, r_1$, and their ratio would be $\frac{1.0413}{1.1824}$. Ans.

(1215) The current in the coil of a tangent galvanometer being directly proportional to the tangent of the angle of deflection (formula **450**), the shunt in this example must reduce the current in the galvanometer from $K \times \tan 58°$ to $K \times \tan 31°$; from the table, $\tan 58° = 1.6003$ and $\tan 31° = .6009$. Then, the shunt reduces the current from $1.6003\ K$ to $.6009\ K$. The multiplying power of the shunt, $n + 1$, may be obtained from formula **454**,

$$C_g = \frac{C}{n+1}.$$

In this example $C_g = .6009\ K$. The total current being the same as when the whole current was passing through the galvanometer, $C = 1.6003\ K$.

Then, $$.6009\ K = \frac{1.6003\ K}{n+1},$$

or $$(n+1) \times .6009\ K = 1.6003\ K,$$

and $$n + 1 = \frac{1.6003\ K}{.6009\ K} = 2.6632.$$

Then, $$n = 1.6632.$$

Also, $$n = \frac{R_g}{R_s}, \text{ or } R_s = \frac{R_g}{n}.$$

From the example, $R_g = 400$ ohms.

Then, $$R_s = \frac{400}{1.6632} = 240.5 \text{ ohms.} \quad \text{Ans.}$$

(1216) (Art. **2507.**) The combined resistance of A and X in parallel is to the resistance of B as 4.1 is to 5.6. B being 77 ohms, then the combined resistance of A and X in parallel is

$$\frac{4.1 \times 77}{5.6} = \frac{315.7}{5.6} = 56.375 \text{ ohms.}$$

(b) The current flowing in the circuit is $\frac{5.6}{77} = .07273$ ampere. Ans.

(a) Since through the resistance A of 196 ohms, enough of this current is passing to cause a drop of 4.1 volts, or $\frac{4.1}{196} = .02092$ ampere, the balance, $.07273 - .02092 = .05181$

ampere, is flowing through the unknown resistance, causing a drop of 4.1 volts. (See Arts. **2323** and **2324.**) The unknown resistance is, then,

$$R = \frac{E}{C} = \frac{4.1}{.05181} = 79.135 \text{ ohms.} \quad \text{Ans.}$$

(1217) The problem may be solved by first finding the value of $A\,t$ in formula **448,** the other quantities being given. The diameter of the coil $= 19\frac{11}{16}$ in. $= 19.6875 \times 2.54 = 50.006$ cm.

Then, in the formula $f = \frac{2\,\pi\,A\,t}{r}$,

$$f = .7,\ r = \frac{50.006}{2} = 25.003,\ 2\,\pi = 6.2832;\ \text{hence,}$$

$$.7 = \frac{6.2832\,A\,t}{25.003},\ \text{or}\ .7 = .2513\,A\,t,\ \text{and}\ A\,t = \frac{.7}{.2513} = 2.7855.$$

In this expression, $A =$ C. G. S. amperes; so this value must be multiplied by 10 to give the correct result. $10 \times 2.7855 = 27.855 =$ product of amperes and turns. Ans.

(1218) See Art. **2507.**

(*a*) The two resistances being directly proportional to the drops through them,

$$X : 41 :: 7.54 : 10.66,$$

or
$$X = \frac{41 \times 7.54}{10.66} = 29 \text{ ohms.} \quad \text{Ans.}$$

(*b*) The current may be found from Ohm's law (Art. **2310**). The known resistance being 41 ohms, and the drop through it 10.66 volts, the current is

$$C = \frac{E}{R} = \frac{10.66}{41} = .26 \text{ ampere.} \quad \text{Ans.}$$

(1219) See Art. **2458.**

(1220) (Art. **2462.**) By bringing to bear upon the poles of the magnet a known constant force in a different direction from that exerted by the coil and measuring the resultant of the two forces.

(1221) (Art. **2546.**) The total resistance of the rod may be found from Ohm's law (Art. **2310**),

$$R = \frac{E}{C}.$$

In this case, $C = 35.4$, $E = .00875$. Then, $R = \frac{.00875}{35.4} = .0002472$ ohm, the resistance of 25 inches of the rod. The resistance of 1 foot of the rod would then be $\frac{.0002472}{25} \times 12 = .00011866$ ohm. Ans.

(1222) (*b*) (Art. **2488.**) In formula **454,** $C_g = \frac{C}{n+1}$; there is given in this example C_g and C. C_g must be the same with or without the shunt, as the deflection is the same in both cases.

Then, $C_g = .062$ ampere,
$C = .558$ ampere.

Hence, $.062 = \frac{.558}{n+1}$, or $\frac{.558}{.062} = n + 1 = 9$, the multiplying power of the shunt. Ans.

(*a*) $n = 9 - 1 = 8$; also, $n = \frac{R_g}{R_s}$, or $R_s = \frac{R_g}{n} = \frac{1,086}{8} = 135.75$ ohms, resistance of the shunt. Ans.

(*c*) The combined resistance of the galvanometer and shunt may be found from formula **412,** Art. **2325.** The joint resistance is

$$R'' = \frac{r_1 r_2}{r_2 + r_1}.$$

In this case, $r_1 = 1,086$ ohms and $r_2 = 135.75$ ohms.

Then, $R'' = \frac{1,086 \times 135.75}{135.75 + 1,086} = \frac{147,424}{1,221.75} = 120.67$ ohms.

As the office of the compensating resistance is to add to the joint resistance of galvanometer and its shunt sufficiently to make the total resistance equal the original resistance of the galvanometer alone, then, in this case, the compensating resistance $= 1,086 - 120.67 = 965.33$ ohms. Ans.

(1223) (Arts. **2493** to **2495.**) 2 hours $= 60 \times 2 \times 60 = 7{,}200$ seconds. We may here use formula **455,** where the given loss in weight $w_1 - w_2$ is 2.1 grams. Then the strength of current is

$$C = \frac{2.1}{.00009324 \times 7{,}200} = 3.128 \text{ amperes.} \quad \text{Ans.}$$

(1224) (Art. **2458.**) On the distance between the conductor and the magnet pole; on the length of the conductor; on the strength of the current; and on the strength of the magnet pole.

(1225) See Art. **2492** and Art. **2506.**

(*a*) The resistance may be found by Ohm's law

$$R = \frac{E}{C}.$$

The answer should be carried to four significant figures, the accuracy of the instruments not being within 1%.

Then, $R = \frac{4.9}{2.3} = 2.13$ ohms. Ans.

(*b*) In this case, the result should be carried out to six significant figures, the accuracy of the instruments being within between .1% and .01%.

Then, $R = \frac{4.9}{2.3} = 2.13043$ ohms. Ans.

(1226) The scale, the beam of light, and the line from the zero of the scale to the mirror form a right-angled triangle, and the angular deflection of the beam of light from its zero position can be calculated from the length of the part of the triangle formed by the scale, and the distance between the center of the scale and the mirror, by applying the rule given in the section on Geometry and Trigonometry:

$$\text{the tangent of the angle} = \frac{\text{side opposite}}{\text{side adjacent}}.$$

Then, when the spot of light is deflected to the end of the scale, the "side opposite" is equal to one-half the total length of the scale, or 24 inches. The side adjacent is the

distance from the mirror to the center of the scale, or 30 inches ($2\frac{1}{2}$ ft.).

Then, $\frac{24}{30} = .8$, the tangent of the angle made by the beam of light, when deflected, with its zero position.

From the table, $.8 = \tan 38° 40'$, nearly. Now, as the angle which the beam of light makes with its zero position is twice the angle of deflection of the needle (Art. **2474**), the deflection of the needle is $\frac{38° 40'}{2} = 19° 20'$.

Similarly, when the deflection of the ray of light extends over 12″ of the scale, the tangent of the angle is $\frac{12}{30} = .4$, the angle is 21° 48′, and the angle of deflection of the needle is $\frac{21° 48'}{2} = 10° 54'$; and when the deflection of the ray of light is 6 inches, the tangent of the angle is $\frac{6}{30} = .2$; the angle is 11° 18′, and the angle of deflection of the needle is $\frac{11° 18'}{2} = 5° 39'$.

To find the current required to cause these deflections, formulas **449** and **448** should be used. In formula **449**,

$$f = H \tan m°,$$

$$H = .184; \quad \tan m° =$$

$$(a) \quad \tan 19° 20' = .35085,$$

$$(b) \quad \tan 10° 54' = .19257,$$

$$(c) \quad \tan \ 5° 39' = .09893.$$

Then,

$$(a) \quad f = .184 \times .35085 = .064556,$$

$$(b) \quad f = .184 \times .19257 = .035433,$$

$$(c) \quad f = .184 \times .09893 = .018203.$$

In formula **448**,

$$f = \frac{2 \pi A t}{r},$$

f is as just given; $t = 6$; $r = 12$ cm.

Then (a), $.064556 = \frac{6.2832 \times A \times 6}{12} = 3.1416 A$,

or

$$A = \frac{.064556}{3.1416} = .02055.$$

Multiplying by 10, to get the amperes, gives .2055. Ans.

(*b*) $.035433 = \frac{6.2832 \times A \times 6}{12} = 3.1416\ A,$

or $$A = \frac{.035433}{3.1416} = .01128.$$

Multiplying by 10, to get the amperes, gives .1128. Ans

(*c*) $.018203 = \frac{6.2832 \times A \times 6}{12} = 3.1416\ A,$

or $$A = \frac{.018203}{3.1416} = .005794.$$

Multiplying by 10, to get the amperes, gives .05794. Ans.

(1227) See Art. **2522.**

(*a*) Formula **464,** for finding the insulation resistance of a line by this method, is

$$I = \frac{R \tan d}{\tan d_1},$$

where R = value of the known resistance, d = angle of deflection when R is in circuit, and d_1 = angle of deflection when the battery is connected to the earth.

In this case, $R = 15{,}000$ ohms, $d = 69°$, $d_1 = 30°\ 30'$.

Then, tan $d = 2.605$ and tan $d_1 = .5891$, and

$$I = \frac{15{,}000 \times 2.605}{.5891} = \frac{39{,}075}{.5891} = 66{,}330 \text{ ohms} =$$

$$\frac{66{,}330}{1{,}000{,}000} = .06633 \text{ megohm.} \quad \text{Ans.}$$

(*b*) (Art. **2522.**) In this case the length of the line is given as 10 miles. The insulation resistance per mile is, then, $10 \times .06633 = .6633$ megohm. Ans.

(*c*) There being 440 poles, the insulation resistance per pole is $440 \times .06633 = 29.2$ megohms. Ans.

NOTE.—This method not being accurate within 1%, four significant figures are sufficient in any calculation or in the result.

(1228) See Art. **2477.**

(1229) (*a*) The diagram should be about like Fig. 100. (See Arts. **2524** and **2526.**)

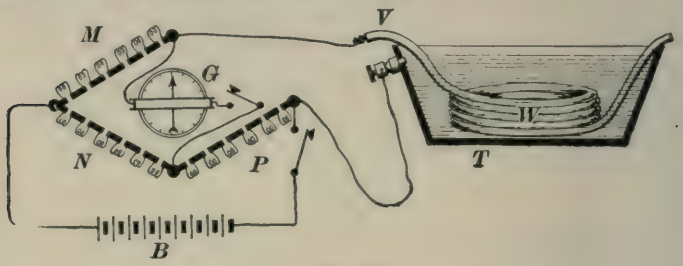

FIG. 100.

(*b*) The diagram should be about like Fig. 101. (See Art. **2546.**)

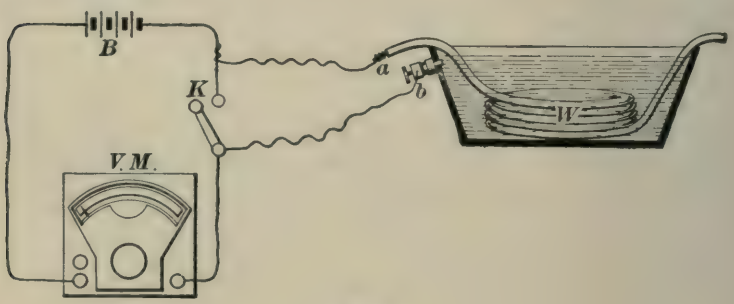

FIG. 101.

BATTERIES.

(QUESTIONS 1230–1292.)

(1230) See Art. **2550.**

(1231) (*a*) See Art. **2570.**
(*b*) See Art. **2568.**
(*c*) See Art. **2569.**

(1232) (*a*) The cause of the E. M. F. of a cell is the affinity which exists between elements of the various substances which make up the cell.

(*b*) The current from a cell is maintained by the combination of various elements of the substances constituting the cell keeping up the E. M. F., which maintains the current. (See Art. **2576.**)

(1233) See Arts. **2599** and **2654.**

(1234) The gravity Daniell cell employs as an anode, zinc; as an electrolyte, a solution of zinc sulphate; as a depolarizer, a solution of copper sulphate; as a cathode, copper. The liquids have different specific gravities, so that they may be kept one over the other in the cell without mixing. The cell gives a constant E. M. F. of about 1.07 volts. The zinc sulphate solution is continually formed while the cell is in action, and must be occasionally removed; the copper sulphate solution is continually reduced, and must be occasionally replaced. The cells are used largely in telegraph and fire-alarm work. (See Arts. **2655** to **2671** and **2706** to **2709,** inclusive.)

(1235) This may be calculated from the electrochemical equivalent as follows: 1 ampere-hour = 1 ampere for

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1 hour, or $60 \times 60 \times 1 = 3{,}600$ coulombs. As the consumption of zinc is .0003382 gram per coulomb (Table 89, column 6), the consumption per ampere-hour is $3{,}600 \times .0003382 = 1.218$ grams per ampere-hour. (See Art. **2718.**)

(1236) The maximum current will flow when the internal resistance equals the external. (Art. **2722.**) In this case, the external resistance is 3 ohms, and the combination of cells which will give this resistance will be found by calculation to be 2 sets in parallel, with 6 cells in series in each set. The internal resistance of a set of 6 cells in series, of 1 ohm resistance each, would be 6 ohms, and the resistance of two such sets would be $\frac{6}{2}$ or 3 ohms. (Art. **2721.**) The E. M. F. of a battery thus arranged would be $6 \times 2 = 12$ volts (Art. **2721**), and the resistance of the circuit is 3 ohms internal + 3 ohms external, or 6 ohms; the current would then be $\frac{12}{6} = 2$ amperes. Ans.

(1237) Hg_2O would be called mercur*ous* oxide, and $Hg\ O$ would be called mercur*ic* oxide. (See Art. **2571.**)

(1238) See Art. **2583.**

(1239) (*a*) The formation of a layer of hydrogen on the cathode of a cell is called polarization.

(*b*) The removal of such a layer by mechanical or chemical means is called depolarization. (Art. **2594.**)

(1240) (*a*) The most important applications of primary batteries are to the telegraph, telephone, signal service, cautery purposes, electroplating, and for running small motors. (See Arts. **2705** to **2716,** inclusive.)

(*b*) See Arts. **2807** to **2815.**

(1241) The positive plates. (Art. **2740.**)

(1242) The battery output is 6,000 ampere-hours, during a day of 24 hours. The total output of the plant is 2,000 amperes multiplied by 24, or 48,000 ampere-hours. The dynamo plant furnishes direct to the line $48{,}000 - 6{,}000 = 42{,}000$ ampere-hours. The dynamo plant furnishes to the line, *through the battery*, $6{,}000 \div .80 = 7{,}500$ ampere-hours.

(*a*) The total daily output of the generating plant is the sum of the direct output to the line and the battery output to the line, or $42{,}000 + 7{,}500 = 49{,}500$ ampere-hours. Ans.

(*b*) The dynamo plant running twenty-four hours at full capacity must furnish $49{,}500 \div 24 = 2{,}062.5$ amperes. Ans.

(1243) See Art. **2554.**

(1244) See Art. **2590.**

(1245) Any cell which answers to the description given in Arts. **2607** to **2621** will do. For example, the zinc, ammonium chloride, and carbon cells, of which the Little Giant, Law cell, etc., are examples.

(1246) (*a*) Because the two solutions will mix and cause local action if the cell is "open-circuited." (Art. **2661.**)

(*b*) Because the cell will polarize if called upon to deliver a current for any considerable time. If used for intermittent currents, the hydrogen layer on the cathode will pass off in the periods of rest of the cell. (Arts. **2594** and **2602.**)

(1247) The formation of a layer of the insoluble white sulphate of lead on the positive plates. (Art. **2736.**)

(1248) The rapid formation of insoluble sulphate in the plugs of active material causes them to expand and thus distort the inelastic lead grid. (See Art. **2741.**)

(1249) The great weight of accumulators necessary; the rapid disintegration of the plates; and the expense of handling the batteries. (See Arts. **2802, 2803,** and **2804.**)

(1250) The E. M. F. of the cell is reduced. (See Art. **2590.**)

(1251) (*a*) Because the cost of the materials consumed makes the cost of the energy many times greater than that obtained from dynamo machinery. (See Art. **2601.**)

(*b*) Because in many instances the cost of the energy is unimportant compared with their constancy and the little attention required. (See Art. **2705.**)

(1252) (*a*) See Art. **2734.**

(*b*) It indicates that the cell is completely charged. (See Art. **2734.**)

(1253) (*a*) The number of cells required is found by dividing the required E. M. F. by the average E. M. F. of each cell (Art. **2822**), or $\frac{52}{1.9} = 28$ cells. The size of each cell is found from the number of ampere-hours required for the devices which consume the current during one charge. (Art. **2825.**) In this case the cells are to be charged once every 7 days, and of the 32 lamps installed, 2 burn for 10 hours per day, and will then use $2 \times 10 \times 7 = 140$ ampere-hours in the week. Of the balance, 6 burn for 5 hours per day, and will then use $6 \times 5 \times 7 = 210$ ampere-hours; the balance, $32 - 8 = 24$, burn for 1 hour per day, and will use $24 \times 1 \times 7 = 168$ ampere-hours. The capacity of the cells must then be $168 + 210 + 140 = 518$ ampere-hours. The plant would then consist of 28 cells, each of a capacity of 518 ampere-hours.

(*b*) Placed in two tiers, 14 in each tier, the floor space occupied would be that of 14 cells, each cell occupying $5\frac{1}{3}^8 = 172\frac{2}{3}$ sq. in. (Art. **2820.**) Then, $14 \times 172\frac{2}{3} = 2{,}417\frac{1}{3}$ sq. in., or 17 sq. ft., practically.

(1254) See Art. **2563.**

(1255) (*a*) Mechanical and chemical depolarization. (See Art. **2595.**)

(*b*) The chemical method. (See Arts. **2596** and **2597.**)

(*c*) Because mechanical depolarizers require some sort of mechanism to perform the required service, which therefore needs attention and is expensive. (See Art. **2596.**)

(1256) Art. **2633.** (*a*) Cells with a liquid depolarizer.

(*b*) See Art. **2604.**

(1257) See Arts. **2740, 2743,** and **2746.**

(1258) See Art. **2742.**

(1259) (*a*) A chemical action is the combination of two or more substances forming another substance or substances of different physical properties, or is the decomposition of some substance with a similar result. Such action is a manifestation of one form of energy.

(*b*) The energy of such actions usually appears in the form of heat.

(*c*) The amount of heat liberated by the formation (or absorbed by the decomposition) of unit weight of a substance is called the heat of formation of that substance. (See Art. **2559.**)

(1260) (*a*) and (*b*) See Arts. **2602** to **2606.**

(1261) See Art. **2663.**

(1262) (*a*) The active material on the negative plate is reduced to spongy lead, sulphuric acid being formed; the positive plate is oxidized, forming lead peroxide. (See Art. **2733.**)

(*b*) The formation of lead oxide and then sulphate by the action of the acid on the negative plate, and the reduction of the peroxide on the positive plate to lead oxide, or to spongy lead. The percentage of acid in the electrolyte is reduced. (See Art. **2735.**)

(1263) See Art. **2752.**

(1264) (*a*) The symbol of an element is the first letter, or the first letter and one other, of its English or Latin name. (Art. **2557.**)

(*b*) Each element has assigned to it a symbol in order that the element may be referred to without constant repetition of its entire name. (Arts. **2557** and **2562.**)

(*c*) The chemical formula of a substance is an expression of the various elements which enter into a substance and the relative amounts of each element in that substance. The presence of the elements is indicated by writing the symbol of each element, and the proportion of each element in the substance is shown by a small number suffixed to the symbol of the element. (Art. **2562.**)

(**1265**) Bichromate cells of the second class are liable to local action, by which the zinc is consumed, if the cells are left with the external circuit open. (Art. **2623.**)

(**1266**) The Faure process consists of coating the surface of a lead plate with a layer of oxide or sulphate of lead, such that on the passage of a charging current this coating is converted into the proper active material for use in accumulators. (Art. **2730.**)

(**1267**) Because at a high rate of discharge all the active material is quickly converted on its surface, and as the acid can only penetrate slowly into the body of the active material, the E. M. F. falls to its limiting value before all the active material is consumed. (Art. **2745.**)

(**1268**) Long-continued overcharging will remove deposits of the white insoluble sulphate from the positive plate. (See Art. **2754.**)

(**1269**) (*a*) The atomic weight of an element is the relative weight of an atom of that substance as compared with an atom of the lightest substance (hydrogen). (See Art. **2561,** and column 3, Table 89.)

(*b*) The chemical equivalent of an element is the weight of that element which will replace unit weight of hydrogen in any substance, or will combine with unit weight of hydrogen. (See Art. **2564,** and column 5, Table 89.)

(*c*) The valency of an element is the ratio of its atomic weight to its chemical equivalent. (See Art. **2565,** and column 4, Table 89.)

(*d*) The atomicity of an element is the same as its valency. (See answer to (*c*).)

(1270) (*a*) See Art. **2621** and Table 92.
(*b*) See Art. **2609** and Table 92.
(*c*) See Art. **2617.**

(1271) Potassium zincate is a compound oxide of potassium and zinc. (See Art. **2683.**)

(1272) See Art. **2699.**

(1273) See Art. **2728.**

(1274) Because the acid will slowly penetrate into the internal portions of the active material, and the original E. M. F. will be restored. (See Art. **2744.**)

(1275) (*a*) They serve to equalize the load on the generators and engines, thus allowing the machinery to be operated at its maximum efficiency. (See Art. **2792.**)

(*b*) See Art. **2793.**

(1276) (*a*) and (*b*) The valency of oxygen is II, while that of iodine is I. Consequently, 2 atoms of iodine will replace 1 atom of oxygen (Art. **2565**), and 1 molecule of H_2O will become 2 molecules of HI.

(1277) Because the free elements all through the liquid recombine as fast as liberated, except at the electrodes, where they either combine with the substances there or appear in the free state. (See Art. **2592.**)

(1278) A standard cell should keep its E. M. F. constant at a certain value throughout the life of the materials of the cell, and should be affected as little as possible by heat, etc. (See Arts. **2693** to **2696,** inclusive.)

(1279) See Art. **2727.**

(1280) The output in ampere-hours on discharge divided by the input in ampere-hours required to charge the cell is its ampere-hour efficiency. (See Art. **2749.**)

(1281) (*a*) See Art. **2778.**
(*b*) See Arts. **2779** and **2782.**

(1282) (*a*) Light weight for a given output, and freedom from buckling or sulphating. (See Arts. **2786** and **2788.**)

(*b*) Its low E. M. F., care and special operation required in charging and discharging, and liability to local action. (See Arts. **2788** and **2789.**)

(1283) (*a*) See Art. **2555.**
(*b*) See Art. **2556.**
(*c*) and (*d*) See Art. **2567.**

(1284) (*a*) As the number of atoms of any two combining elements which make up a molecule are inversely proportional to the valencies of those elements (Art. **2565**), in this case the valency of the copper in the $Cu\ Cl$ solution must be I, and in the $Cu\ Cl_2$ solution must be II. The amount of copper which will be deposited per coulomb being its electrochemical equivalent, then from the $Cu\ Cl$ solution will be deposited .0006568 gram per coulomb, while from the $Cu\ Cl_2$ solution will be deposited .0003284 gram per coulomb. (Column 6, Table 89.) In this case the current is 4 amperes, maintained for $10 \times 60 = 600$ seconds, and is thus equal to $4 \times 600 = 2{,}400$ coulombs. Then, from the first bath ($Cu\ Cl$) will be deposited $2{,}400 \times .0006568 = 1.576$ grams of copper.

(*b*) From the second bath ($Cu\ Cl_2$) will be deposited $2{,}400 \times .0003284 = .7882$ gram of copper.

(*c*) This is answered above. (See Art. **2571.**)

(1285) The bichromate cell is the best example. (See Arts. **2622** to **2631,** inclusive.)

(1286) See Arts. **2689** and **2725.**

(1287) The plate at which the current *enters* on charging and leaves on discharging. (Art. **2726.**)

(1288) The Lalande-Chaperon and Edison-Lalande. (Art. **2783.**)

(1289) The materials used are as follows: Anode, zinc; electrolyte, solution of caustic potash (potassium hydrate;)

depolarizer and cathode, oxide of copper. The action is the oxidation of the zinc and its formation with the potassium hydrate of potassium zincate; the depolarization is effected by the combination of the evolved hydrogen with the oxygen of the copper oxide. (See Art. **2684.**)

(1290) Oxygen. (See Art. **2597.**)

(1291) From the rule given in Art. **2724,** the current in the external circuit $= \frac{E}{R} = \frac{1.6}{4.5} = .3555$ ampere. The drop in the cell is $1.9 - 1.6 = .3$ volt, and the resistance of the cell $= \frac{E}{C} = \frac{.3}{.3555} = .844$ ohm. Ans.

(1292) An accumulator stores energy in the form of potential chemical energy. (See Art. **2748.**)

APPLIED ELECTRICITY.

(QUESTIONS 1293–1359.)

(1293) (*a*) The purpose of the register is to make a record of the signals received.

(*b*) A steel point records the message by alternately pressing and releasing a grooved roller over which a strip of paper is passing. This point is carried by a lever which revolves upon an arbor. The projection of this lever beyond the arbor carries an armature which is attracted by two electromagnets, and its motion is limited by two screws. The main current, passing through these magnets, attracts the armature, so that the steel point is pressed against the paper. When the current is broken, the armature is pushed away by a spring, and the steel point is removed from the paper.

(*c*) Dots, dashes, and spaces.

(*d*) By different combinations of these dots, dashes, and spaces.

(1294) See Art. **2863,** Fig. 1075.

(1295) See Art. **2872,** Fig. 1079.

(1296) (*a*) Two U-shaped springs.

(*b*) Closed.

(*c*) When the key of the western station is opened, the western relay of the repeater opens the local circuit through its sounder. This sounder breaks two contacts, one slightly before the other. The contact which is thus broken first opens a local circuit, which releases the armature of an extra

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local magnet. This armature when released holds the eastern line relay closed. The second contact broken by the repeater sounder opens the eastern main line, but the eastern relay does not open its contact, because its armature is held back by the armature of the extra magnet, as just described. The relay of the eastern line is therefore always held closed while the western key is being operated.

(1297) (*a*) See Art. **2881.**

(*b*) See Arts. **2884, 2885,** and **2886;** also Fig. 1082.

(*c*) The resistance through the artificial line must be equal to the line resistance.

(*d*) See Art. **2886.**

(1298) (*a*) By dynamos.

(*b*) See Arts. **2902** to **2908,** inclusive.

(1299) (*a*) See Arts. **2954** and **2955.**

(*b*) See Art. **2957.**

(*c*) and (*a*) See Art. **2956.**

(1300) See Art. **2980,** Fig. 1111.

(1301) Suppose in starting that the keys at the terminal stations are closed. When the eastern key is opened, the eastern relay R_1 of the repeater releases its armature, thus opening its local circuit through the sounder S_1. This sounder releases its armature, which first causes the contact x_1 to open and immediately afterwards the contact f_1. When contact x_1 opens, the armature of the extra magnet M is released and is pulled by the spring s into contact with the armature g of the eastern line relay R. This holds the local circuit of relay R closed, when the eastern line circuit is broken immediately afterwards by the opening of the second contact of the sounder S_1, as just described. Thus the opening of the eastern key has no effect on the western repeater relay R, as the contacts of this relay are either held closed by the line current when the line circuit is closed, or

by the armature of the extra magnet M when the line circuit is open. The opening of the contact j_1 of the sounder S_1 releases the armature of the relay at the western terminal station and causes its sounder to operate. The closure of the key at the western terminal station will cause sounder S_1 to close contact j_1 in the western line, thus operating the relay and sounder at the western station.

(1302) (*a*) and (*b*) See Art. **2886.**

(1303) (*a*) See Art. **2917.**

(*b*) Length and diameter at the top. Usually 7-inch tops are specified.

(1304) (*a*) Galvanized iron and copper.

(*b*) See Art. **2912.**

(1305) (*a*) See Art. **2837.**

(*b*) See Fig. 1067 and description.

(*c*) Because it saves running another wire for the return. By the use of an earth return, just one-half of the amount of wire is employed that would be necessary were a metallic return used.

(1306) (*a*) Because it is found that operators can receive with greater facility and reliability by sound than by the visible dots and dashes on the tape of a register. The simplicity of the sounder is also in its favor.

(*b*) By the sound of the armature lever striking against its stops.

(*c*) See Art. **2840.**

(*d*) See Art. **2847.**

(*e*) By mounting the instrument upon a sounding-board and by giving the armature more play.

(1307) In false cables the capacity is distributed, while in the artificial line it is localized in one adjustable condenser. The method of obtaining distributed capacity is shown in Fig. 1083 and described in Art. **2887.**

(1308) The coating of an article with a layer of varying thickness, so that the heaviest deposit shall be on the part exposed to most wear. (See latter part of Art. **3000.**)

(1309) (*a*) See Arts. **2841** and **2987.**

(*b*) See Art. **2987.**

(*c*) See Art. **2990.**

(1310) (*a*) See Art. **2989.**

(*b*) See Art. **2989.**

(*c*) By the use of properly transposed metallic circuits.

(1311) (*a*) See Art. **2878.**

(*b*) To adjust the instruments and care for the batteries.

(1312) (*a*) Condensers.

(*b*) A condenser.

(*c*) The key is so arranged that, by pressing one lever, the plus pole of the battery is put to line, while the minus pole is to earth; and by pressing the other, the minus pole is put to line, while the plus pole is to earth.

(*d*) The receiving instrument is a simple galvanometer or a siphon register. The latter instrument consists of a coil of wire swinging upon a pivoted axis between the poles of a magnet. The current passes through this coil and swings it according to the direction of flow. A glass siphon is attached to the coil, one end of which dips into a vessel of ink and the other spurts ink upon a strip of paper which is drawn across. The ink is charged positively, while the plate over which the paper passes is given a negative charge. Therefore the ink is splashed upon the paper, and a record is obtained of the movements of the coil.

(*e*) A movement of the siphon in one direction is taken as a dot, and one in the other direction as a dash.

(1313) (*a*) See Arts. **2915** and **2916.**

(*b*) The McIntire sleeve joint.

(*c*) They do not require the use of solder to maintain good contact and the joint is made more easily.

(1314) A cold bath will give a lighter color than a hot bath. If too much free cyanide is used the deposit will be brown, while if too little is used the deposit will be pale. A strong current will produce a darker color than a weak current.

(1315) See Art. **2926.**

(1316) See Art. **2856.**

(1317) (*a*) See Art. **3011.**
(*b*) See Art. **3011.**
(*c*) See Art. **3012.**

(1318) (*a*) See Art. **2975.**
(*b*) Series and bridged.
(*c*) See Arts. **2976** and **2978.**

(1319) (*a*) See Art. **2979.**
(*b*) See Arts. **2982** and **2983.**
(*c*) See Art. **2984.**

(1320) (*a*) A device for normally maintaining a short circuit around the winding of a generator armature, but which opens this short circuit automatically when the generator is put into operation.

(*b*) See Art. **2961.**

(*c*) It removes the resistance of the generator armature from the line, so that it is not in the path of incoming currents from other stations. It also reduces the liability to armature burn-outs from lightning.

(1321) (*a*) See Art. **2959.**
(*b*) See Art. **2963.**

(1322) See Arts. **2966** and **2967.**

(1323)

	Keys.				Pressure at Point *e*.		Current in			Effective Current.		Relays Operated.			
	West.		East.		West.	East.	West.	Line.	East.	West.	East.	West.		East.	
	P.	*P'.*	*P.*	*P'.*			*R G'.*	*R R.*	*R G'.*	Relays *W* and *W'*.	Relays *W* and *W'*.	*W.*	*W'.*	*W.*	*W'.*
9	Up	Up	Up	Down	− 100	− 300	$\frac{100}{R}(a)$	$\frac{200}{R}(a)$	$\frac{300}{R}(b)$	$\frac{300}{R}(a\ L)$	$\frac{100}{R}(b\ G')$		×		
10	Down	Up	Up	Down	+ 100	− 300	$\frac{100}{R}(b)$	$\frac{400}{R}(a)$	$\frac{300}{R}(b)$	$\frac{300}{R}(a\ L)$	$\frac{100}{R}(a\ L)$		×	×	
11	Up	Down	Up	Down	− 300	− 300	$\frac{300}{R}(a)$	0	$\frac{300}{R}(b)$	$\frac{300}{R}(a\ G')$	$\frac{300}{R}(b\ G')$		×		×
12	Down	Down	Up	Down	+ 300	− 300	$\frac{300}{R}(b)$	$\frac{600}{R}(a)$	$\frac{300}{R}(b)$	$\frac{300}{R}(a\ L)$	$\frac{300}{R}(a\ L)$		×	×	×
13	Up	Up	Down	Down	− 100	+ 300	$\frac{100}{R}(a)$	$\frac{400}{R}(b)$	$\frac{300}{R}(a)$	$\frac{300}{R}(b\ L)$	$\frac{100}{R}(b\ L)$	×	×		
14	Down	Up	Down	Down	+ 100	+ 300	$\frac{100}{R}(b)$	$\frac{200}{R}(b)$	$\frac{300}{R}(a)$	$\frac{300}{R}(b\ L)$	$\frac{100}{R}(a\ G')$	×	×	×	
15	Up	Down	Down	Down	− 300	+ 300	$\frac{300}{R}(a)$	$\frac{600}{R}(b)$	$\frac{300}{R}(a)$	$\frac{300}{R}(b\ L)$	$\frac{300}{R}(b\ L)$	×	×		×
16	Down	Down	Down	Down	+ 300	+ 300	$\frac{300}{R}(b)$	0	$\frac{300}{R}(a)$	$\frac{300}{R}(b\ G')$	$\frac{300}{R}(a\ G')$	×	×	×	×

(1324) The magneto-transmitter serves as a generator of electricity, the movements of the iron diaphragm in front of the pole-pieces inducing currents in the coils. The battery transmitter serves simply as a valve for controlling the flow of current from a battery. This it does by varying the resistance of the circuit, in accordance with the sound-waves received by the diaphragm.

(1325) See Art. **2946.**

(1326) (*a*) The magneto-transmitter needs no battery, because it generates its own current, as explained in Art. **2939.**

(*b*) The transmitter itself.

(1327) (*a*) See Art. **2927.**

(*b*) See Art. **2928.**

(1328) Any good form of sal ammoniac battery, the Fuller battery, and also storage-batteries.

(1329) Insulators of glass are mounted on pins which are carried on cross arms or brackets firmly secured to the poles. The wire is laid in a groove in the insulator and firmly tied in place by the method shown and described in Art. **2925.** All joints in the wire are made according to one of the methods given in Arts. **2915** and **2916.**

(1330) (*a*) 150 ohms.

(*b*) See Rules **I** and **II,** Art. **2850.**

(*c*) One thirty-second of an inch.

(*d*) See Rule **IV,** Art. **2850.**

(1331) (*a*) See Art. **2892.**

(*b*) Upon changes in the direction of current, and also in its strength.

(*c*) One key at each end of the line governs the direction of the current, and the other its strength. At each end are two relays, one of which is operated only by the changes in the strength of the current, while the other is operated only by changes in the direction of the current. The key

at one end governing the strength of current therefore produces no effect on the relay at the other end which operates on changes in direction, while the other relay responds to all its movements. Similarly, the key governing the direction of current operates only the relay at the other end which is affected by changes in direction.

(1332) (*a*) See Art. **2938.**
(*b*) See Art **2939.**

(1333) (*a*) See Art. **2887.**
(*b*) In duplex submarine telegraphy.

(1334) See Art. **2862,** Fig. 1074.

(1335) (*a*) and (*b*) See Art. **2847.**

(1336) (*a*) See Art. **2839.**
(*b*) See Art. **2845.**

(1337) A local circuit is a circuit containing a local battery and receiving instrument, and controlled by contacts actuated by an electromagnet in the line-wire. (See Fig. 1068.)

(1338) (*a*) Upon the principle of the Wheatstone bridge.
(*b*) The battery, key, and artificial line.
(*c*) The galvanometer.

(1339) See Art. **3008.**

(1340) See Art. **3014.**

(1341) See Arts. **2911** and **2913.**

(1342) (*a*) See Art. **2924** and Fig. 1093.
(*b*) See Art. **2924.**

(1343) See Art. **2947.**

(1344) See Art. **2907.**

(1345) When the eastern circuit is repeating into the western, the lever *k* (see Fig. 1079) connects *c* and *b*. When the eastern operator closes his key, a current passes along the eastern line, through relay R_1, battery B_1, points 5 and

o_3, contact c, lever k, ground switch l, and back by ground return to the eastern station. This current operates relay R_1, which closes its local circuit, containing battery B_1 and sounder S_1. This sounder closes the western line circuit, which may be traced as follows: From ground G, switch l, lever k, contact b, point o_1, through contacts of sounder S_1, point *2*, battery B, relay R, line W, and to ground at the western station.

(1346) (*a*) See Art. **2875.**

(*b*) Because the sounder is of too low a resistance to be affected by the currents passing through the high-resistance relay magnets.

(*c*) The lever l (see Fig. 1080) is moved to contact a. Currents from line W then pass through relay R, lever l, sounder S, and battery B to ground. These currents operate relay R, but not sounder S. When relay R closes its contacts c, d, it closes the eastern line E, the circuit of which is from ground G, points *7* and *10*, contacts c, d, points b, and *2*, relay R_1 and line E to ground through the instruments at the eastern station. The relay R_1 closes the local circuit traced from the sounder S, point *3*, lever l, contact a, contacts d_1, c_1, points *6*, *7*, and *5*, battery B, points *12* and *4* back to sounder S.

(1347) A system invented by Morse, using a code composed of combinations of dots and dashes.

(1348) To prevent corrosion.

(1349) (*a*) See Art. **2871.**

(*b*) Wood's, Edison's, and Milliken's.

(1350) See Fig. 1079 and answer to Question 1345.

(1351) (*a*) See Art. **2877.**

(*b*) By sending "heavy."

(1352) (*a*) See first part of Art. **2868.**

(*b*) See latter part of Art. **2868.**

(*c*) See Art. **2869.**

(1353) The problems of neutralizing cross-talk and other forms of induction.

(1354) This condition exists for four different combinations of the keys, which are as follows: All four keys open; all four keys closed; key P down and key P' up, at each station; and key P' down and key P up, at each station. (See Table 95 and also answer to Question 1323.)

(1355) (*a*) and (*b*) See Art. **2861.**

(1356) (*a*) A relay serves the purpose of receiving feeble currents from a line and causing them, by the movement of a light armature, to open and close a local circuit containing a battery and local receiving-instrument.

(*b*) An electromagnet with a very light armature, suitable contact points, and a retractile spring.

(*c*) See Art. **2844.**

(1357) (*a*) See Art. **2981.**

(*b*) To facilitate making connections between lines.

(*c*) See Art. **2867.**

(*d*) See Fig. 1113 and description.

(1358) (*a*) Loudness, pitch, and timbre.

(*b*) and (*c*) See Arts. **2930, 2931,** and **2932.**

(1359) See Art. **2997.**

APPLIED ELECTRICITY.

(QUESTIONS 1360–1429.)

(1360) (*a*) See Art. **3041.**

(*b*) Unipolar induction. (See Art. **3059.**)

(1361) (*a*) The E. M. F. generated in the conductor may be found by applying formula **477.** Since all the angles in this example are 90°, so that their sines are each equal to 1, this may be written

$$E = \frac{\mathbf{B}\, L\, M}{10^8}.$$

In this example, $\mathbf{B} = 8{,}000$, $L = 22$, and $M = \frac{90{,}000}{60} = 1{,}500$. Substituting these values in the formula,

$$E = \frac{8{,}000 \times 22 \times 1{,}500}{10^8} = \frac{264{,}000{,}000}{10^8} = 2.64 \text{ volts.}$$

Since the resistance of the circuit is 0.2 ohm, the current flowing is $\frac{2.64}{0.2} = 13.2$ amperes, and the rate at which the electrical energy is expended is $13.2 \times 2.64 = 34.848$ watts. Ans.

(*b*) (See Art. **3028.**) Using formula **478,** $f = A\, \mathbf{B}\, L$, in this example, $A = \frac{13.2}{10} = 1.32$ absolute units, $\mathbf{B} = 8{,}000$ lines of force per square centimeter, and $L = 22$ centimeters. Substituting these values,

$$f = 1.32 \times 8{,}000 \times 22 = 232{,}320 \text{ dynes.} \quad \text{Ans.}$$

(*c*) Since the force necessary to apply to the conductor is 232,320 dynes, and it is moved with a velocity of 1,500 cm per second, the rate of doing work is equal to 232,320 ×

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$1,500 = 348,480,000$ ergs per second. (See Art. **3030.**) Since 10^7 ergs per second equal 1 watt, the rate at which energy is expended in moving the conductor is $\frac{348,480,000}{10^7} =$ 34.848 watts. Ans.

(1362) (*a*) See Art. **3054.**

(*b*) See Arts. **3050** to **3053.**

(1363) (*a*) Electromagnets.

(*b*) See Art. **3017.**

(1364) A part ($C^2 R_i$) is converted into heat, and the remainder ($e C$) into mechanical work done. (See Arts. **3033** and **3034.**)

(1365) (See Art. **3046.**) For this case formula **479,** $E_m = \frac{N \pi S}{10^8 \times 60}$, would be used. In this example, $N = \mathbf{B} L 2r$; as $\mathbf{B} = 51,000$, $L = 13$, and $r = 8$, $N = 51,000 \times 13 \times 16 =$ 10,608,000 lines of force; $S = 900$. Then,

$$E_m = \frac{10,608,000 \times 3.1416 \times 900}{10^8 \times 60} = 4.99891392 \text{ volts, or 5 volts,}$$

practically. Ans.

(1366) (*a*) and (*b*) See Art. **3068.**

(1367) See Arts. **3063** and **3064.**

(1368) (*a*) The E. M. F. would be reduced in the ratio $\frac{\sin 62^\circ}{\sin 90^\circ}$. (See Arts. **3020** and **3021.**) Sin $90^\circ = 1$, and from the table of Natural Sines and Tangents, sin $62^\circ =$ .88295. Hence, the E. M. F. under these circumstances would be reduced to $2.64 \times .88295 = 2.33$ volts, practically. The E. M. F. would therefore be reduced $2.64 - 2.33 = .31$ volt. Ans.

(*b*) See Arts. **3020** and **3021.**

(1369) See Arts. **3044** and **3046.**

(1370) See Art. **3036.**

(1371) (*a*) Direct currents and alternating currents.

(*b*) See Arts. **3056** to **3058.**

(1372) (*a*) In a direction at right angles to the lines of force and to the length of the conductor.

(*b*) Yes; provided this direction is any other than parallel to the length of the conductor or to the lines of force. (See Art. **3032.**)

(1373) See Art. **3015.**

(1374) One in which the divisions of the ordinates represent the sines of the angles which are represented by the divisions of the abscissas. (See Art. **3045.**)

(1375) (*a*) See Art. **3053.**

(*b*) See Art. **3055.**

(1376) (*a*) See Arts. **3063** to **3066.**

(*b*) See Art. **3068.**

(1377) See Art. **3031.**

(1378) By reason of the drop in the armature conductors, due to the current flowing. (See Art. **3027.**)

(1379) (*a*) and (*b*) See Art. **3061.**

(1380) (*a*) Because the connecting wires would themselves cut the lines of force in such a way as to set up an opposing E. M. F. in them equal to the E. M. F. set up in the same number of armature conductors. (See Art. **3070.**)

(*b*) In parallel. (See Art. **3071.**)

(1381) See Art. **3080.**

(1382) See Art. **3095.**

(1383) (*a*) See Arts. **3105** and **3106.**

(*b*) See Art. **3107.**

(1384) (*a*) See Art. **3118.**

(*b*) See Arts. **3116, 3117,** and **3118.**

(1385) (See Arts. **3129** and **3130.**) Formula **481** would be used in this case. In this example the pitch y equals 41, and the number of pairs of poles p equals 3. Substituting these values in the formula to find the number of coils,

$$s = (3 \times 41) \pm 1 = 122 \text{ or } 124. \quad \text{Ans.}$$

(1386) In this example, $\frac{w}{2} = 52$; the back pitch being but 49, it is less than $\frac{w}{2} - 1$; therefore, the winding would be called a chord winding. (See Arts. **3146** and **3147.**)

(1387) See Art. **3157.**

(1388) (*a*) The first, with 111 coils, will have 3 reentrant systems, and the other will have 1.

(*b*) See Arts. **3171, 3172,** and **3173.**

(1389) (See Arts. **3181** and **3183.**) (*a*) Since there are two conductors per slot and per coil, the number of winding spaces w equals $2 \times 182 = 364$; the number of pairs of poles p equals 3. Substituting these values in formula **488,** $364 = 6y \pm 2m$. If $m = 1$, as would be the case in a single-wound armature, then $y = \frac{362}{6} = 60+$, or $\frac{366}{6} = 61$, which latter value would give a single-wound two-circuit drum winding. If $m = 2$, giving a double-wound armature, then $y = \frac{360}{6} = 60$, or $\frac{368}{6} = 61+$. The former value would necessarily be used. If $m = 3$, giving a triple-wound armature, then $y = \frac{358}{6} = 59+$, or $\frac{370}{6} = 61+$. Neither of these could be used. If $m = 4$, giving a quadruple winding, then $y = \frac{356}{6} = 59+$, or $\frac{372}{6} = 62$, which latter value would be used. With this particular core, then, a single, double, or quadruple two-circuit winding could be used. Ans.

(*b*) The above pitches are, of course, in each case the average pitch. Since each pitch must be odd (Art. **3181**), for the single winding, both front and back pitch would be 61; for the double winding, where the average pitch is even (60), the back pitch might be 61 and the front pitch 59, or *vice versa*. In the quadruple winding, the average pitch

being again even (62), the back pitch might be 63 and the front pitch 61, or *vice versa*. Ans.

(*c*) The relative E. M. F.'s of the several windings would be inversely as the number of separate windings (Art. **3168**). Therefore if the E. M. F. of the single winding be taken as 1, the E. M. F. of the double winding would be $\frac{1}{2}$, and of the quadruple winding, $\frac{1}{4}$. Ans.

(1390) See Art. **3197.**

(1391) (*a*) See Art. **3210.**

(*b*) See Arts. **3208** and **3210.**

(1392) (*a*) and (*b*) See Art. **3215.**

(1393) (*a*) This is a combination magnet, like that described in Art. **3200.** The N poles are salient and the S poles consequent.

(*b*) See Art. **3192.**

(*c*) The magnet would become a salient-pole magnet of the form described in Art. **3201.**

(1394) Each coil would be connected to two segments separated by $m - 1 = 3 - 1 = 2$ others. (See Art. **3177.**)

(1395) See Art. **3167.**

(1396) See Arts. **3150, 3151,** and **3152.**

(1397) (*a*) Formula **481,** $s = p\,y \pm 1$, would be used for this winding, using only the $-$ sign. (Art. **3133.**)

Thus, since $s = 109$ and $p = 2$,

$$109 = 2\,y - 1, \text{ or } y = 54\tfrac{1}{2} = 55. \quad \text{Ans.}$$

(*b*) Formula **482,** $E = \dfrac{c\,p\,N\,S}{60 \times 10^8}$, would be used in this case. (See Art. **3136.**)

In this example, $c = 109 \times 4 = 436$, $p = 2$, $N = 3{,}000{,}000$, and $S = 600$.

Then, $E = \dfrac{436 \times 2 \times 3{,}000{,}000 \times 600}{60 \times 10^8}$; or, canceling and multiplying, $E = \dfrac{872 \times 3}{10} = 261.6$ volts. Ans.

(1398) See Art. **3120.**

(1399) See Arts. **3109** and **3110.**

(1400) See Art. **3102.**

(1401) See Art. **3089.**

(1402) See Arts. **3082** and **3102.**

(1403) In this example the values of E, c, and S in formula **480** are given, and the problem is to find the fourth quantity N. (See Art. **3104.**)

In this case, $E = 118$, $c = 208$, and $S = 1{,}500$.

Then (formula **480**), $118 = \dfrac{208 \times N \times 1{,}500}{10^8 \times 60}$,

or $118 = .000052\ N$,

and $N = \dfrac{118}{.000052} = 2{,}270{,}000$ lines of force, practically.

Ans.

(1404) See Arts. **3113** and **3114.**

(1405) Six sets of brushes and six paths for the current. (See Arts. **3123** and **3124.**)

(1406) (*a*) See Art. **3139.**

(*b*) Loop winding. (See Art. **3157.**)

(*c*) Wave winding. (See Art. **3159.**)

(1407) Nearly or quite the full difference of potential that exists between brushes. (See Art. **3154.**)

(1408) (*a*) No; the average difference of potential should not be more than about 15 volts. (See Art. **3114.**)

(*b*) By using an "interpolated segment" commutator with as many segments as there are winding spaces. (See Art. **3162.**)

(1409) (*a*) (See Art. **3168.**) This being a two-circuit multiple-wound armature, formula **486,**

$$E = \frac{c\ p\ N\ S}{60 \times 10^8\ m},$$

would be used.

In this example, $c = 554$, $p = 3$, $N = 5{,}400{,}000$, $S = 300$, and $m = 2$.

Then, $E = \frac{554 \times 3 \times 5{,}400{,}000 \times 300}{60 \times 10^8 \times 2}$; or, canceling and dividing, $E = 224.4$ volts. Ans.

(*b*) (See Art. **3181.**) For this example, formula **488**, $w' = 2\,p\,y \pm 2\,m$, would be used. In this case, $w = 554$, $p = 3$, and $m = 2$. Then, $554 = 2 \times 3 \times y \pm 4$, or $y = 5\frac{5}{6}^0 = 91+$, or $5\frac{5}{6}^8 = 93$, which latter value would necessarily be used. This being an odd number, both front and back pitches would be 93. Ans.

(*c*) Formula **485**, $E = \frac{c\,N\,S}{60 \times 10^8\,m}$, would be used in this case, the winding being a multiple circuit, multiple winding. (See Art. **3168.**)

In this example, $c = 554$, $N = 5{,}400{,}000$, $S = 300$, and $m = 2$.

Then, $E = \frac{554 \times 5{,}400{,}000 \times 300}{60 \times 10^8 \times 2}$. Canceling and dividing, $E = 74.8$ volts, practically. Ans.

(*d*) In the two-circuit winding, the number of reentrant systems found would be equal to the greatest common factor of y and m (see Art. **3181**), which in this case, y being odd and m even, is 1. In the multiple-circuit winding, the number of reentrant systems is equal to the greatest common factor of $\frac{w'}{2}$ and m (see Art. **3179**), which in this case, $\frac{w'}{2}$ being odd (277) and m even, is also 1.

(*e*) (See Art. **3183.**) Two E. M. F.'s (224.4 and 74.8 volts) have been given. By changing the two-circuit winding from a double to a single winding, the E. M. F. would be $m \times 224.4 = 448.8$ volts. (Art. **3168.**) This could be done, the pitch being (formula **484**) $554 = 2 \times 3 \times y \pm 2$, or $y = 5\frac{5}{6}^6 = 92+$, or $5\frac{5}{6}^2 = 92$, which latter value would be used, making the front pitch 91 and the back 93, or *vice versa*. (Art. **3181.**) Changing the multiple-circuit winding from a double to a single winding would also double its E. M. F., making it $2 \times 74.8 = 149.6$ volts.

Making the two-circuit winding a triple winding would make its E. M. F. one-third of that developed in the single winding, or $\frac{448.8}{3} = 149.6$ volts, the same as the multiple-circuit, single winding. This could not be done, however, since from formula **488,** $w = 2\,p\,y \pm 2\,m$, y would equal either $\frac{548}{6} = 91+$, or $\frac{560}{6} = 93+$, neither of which values could be used. Making the multiple-circuit winding a triple winding would make its E. M. F. one-third of that developed in the single winding, or $\frac{149.6}{3} = 49.9$ volts, practically. This winding could of course be used. (See Art. **3179.**) The answer to this question would then be 5 E. M. F.'s; namely, 49.9, 74.8, 149.6, 224.4, and 448.8 volts.

(1410) (*a*) To deliver the lines of force that are generated in the magnetizing coil to the armature at the proper point. (See Art. **3186.**)

(*b*) It need be of no particular form to accomplish this purpose. (See Art. **3186.**)

(*c*) See Art. **3185.**

(1411) See Art. **3192.**

(1412) (*a*) and (*b*) See Art. **3203.**

(1413) (*a*) and (*b*) See Art. **3214.**

(1414) See Arts. **3205** and **3208.**

(1415) There is of course no specific answer to this question, but a comparison of the answer given with the matter given in Art. **3192** and those following will show whether it is correct or not.

(1416) Because each of the two circuits of the consequent-pole magnet supplies only half the total number of lines of force, and can therefore be of half the area. (See Art. **3194.**)

(1417) (*a*) Unstable.

(*b*) Because accidental changes in the magnetizing force, even if small, would produce great changes in the number of lines of force in the circuit. (See Art. **3188.**)

(1418) (*a*) For this example, formula **487,** $s = p\,y \pm m$, would be used, this being a multiple-wound two-circuit ring armature. (See Art. **3174.**) We have $s = 105$, $p = 2$, and $m = 3$. Then, $105 = 2 \times y \pm 3$, or $y = \frac{1}{2}^{0}{}^{8} = 54$, or $\frac{1}{2}^{0}{}^{2} = 51$. Either pitch, 54 or 51, could be used. Ans.

(*b*) The same formula applies also to this case, in which $s = 105$ and $m = 3$, as before. Here, however, $p = 3$, and $105 = 3 \times y \pm 3$, or $y = \frac{1}{3}^{0}{}^{8} = 36$, or $\frac{1}{3}^{0}{}^{2} = 34$. Either pitch, 36 or 34, could be used. Ans.

(*c*) For this example, formula **486,** $E = \dfrac{c\,p\,N\,S}{60 \times 10^{8}\,m}$, would be used, this being a two-circuit multiple winding. (See Art. **3168.**) There being 105 coils, each of 3 turns, in the example, $c = 3 \times 105 = 315$. The *total number* of lines of force entering the armature being in each case 12,000,000, for the four-pole field, $N = \dfrac{12{,}000{,}000}{2} = 6{,}000{,}000$, and for the six-pole field, $N = \dfrac{12{,}000{,}000}{3} = 4{,}000{,}000$. (See Art. **3126.**) Since when $N = 6{,}000{,}000$, $p = 2$, and when $N = 4{,}000{,}000$, $p = 3$, it is evident that the product of the two terms p and N in the formula is the same (12,000,000) in each case. $S = 600$ and $m = 3$, from the example.

Then, $E = \dfrac{315 \times 12{,}000{,}000 \times 600}{60 \times 10^{8} \times 3} = 126$ volts. Ans.

(1419) (*a*) A multiple winding having three separate windings. (See Art. **3167.**)

(*b*) A multiple winding having two separate windings (Art. **3167**), and having the end of the first winding joined to the beginning of the second, and the end of the second to the beginning of the first. (See Art. **3169.**)

(1420) (*a*) 2.

(*b*) Because, from formula **484,** when p is even w must be twice an odd number. (See Art. **3165.**)

(1421) (*a*) Yes.

(*b*) No.

(*c*) See Arts. **3123** and **3156.**

(1422) See Art. **3149.**

(1423) By combining the commutators of the various windings into one and using brushes of sufficient angular span to be in contact with at least one segment belonging to each commutator. (See Art. **3167.**)

(1424) (*a*) Formula **481,** $s = p\,y \pm 1$, would be used for this example, using -1, only. (See Art. **3133.**) Then, as $s = 171$ and $p = 4$, $171 = 4\,y - 1$, or $y = \frac{172}{4} = 43$. Ans.

(*b*) The number of commutator segments would be p times the number of coils (see Art. **3134**), or, as $p = 4$, $4 \times 171 = 684$ segments. Ans.

(*c*) The angular distance between commutator segments would be $\frac{360^\circ}{p}$ (Art. **3134**); or, since $p = 4$, $\frac{360^\circ}{4} = 90^\circ$. Ans.

(1425) (*a*) The product of the difference of potential between the terminals of the dynamo and the current flowing in the external circuit is the output in watts. (Art. **3121.**)

(*b*) The total power required to drive the armature, made up of the power required to move the conductors through the magnetic field and that necessary to overcome hysteresis and eddy-current losses in the armature core, the friction of the journals, and other mechanical losses, is the input. (Art. **3121.**)

(*c*) The ratio of the output to the input, that is,

$$\frac{\text{output}}{\text{input}} = \text{efficiency.}$$

(1426) (*a*) and (*b*) See Art. **3120.**

(1427) See Art. **3119.**

(1428) (*a*) Low-resistance brushes require the greater amount of shifting.

(*b*) See Art. **3112.**

(1429) (*a*) Formula **484,** $w = 2\,p\,y \pm 2$, applies to this form of winding. In this case, $w = 38$ and $p = 2$; therefore, $38 = 4\,y \pm 2$; whence, $y = \frac{36}{4} = 9$, or $y = \frac{40}{4} = 10$. That is, y, the *average* pitch, may be either 9 or 10. Fig. 102

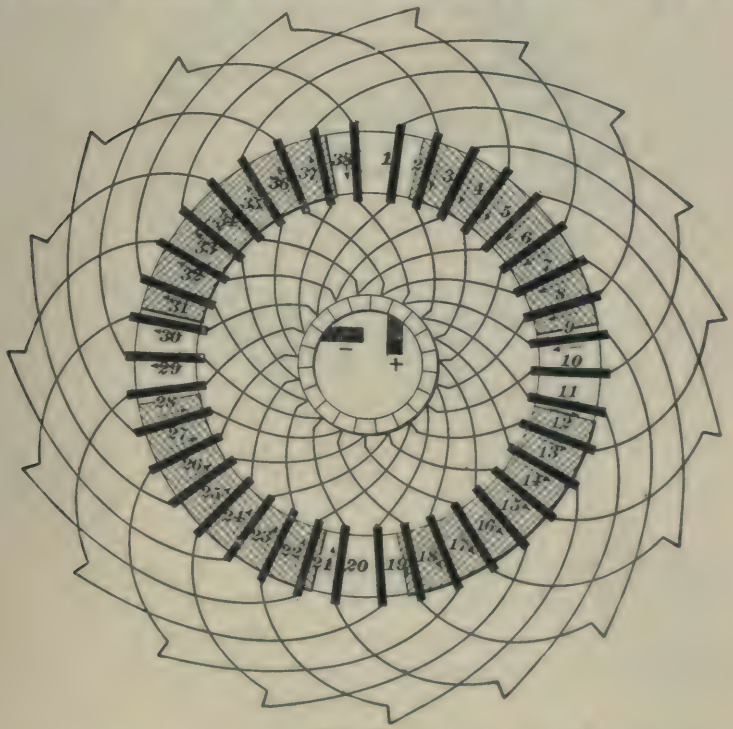

FIG. 102.

shows the diagram that would be drawn for this winding, using 9 for both front and back pitch. The coils short-circuited by the brushes are indicated by the omission of the arrow-heads, while on the other conductors the arrow-heads indicate the direction of the E. M. F.

In order to use 10 as the average pitch, the actual pitches must be 9 and 11. Fig. 103 indicates the diagram that would be drawn for this winding, using for the front pitch 9 and for the back pitch 11. These might, without error, be

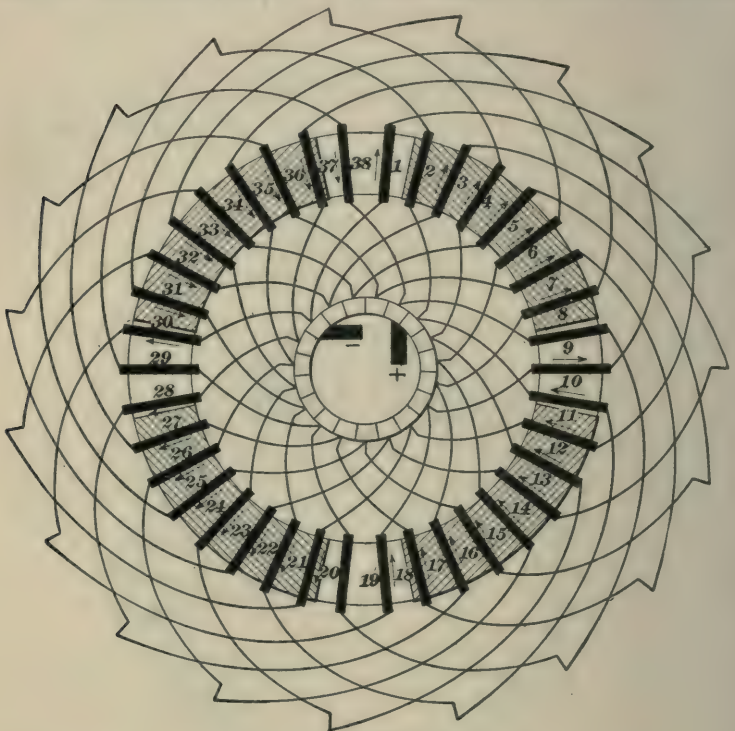

FIG. 103.

taken the other way, i. e., front pitch 11 and back pitch 9, if desired. The short-circuited coils are indicated as in Fig. 102.

In either of these diagrams the brushes need not be in the position shown; either may be located diametrically opposite the position given. This would change the short-circuited coils to some extent, as would be indicated by following out the circuits through the armature from brush to brush.

(*b*) The winding shown in Fig. 102 has some advantage over that shown in Fig. 103, in that the end connections on the back do not make so many crossings, and the length of the circuits through the armature is slightly shorter.

ELECTRIC TRANSMISSION.

(QUESTIONS 1430–1505.)

(1430) (*a*) Because by the use of transformers an alternating current may be given a very high voltage and correspondingly low amperage, under which conditions it is more economically transmitted over the line, by virtue of the smaller heating effect.

(*b*) The heating effect in a conductor is equal in watts to $C^2 R$. This means the loss due to heating varies as the square of the current. It is therefore much more economical to transmit the same amount of energy at a high voltage and small current than at a low voltage and heavy current.

(1431) (*a*) The percentage of loss in a conductor is the percentage which the drop in voltage in the conductor is of the voltage at the dynamo.

(*b*) By dividing the drop in voltage by the initial voltage and multiplying by 100.

(1432) See Art. **3289.**

(1433) See Art. **3242.**

(1434) (*a*) The amount of current a conductor is capable of carrying without excessive heating.

(*b*) See Arts. **3307** and **3310.**

(*c*) By adding 3 to the gauge number of the single wire. (See Art. **3279** and Table 105.)

(1435) (*a*) See Art. **3279.**

(*b*) A wire with a gauge number three larger than that of a given size will have a cross-sectional area one-half as great. (See also Art. **3279.**)

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(*c*) Remember that the resistance of a No. 10 B. & S. gauge wire is approximately 1 ohm. For every three sizes larger, the resistance will be one-half as great, and for every three sizes smaller the resistance will be twice as great. The resistance of the size next smaller or larger than a given size is found by multiplying or dividing the resistance of the given size by 1.26. In like manner the cross-sectional area may be found, remembering that the area is in inverse proportion to the resistance. If it is remembered that the diameter of a No. 10 B. & S. wire is one-tenth of an inch, or 100 mils, its cross-sectional area is known at once to be 10,000 circular mils; and the corresponding area of any other size of wire may be determined by the foregoing method. Having found the cross-sectional area in circular mils, extract the square root to obtain the diameter in mils.

(*d*) The resistance of a No. 10 B. & S. wire 1,000 feet long is 1 ohm. A No. 4 wire is six sizes larger and will have one-quarter the resistance, or .25 ohm. The resistance of a No. 3 wire will be $\frac{.25}{1.26} = .198$ ohm. Ans.

(1436) See Art. **3330.**

(1437) See Art. **3327.**

(1438) (*a*) See Art. **3328.**

(*b*) To provide convenient means of access to the cables and conduits.

(1439) (*a*) Copper and iron.

(*b*) Copper, because of its high conductivity, its durability, and its moderate cost. Iron, because of its cheapness, and in some cases because of its greater tensile strength.

(1440) (*a*) See Art. **3329.**

(*b*) See Art. **3329.**

(1441) See Art. **3332.**

(1442) See Art. **3339,** Figs. 1225 and 1226.

(1443) (*a*) and (*b*) See Arts. **3219** and **3220.**

(1444) See Art. **3225.**

(1445) In formula **503,** Art. **3339,** $C = 2$, $D' = 25$, and $D = 69$. The capacity of the cable will therefore be

$$C' = \frac{2 \times 25}{69} = .7246 \text{ microfarad.} \quad \text{Ans.}$$

(1446) See Art. **3340.**

(1447) (*a*) See Art. **3231.**

(*b*) With the handle up when closed.

(1448) In formula **504,** Art. **3340,** $D = 27$, $D' = 67$, $L' = 2{,}750$. The distance to the break will therefore be

$$L = \tfrac{27}{67} \times 2{,}750 = 1{,}075 \text{ feet.} \quad \text{Ans.}$$

(1449) See Art. **3272.**

(1450) See Art. **3216.**

(1451) See Art. **3226.**

(1452) See Art. **3223.**

(1453) (*a*) See Art. **3231,** Fig. 1205.

(*b*) A single-throw switch has but one set of contact points for each blade, as in Fig. 1205, while a double-throw switch has two sets of contact points, so placed that in passing from one set to the other the lever must swing through 180°.

(1454) (*a*) See Art. **3236.**

(*b*) By connecting two incandescent lamps in series across the mains, and grounding the point between them.

(1455) According to formula **493,**

$$R = \frac{360{,}000}{(162)^2} = 13.71 \text{ ohms.} \quad \text{Ans.}$$

(1456) (*a*) See Art. **3273.**

(*b*) See Art. **3274.**

(1457) (*a*) See Art. **3291.**

(*b*) See Arts. **3293, 3295,** and **3296.**

(1458) See Art. **3244** and Fig. 1209.

(1459) See Arts. **3246, 3247,** and **3258.**

(1460) It is a better conductor than any other metal except silver; it is not excessively expensive; it is capable of being drawn into wire possessing great tensile strength; and, lastly, it is non-corrosive under ordinary conditions.

(1461) Referring to Table 101, we find that a wire 1 foot long weighing 1 grain will have a resistance of .2271 B. A. ohm at 77° F., which is equivalent to

$$.2271 \times .9889 = .22458 \text{ legal ohm.}$$

In formula **495,** $L = 1$, $R = .22485$, and $w = 240$;

$$r' = \frac{L^2 R}{w} = \frac{.22458 \times 1^2}{240} = .00093.$$

In formula **497,** $R' = .0002$, $D = 25$, and $D' = 5$;

hence, $$R = \frac{D}{D'} R' = \frac{25}{5} \times .0002 = .001.$$

The resistance of the sample is therefore .001 ohm, and that of a pure wire of the same size and dimensions is, as above, .00093.

The percentage conductivity is therefore, according to formula **496,**

$$x = 100 \times \frac{.00093}{.001} = 93\%. \quad \text{Ans.}$$

(1462) See Art. **3278.**

(1463) See Fig. 1207.

(1464) (*a*), (*b*), and (*c*) See Art. **3234.**

(1465) By formula **490,**

$$W = \frac{(265)^2}{62.5} = 1{,}123.6 \text{ lb.} \quad \text{Ans.}$$

(1466) (*a*) See Art. **3266.**

(*b*) An unbalanced armature.

(1467) (*a*) One-thousandth of an inch.

(*b*) The area of a circle whose diameter is 1 mil.

(*c*) The ratio is as .7854 is to 1. (See Art. **3275.**)

(1468) See Art. **3290.**

(1469) (*a*) and (*b*) See Art. **3286.**

(1470) (*a*) A building containing all apparatus for generating power and converting it into electrical energy, to be distributed over the various circuits radiating from it.

(*b*) See Art. **3218.**

(1471) See Art. **3227.**

(1472) See Art. **3223.**

(1473) A circuit breaker acting automatically by causing an electromagnet to attract an armature when the current becomes excessive.

(1474) When power may be obtained cheaply at some point distant from the distributing center, the high-pressure alternating current is most suitable for its transmission.

(1475) (*a*) See Art. **3292.**

(*b*) Whenever the greatest possible mechanical strength is required, as in very long spans.

(1476) See Art. **3298.**

(1477) (*a*) High-speed automatic cut-off engines.

(*b*) Because they give close speed regulation, which is needed especially in view of the fact that in small stations the variations in load are extreme.

(1478) See Arts. **3243** and **3244.**

(1479) One would run as a motor. (See Art. **3250.**)

(1480) (*a*) See Art. **3306.**

(*b*) See derivation of formula **501**, Art. **3306.**

(1481) See Art. **3344.**

(1482) When there are but few cars, and also when the road is very hilly.

(1483) All the units should be large, except two or three, aggregating in output one large machine, so that at all times the dynamos may be run at a high efficiency.

(1484) See Art. **3258.**

(1485) (*a*) Low-speed Corliss engines, run with condensers, if condensing water is available.

(*b*) High-speed automatic cut-off engines.

(1486) See Art. **3345.**

(1487) To maintain the fields of the two machines at their proper strength.

(1488) Any desired combination of engines and dynamos may be readily obtained.

(1489) (*a*) 10° C. or 18° F.

(*b*) A change in resistance of .21 per cent. takes place for every degree change in temperature. The resistance becomes higher as the temperature increases.

(1490) (*a*) A short-circuited field coil will remain cold.

(*b*) A short-circuited armature coil will become excessively hot.

(1491) See Art. **3226.**

(1492) (*a*) The allowable loss in the line and the heating effect.

(*b*) When the cost of power is very small, so that the loss is to some extent compensated by the small first cost of the conductor.

(1493) It should be impregnated with oil or paraffin.

(1494) See Art. **3248.**

(1495) See Arts. **3260, 3261,** and **3262.**

(1496) Be sure that the positive brush of the dynamo that is being switched into circuit will be connected with the positive side of the line. The dynamo should then be brought up to speed, so as to generate a pressure equal to,

or a little above, that of the mains across which it is to be connected.

(1497) Eighty degrees Fahrenheit above the surrounding atmosphere.

(1498) (*a*) See Fig. 1212.

(*b*) See latter part of Art. **3252.**

(1499) Because the better ventilation, due to the rapid rotation, will tend to cool the armature.

(1500) Applying the notation used in Art. **3342** to this problem, we have $B = 460$ ohms, $G = 5{,}980$ ohms, $s = 920$ ohms, $r = 1{,}030{,}000$ ohms, $d_1 = 308$ divisions, and $d_2 = 203 + 3 = 206$ divisions.

The resistance of the circuit in megohms is

$$R = \frac{1{,}030{,}000 + \left(\frac{5{,}980 \times 920}{5{,}980 + 920}\right) + 460}{1{,}000{,}000} = 1.03126 \text{ megohms.}$$

According to formula **505,**

$$I = \frac{1.03126 \times 308 \times \frac{5{,}980 + 920}{920}}{206} = 11.56 \text{ megohms.} \quad \text{Ans.}$$

(1501) The resistances of the series windings must be inversely proportional to the current-carrying capacity of the machines.

(1502) The resistance of entire loop is

$$R = \frac{1.284 \times 13{,}405 \times 2}{1{,}000} = 34.424 \text{ ohms.}$$

Using formula **510,**

$$x = \frac{1{,}000 \times 34.424 - 100 \times 222.4}{100 + 1{,}000} = 11.0764 \text{ ohms.}$$

Distance from testing station =

$$\frac{11.0764}{1.284} \times 1{,}000 = 8{,}626 \text{ ft.} \quad \text{Ans.}$$

(1503) (*a*) A tube or duct in which cables may be laid
(*b*) Wood, clay, iron, asphalt, and cement
(*c*) See Art. **3324.**

(1504) See Arts. **3334** and **3335.**

(1505) See Art. **3333.**

ELECTRIC RAILWAYS.

(QUESTIONS 1506–1581.)

(1506) The motor is entirely enclosed in a frame forming part of the magnetic circuit, and supporting shaft and armature. (See Arts. **3375** and **3376.**)

(1507) (*a*) See Art. **3378.**

(*b*) Those in which the dead resistance is cut out, and the motors are either in series or parallel.

(1508) (*a*) An overhead insulated wire carrying current and a return path through the rails; cars propelled by motors geared to the axles, and supplied with current through a contact wheel held against the overhead wire. (See Art. **3358.**)

(*b*) The first cost is comparatively small, and the insulation can be made higher than in other systems.

(1509) See Art. **3394.**

(1510) By comparison with a table showing the safe current capacity of the wire. (Art. **3400.**)

(1511) See Art. **3402.**

(1512) See Art. **3363.**

(1513) Center poles, or side poles with brackets, or side poles with span wires. (Art. **3388.**)

(1514) (*a*) The cables must be tested for insulation resistance and for faults.

(*b*) See Art. **3414.**

(1515) See Art. **3421.**

(1516) See Art. **3423.**

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(1517) By formula **525**, Art. **3431**,

$f' = [70 + (20 \times 4)] \times 7 = 1{,}050$ pounds. Ans.

(1518) See Art. **3368.**

(1519) Storage-battery cars; electromagnetic devices in connection with closed conduits; open conduits; third rail and overhead trolley systems.

(1520) Sixteen or eighteen feet, single trucks. Longer cars usually have double trucks.

(1521) Those with salient and those with consequent-salient poles.

(1522) (*a*) Due to eddy currents set up in the revolving disk.

(*b*) No; the increased air-gap would cut down the lines of force and reduce the action for a given magnetizing current.

(1523) This is one in which a starting resistance is inserted in the armature circuit and cut out as the speed increases. (Art. **3383.**)

(1524) See Art. **3389.**

(1525) See Art. **3395.**

(1526) (*a*) See Art. **3404.**

(*b*) Reduction of track resistance and consequent saving of power.

(1527) They are required to steady the engine during sudden variations of load, which are of frequent occurrence.

(1528) In order to prevent the salts from creeping.

(1529) The engines may always be run at full load, and a smaller plant will suffice than would otherwise be required.

(1530) The great weight of the battery requires a large proportion of the available energy for transportation; deterioration is also in most cases very rapid. (Art. **3347.**)

(1531) (*a*) See Art. **3355.**

(*b*) See Art. **3356.**

(1532) The wheel-base is the distance between points of contact of wheels, measured along the rail.

(1533) See Art. **3367.**

(1534) (*a*) Point 1:
First half of Rheostat
to Field, No. 1 Motor;
to Armature, No. 1 Motor;
to Second half of Rheostat;
to Field, No. 2 Motor;
to Armature, No. 2 Motor.
Point 2: First half of Rheostat cut out.
Point 3: Second half of Rheostat cut out.
Point 4:
Field, No. 1 Motor;
to Armature, No. 1 Motor;
to First half of Rheostat.

In parallel with the above:
Second half of Rheostat
to Field, No. 2 Motor;
to Armature, No. 2 Motor.

Point 5 cuts out one-half, point 6 the whole, of Rheostat.

(*b*) Point 1:
Whole Rheostat
to Armature, No. 1 Motor;
to Field, No. 1 Motor;
to Armature, No. 2 Motor;
to Field, No. 2 Motor.
Point 2: One-half of Rheostat cut out.
Point 3: Entire Rheostat cut out.
Point 4: Rheostat cut out and Fields shunted.
Point 5: Same as Point 2.
Point 6: Second Motor cut out.

Point 7: Same as Point 6.
Point 8: One-half of Rheostat
to Armature, Motor No. 1;
to Field, Motor No. 1.
Also, Armature, Motor No. 2;
to Field, Motor No. 2;
in parallel with Armature and Field of No. 1.
Point 9: Rheostat cut out.
Point 10: Rheostat cut out and Fields shunted.

(1535) Trolley-wire not less than No. 0, B. & S. hard-drawn copper; span wire No. 5, B. & S. galvanized steel wire.

(1536) The difference of potential between trolley-wire and track shall have a certain minimum and a certain average value; also, the conductor must not heat beyond a safe limit.

(1537) From formula **515,** Art. **3399,** the drop in potential $e = \frac{5.4\ C\ D}{d^2}$ for a temperature of 75° F. The current in this case is 120 amperes, and by reference to the table of carrying capacity it will be seen that No. 00 wire will rise in temperature about 13° C., or say 23° F. Then, by formula **517,** the resistance of one mil-foot $= 10.79 + (.00222 \times 10.79 \times 23) = 11.34$ ohms. For a distributed load, one-half of this amount is taken, and

$$e = \frac{5.67 \times 120 \times 10{,}560}{133{,}079} = 54 \text{ volts.} \quad \text{Ans.}$$

(1538) A reflecting galvanometer (Thomson) may be used when there are no disturbing currents in neighboring conductors; when there are disturbing currents, only the ballistic galvanometer can be used.

(1539) The battery would be charged when the external load was heavy, because the generator E. M. F.

would then be the highest, and would discharge when the external load was light.

(1540) See Art. **3440.**

(1541) By changing the battery connection by means of a controller.

(1542) See Art. **3363.**

(1543) Between $3\frac{1}{3}$ and 5 to 1.

(1544) See Art. **3384.**

(1545) (*a*) See Art. **3393.**

(*b*) 6-inch offset.

(1546) For a uniformly distributed load the cross-section of conductor need be only one-half that required when all the load is concentrated at the end of the line.

(1547) See Art. **3405.**

(1548) (*a*) By incorrect use of the brakes, which allows the wheels to skid; sometimes flats are due to soft spots on the wheel tread.

(*b*) See Art. **3410.**

(1549) See Art. **3415.**

(1550) See Arts. **3426** and **3439.**

(1551) By formula **524,** Art. **3429,** the horsepower $H = \frac{h w + D f}{33{,}000\ \mathrm{E}}$. The speed of the car is 9 miles per hour, therefore the distance covered in 1 minute $= D = \frac{9 \times 5{,}280}{60} =$ 792 feet, corresponding, on a 7 per cent. grade, to a vertical distance $h = 792 \times .07 = 55.44$ feet. The weight of car $w = 9 \times 2{,}000 = 18{,}000$ pounds. The force required for propulsion is, by formula **522,**

$$f = 30 \times 9 = 270 \text{ pounds};$$

and, $H = \frac{(55.44 \times 18{,}000) + (792 \times 270)}{33{,}000 \times .7} = 52.46$ horsepower. Ans.

(1552) See Art. **3349.**

(1553) By an extension of the yokes under the rails, and the use of a heavy cross-section of metal in their construction.

(1554) In order to prevent oscillation of the car body, which strains the car unnecessarily and also tends to cause slipping of the driving wheels.

(1555) See Art. **3369.**

(1556) Between the canopy switch and controller. (See Fig. 1260.)

(1557) (*a*) See Art. **3390.**

(*b*) No. 8 B. & S. galvanized steel wire.

(1558) (*a*) See Art. **3403.**

(*b*) To avoid the shock to the car on entering curves and to save wear on the track.

(1559) See Art. **3411.**

(1560) See Art. **3417.**

(1561) The answer is obtained from formula **521,** Art. **3422,**

$$D = C' \frac{R_1}{R_1 + R + w}.$$

In this case $C' = 14{,}500$, $R_1 = 760$, $R = 950$, and $w = 7.3$. Then, the distance to the fault is

$$D = 14{,}500 \times \frac{760}{760 + 950 + 7.3} = 6{,}417 \text{ feet.} \quad \text{Ans.}$$

(1562) See Art. **3437.**

(1563) The horizontal effort required is greater as the radius of the curve decreases and as the length of wheel-base increases.

(1564) (*a*) and (*b*) See Arts. **3390** and **3391.**

(1565) (*a*) From Table 107, the wire to use under these conditions is .6 inch in diameter, when only one is used; if three are used, each wire carries 116.6 amperes, and will be, say, No. 2 wire. By formula **513,** Art. **3399,** modified for a temperature rise of 30°, the drop will be, in the first case,

$$e = \frac{12.08 \times 350 \times 1,200}{360,000} = 14.09 \text{ volts.} \quad \text{Ans.}$$

In the second case,

$$e = \frac{12.08 \times 350 \times 1,200}{3 \times 66,373} = 25.48 \text{ volts.} \quad \text{Ans.}$$

(*b*) If the three wires are to equal the single one in area, they will each be $\frac{360,000}{3} = 120,000$ circ. mils. The nearest size of wire is No. 00 = 133,079 circ. mils, and the temperature rise will be 12° C. for 116.6 amperes. Then, the drop is

$$e = \frac{11.3 \times 350 \times 1,200}{3 \times 133,079} = 11.9 \text{ volts.} \quad \text{Ans.}$$

(1566) See Art. **3409.**

(1567) By formula **523,** Art. **3428,** $f = r\,W = 25 \times 11 = 275$ pounds.

By formula **527,** the force to be applied is

$$F = 275 + \frac{91.1 \times 11 \times 12}{30} = 675.84 \text{ pounds.} \quad \text{Ans.}$$

(1568) See Art. **3425.**

(1569) (*a*) and (*b*) See Art. **3426.**

(1570) Nine H. P. per car.

(1571) The motors are used as generators, and a small current is taken from them which is used to energize an electromagnetic brake on the car-axle, operating directly on the wheels.

(1572) Feeders are intended to carry current from the power-house to centers of distribution, and are tapped at no intermediate point.

(1573) The first point to determine is the drop in potential through the known resistance. In the first division, the length of track from the feeder to the end of the line is 2,400 feet. The resistance is 0.03 ohm per mile, and for 2,400 feet will be $\frac{2,400}{5,280} \times .03 = .0136$ ohm. The total length of road is 24,600 feet, and there being 50 cars, there will be 1 car for each 492 feet. In 2,400 feet there may be $\frac{2,400}{492} =$ 5 cars, and the current will be $5 \times 16 = 80$ amperes, distributed load. The drop in potential will be

$$e = \frac{80 \times .0136}{2} = 0.54 \text{ volt.} \qquad \textit{Track I.}$$

For the second division, the length is 2,700 feet, and the resistance is $\frac{2,700}{5,280} \times .03 = .0153$ ohm. On this length of road there may be $\frac{2,700}{492} = 6$ cars, and the current will be 96 amperes, distributed load. The drop in potential is

$$e = \frac{96 \times .0153}{2} = 0.73 \text{ volt.} \qquad \textit{Track II.}$$

For the third division, the length is 3,900 feet, and the resistance $= \frac{3,900}{5,280} \times .03 = .022$ ohm. Here, there may be $\frac{3,900}{492} = 8$ cars, and the current $= 128$ amperes. The drop is

$$e = \frac{128 \times .022}{2} = 1.41 \text{ volts.} \qquad \textit{Track III.}$$

For the fourth division, the length is 3,300 feet, and the resistance $= \frac{3,300}{5,280} \times .03 = .019$ ohm. On this length of road there may be $\frac{3,300}{492} = 7$ cars, and the current $= 112$

amperes. The drop

$$e = \frac{112 \times .019}{2} = 1.06 \text{ volts.} \qquad \textit{Track IV.}$$

The sections of trolley-wire are each 600 feet; then, from the sub-feeder to the end is 300 feet. In this length there can be but one car, and it may be at any point. The size of wire is No. 0, B. & S.; then the drop of potential is

$$e = \frac{5.4 \times 16 \times 300}{105{,}592} = 0.24 \text{ volt.} \qquad \textit{Trolley I, II, III, IV.}$$

Division I. The main carries in each half a current of 80 amperes, distributed load. Allowing a temperature rise of 10° C., the size of wire will be No. 1; the main being 600 feet shorter than the trolley, its half-length is $\frac{1}{2}(4800 - 600) = 2{,}100$ feet. The area of wire, by Table 107, is 83,694 circ. mils, and the drop, by derivation from formula **515,** Art. **3399,** is

$$e = \frac{5.6 \times 80 \times 2{,}100}{83{,}694} = 11.2 \text{ volts.} \qquad \textit{Main I.}$$

The feeders, both outgoing and return, are alike for any division; the current is double that considered for one-half of the main, and the available drop of potential is $75 - (.54 + .24 + 11.2) = 63.02$, or 31.5 volts each. Then, by formula **512,** Art. **3399,**

$$d^2 = \frac{10.79 \times 160 \times 5{,}000}{31.5} = 274{,}031 \text{ circ. mils.}$$

The rise of temperature will be about 8° C., and

$$d^2 = \frac{11.13 \times 160 \times 5{,}000}{31.5} = 282{,}667 \text{ circ. mils.} \quad \text{Ans.}$$

Feeders I.

Division II. The main carries in each half a current of 96 amperes, distributed load, and the length is 2,400 feet. With a temperature rise of 10° C., the size wire will be No. 0 = 105,592 circ. mils. The drop in potential is

$$e = \frac{5.6 \times 96 \times 2{,}400}{105{,}592} = 12.2 \text{ volts.} \qquad \textit{Main II.}$$

This leaves for the feeders a drop of potential of 75 $-(.73+.24+12.2)=61.83$, or 30.9 volts each. Then, the area of feeder is

$$d^2=\frac{10.79\times 192\times 4{,}500}{30.9}=301{,}701 \text{ circ. mils.}$$

The rise of temperature will be about 12° C. Corrected for this,

$$d^2=\frac{11.3\times 192\times 4{,}500}{30.9}=315{,}961 \text{ circ. mils.}$$ Ans.

Feeders II.

Division III. The current in each half of the main is 128 amperes, distributed load, and the length is 3,600 feet. With a temperature rise of 10° C., the size wire will be No. 000, having an area of 167,805 circ. mils. Then, the drop is

$$e=\frac{5.6\times 128\times 3{,}600}{167{,}805}=15.3 \text{ volts.}$$ *Main III.*

This leaves for the feeders $75-(1.41+.24+15.3)=58.05$ volts, or a drop of 29 volts in each; then,

$$d^2=\frac{10.79\times 256\times 3{,}800}{29}=361{,}948 \text{ circ. mils.}$$

This will involve a temperature rise of about 18° C., and corrected for this,

$$d^2=\frac{11.56\times 256\times 3{,}800}{29}=387{,}778 \text{ circ. mils.}$$ Ans.

Feeders III.

Division IV. The main carries a distributed load of 112 amperes in each half, and the length is 3,000 feet. For a temperature rise of 10° C., the wire to use is No. 00, in area 133,079 circ. mils. The drop in potential is

$$e=\frac{5.6\times 112\times 3{,}000}{133{,}079}=14.1 \text{ volts.}$$ *Main IV.*

There is available for the feeders $75-(1.06+.24+14.1)=59.6$ volts for the two, or 29.8 volts each. Then,

$$d^2=\frac{10.79\times 224\times 4{,}000}{29.8}=324{,}424 \text{ circ. mils.}$$

This introduces a rise of about 15° C. in temperature, and the area becomes, by correction,

$$d^2 = \frac{11.44 \times 224 \times 4{,}000}{29.8} = 343{,}967 \text{ circ. mils.} \quad \text{Ans.}$$

Feeders IV.

(1574) (*a*) and (*b*) See Art. **3412.**

(1575) See Art. **3373.**

(1576) See Arts. **3419** and **3420.**

(1577) (*a*) The ratio of adhesive force to weight on drivers is $a = \frac{1}{7}$; one-half the weight of car is on drivers. Then, by formula **533,** Art. **3436,** the maximum grade will be

$$G_r = \frac{\left(\frac{2{,}000}{2} \times \frac{1}{7}\right) - 30}{20} = 5.64 \text{ per cent.} \quad \text{Ans.}$$

(*b*) By formula **532,**

$$G_s = \frac{\left(\frac{2{,}000}{1} \times \frac{1}{7}\right) - 70}{20} = 10.78 \text{ per cent.} \quad \text{Ans.}$$

(1578) See Art. **3392.**

(1579) Formula **519,** Art. **3420,** is applicable here.

The insulation resistance $I = \dfrac{26.06 \times 4}{1.89 \times \log \frac{283}{86}}$.

$\frac{283}{86} = 3.29$, and by reference to the table of logarithms, the logarithm of 3.29 is found to be 0.517.

Substituting this value for the ratio of the deflections, we have

$$I = \frac{104.24}{1.89 \times 0.517} = 106.6 + \text{ megohms.} \quad \text{Ans.}$$

(1580) See Art. **3410.**

(1581) The average speed of the cars is 5 miles in one-half hour, or 10 miles an hour, and the distance traveled in one minute is $\frac{10 \times 5{,}280}{60} = 880$ feet. The force required

to propel the car on the level is 30 pounds per ton, and the weight being 8 tons, $\frac{8 \times 30 \times 880}{33{,}000} = 6.4$ horsepower will be required for one car. There are four cars on the level, and for these the power will be $6.4 \times 4 = 25.6$ horsepower. For starting on the level we require a force of 70 pounds per ton, and the power required may be determined by the proportion $30 : 70 = 6.4 : x$; whence, $x = 14.9$ H. P., and for 2 cars, 29.8 H. P. The force necessary for starting on a 3 per cent. grade is, by formula **525,** Art. **3431,**

$$f' = 70 + (20 \times 3) \times 1 = 130 \text{ pounds per ton.}$$

Then, the power required may be determined by the proportion $30 : 130 = 6.4 : x$; whence, $x = 27.7$ H. P. The power required in ascending a 5 per cent. grade is found from formula **524,** Art. **3429,** where, disregarding for the present the question of efficiency,

$$H = \frac{h\,w + D\,f}{33{,}000} = \frac{(44 \times 16{,}000) + (880 \times 240)}{33{,}000} = 27.7 \text{ H. P.}$$

The car descending the grade and that at rest require no power, and the total horsepower to be delivered at the wheels will be:

4 cars on the level.....	25.6 H. P.
2 cars starting on the level.....	29.8 H. P.
1 car starting on a 3 per cent. grade..	27.7 H. P.
1 car ascending a 5 per cent. grade...	27.7 H. P.
	110.8 H. P.

The efficiency of transmission being 60 per cent., the total horsepower required is $\frac{110.8}{0.6} = 184.6$ H. P. Ans.

ELECTRIC LIGHTING.

(QUESTIONS 1582–1647.)

(1582) See Art. **3489.**

(1583) The current in the neutral wire is equal to the difference between the values of the current flowing in the two sides of the system.

(1584) 10 amperes.

(1585) The size of the crater is increased. (See Art **3445.**)

(1586) By formula **547,** Art. **3533,** the resistance per foot of the conductor is

$$R_f = \frac{12}{15 \times 610} = .00131 \text{ ohm},$$
$$= 1.31 \text{ ohms per } 1{,}000 \text{ feet}.$$

By reference to Table 98, it will be seen that this corresponds to No. 11 wire. Ans.

(1587) See Art. **3538.**

(1588) (*a*) The arc lamps should be connected in groups of two in series and each group connected in parallel across the 110-volt mains.

(*b*) Two arcs require $2 \times 45 = 90$ volts, leaving $110 - 90 = 20$ volts to be taken up by resistance. Since the current is 9.5 amperes through the lamps and resistance, and the drop around the resistance is 20 volts,

$$R = \frac{20}{9.5} = 2.1 \text{ ohms}.$$

Hence, the auxiliary resistance must be 2.1 ohms, and its carrying capacity, 9.5 amperes. Ans.

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(1589) About 3,500° C.

(1590) See Art. **3504.**

(1591) (*a*) Formula **545,** Art. **3484,** is to be used $N = 60$, $v = 1{,}175$, and $v_1 = 950$. Substituting,

$$x = 60 \times \frac{1{,}175}{1{,}175 + 950} = 33.$$

Hence the fault was situated beyond the 33d lamp, that is, between the 33d and 34th. Ans.

(*b*) In formula **546,** $R = 145{,}000$, $v = 1{,}175$, $x = 33$, and $e = 45$.

Then, $R_1 = \frac{145{,}000}{1{,}175} \times (45 \times 33 - 1{,}175) = 38{,}255$ ohms. Ans.

(1592) See Art. **3487.**

(1593) (*a*) Applying Ohm's law,

$$C = \frac{E}{R} = \frac{110}{220} = .5 \text{ ampere.} \quad \text{Ans.}$$

(*b*) Watts = volts × amperes = $110 \times .5 = 55$ watts; or, watts = current^2 × resistance = $.5^2 \times 220 = 55$ watts. Ans.

(*c*) 55 watts for 16 candles, or $\frac{55}{16} = 3.44$ watts per candle. Ans.

(1594) Yes. (See Art. **3469.**)

(1595) (*a*) See Art. **3450.**

(*b*) The positive carbon.

(1596) See Art. **3538.**

(1597) (*a*) See Art. **3508.**

(*b*) Upon the current transmitted and the resistance of the circuit.

(1598) See Art. **3443.**

(1599) Impurities volatilize at comparatively low temperatures, and do not affect the brightness of the crater. (See Art. **3446.**)

(1600)

Three 100 c. p. lamps = $3 \times 6 = 18$ lamps of 16 c. p
Eight 50 c. p. lamps = $8 \times 3 = 24$ lamps of 16 c. p
Twelve 32 c. p. lamps = $12 \times 2 = 24$ lamps of 16 c. p.
Total = 66 lamps of 16 c. p

Ans.

(1601) See Art. **3507.**

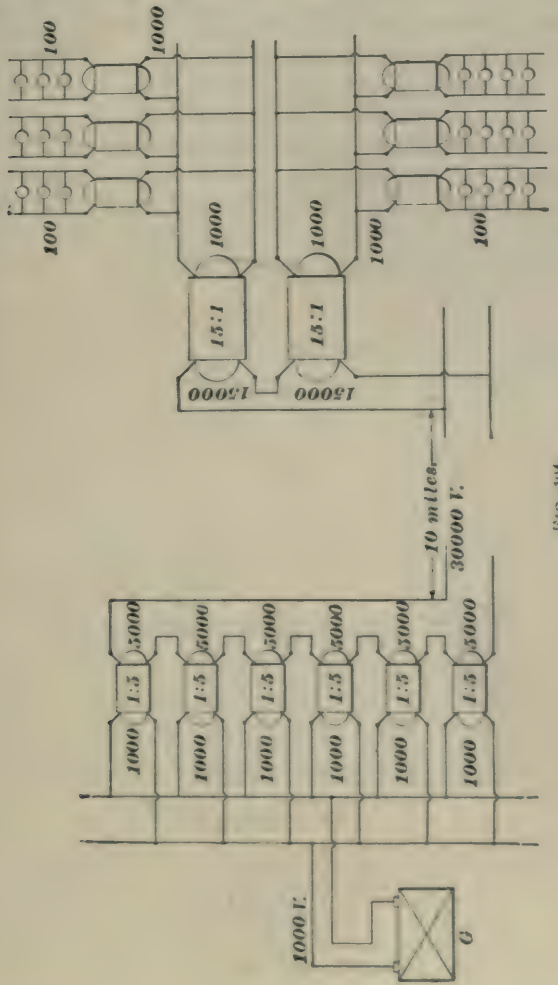

FIG. 104.

(1602) (*a*) See Art. **3457.**

(*b*) Because very little oxygen is supplied to the carbons in an enclosed arc lamp.

(*c*) See Art. **3459.**

(1603) See Art. **3514.** 220 volts. Ans.

(1604) On the multiple-arc system. (See Art. **3521.**)

(1605) When one lamp of a series burns out, all other lamps on that series are also extinguished.

(1606) See Art. **3445.**

(1607) See Fig. 104 and Arts. **3521** and **3522.**

(1608) See Art. **3517.**

(1609) The core has a higher conductivity than the surrounding material, thus tending to hold the arc in the center of the rod.

(1610) Apply Ohm's law:

$$R = \frac{E}{C}, \text{ or } R = \frac{100}{.64} = 156.25 \text{ ohms.} \quad \text{Ans.}$$

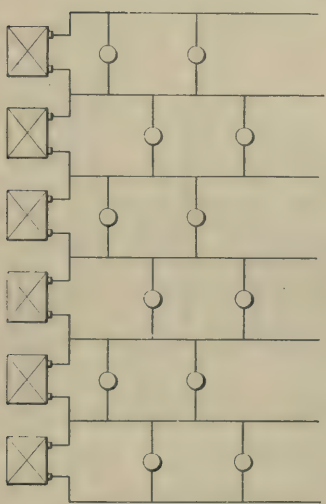

FIG. 105.

(1611) (*a*) Chiefly from the crater of the positive carbon.

(*b*) See Art. **3445.**

(1612) (*a*) See Art. **3462.**

(*b*) When the lamps are connected in series.

(1613) (*a*) Six dynamos. (See Art. **3517.**)

(*b*) See Fig. 105.

(1614) See Art. **3518.**

(1615) (*a*) About 39 volts.

(*b*) See Art. **3446.**

(1616) By formula **502** we find that the actual drop on the line is

$$E = \frac{100 \times 110}{95} - 110 = 5.79 \text{ volts.}$$

Then, we may use formula **549** to determine the size of cable. $N = 350$, $F = 600$, and $E = 5.79$.

Then, $A = \frac{10.8 \times 210{,}000}{5.79} = 391{,}710$ circular mils.

Current per lamp, $\frac{1}{2}$ ampere; 350 lamps require $350 \times \frac{1}{2} = 175$ amperes.

Referring to Table 106, we find 391,383 circ. mils as having more than the safe carrying capacity for 175 amperes, and, hence, adopt this conductor and require no revision. Ans.

(1617) $\frac{1}{4}$ of $20 = 5$. Ans. (See Art. **3472.**)

(1618) See Art. **3479.**

(1619) See Art. **3449.**

(1620) The voltage at the dynamo, by formula **502,** is

$$E = \frac{100 \times 110}{94} - 110 = 7.02 \text{ volts.}$$

The lamps take $\frac{1}{2}$ ampere, and are therefore of 16 c. p. Then $N = 150$, $F = 600$, and the resistance per foot of the wire will be, by formula **547,**

$$R_f = \frac{7.02}{150 \times 600} = .000078 \text{ ohm,}$$

$$= .078 \text{ ohm per 1,000 feet.}$$

By reference to Table 98 it will be found that this corresponds closely to No. 00 wire, which should therefore be used. Ans.

(1621) See Art. **3515.**

(1622) Regulators are used to keep the current at a constant value. (See Art. **3492.**)

(1623) 45 volts. (See Art. **3488.**)

(1624) (*a*) Two open arc lamps, or one enclosed lamp, with suitable resistances.

(*b*) One open arc lamp.

(1625) (*a*) and (*b*) See Art. **3447.**

(1626) (*a*) Enclosed arc lamps are usually run on a 110-volt direct-current lighting circuit.

(*b*) In order that deposits may not be made upon the inner globe.

(1627) See Art. **3497.**

(1628)

First circuit requires $10 \times 1{,}000 = 10{,}000$ circular mils.
Second circuit requires $60 \times 1{,}000 = 60{,}000$ circular mils.
Third circuit requires $30 \times 1{,}000 = 30{,}000$ circular mils.
Fourth circuit requires $80 \times 1{,}000 = 80{,}000$ circular mils.
Fifth or main circuit requires ..... $190 \times 1{,}000 = 190{,}000$ circular mils.

Diameter for 10,000 circular mils $= \sqrt{10{,}000} = 100$ mils.

Diameter for 60,000 circular mils $= \sqrt{60{,}000} = 245$ mils.
Diameter for 30,000 circular mils $= \sqrt{30{,}000} = 174$ mils.
Diameter for 80,000 circular mils $= \sqrt{80{,}000} = 283$ mils.
Diameter for 190,000 circular mils $= \sqrt{190{,}000} = 435$ mils.

According to Table 98, the following are the nearest sizes of wire in each case:

100 mils equivalent to No. 10 B. & S. wire.
245 mils equivalent to No. 2 B. & S. wire.
174 mils equivalent to No. 5 B. & S. wire.
283 mils equivalent to No. 1 B. & S. wire.
435 mils equivalent to No. 0000 B. & S. wire.
} Ans.

(1629) See Art. **3508.** Drop $= 10 \times 3 = 30$ volts. Ans.

(1630) See Art. **3505.**

(1631) When the carbons are widely separated, the shunt coils would be burnt out if the current were diverted

wholly through them. The cut-out prevents this from occurring. (See Art. **3453.**)

(1632) See Art. **3486.**

(1633) The economy of copper obtained in the feeders.

(1634) Regulation is obtained by the differential action of the series coil and gravity. (See Art. **3464.**)

(1635) The efficiency is the mean spherical candle-power divided by the electrical work done, as measured between the terminals of the lamp.

(1636) (*a*) See Art. **3474.**

(*b*) By the rule in Art. **3474,** the mean spherical candle-power $= \frac{421}{2} + \frac{2,070}{4} = 728$ c. p. Ans.

(1637) (*a*) See Arts. **3502** and **3503.**

(*b*) The controlling magnet puts in or out of circuit the regulating magnet.

(*c*) The regulating magnet moves the brushes to the proper positions. (See Art. **3503.**)

(1638) (*a*) See Art. **3511.**

(*b*) See Art. **3512.**

(1639) The average life of an incandescent lamp is about 800 hours.

(1640) See Art. **3485.**

(1641) The voltage is automatically changed as lamps are cut in or out. (See Art. **3492.**)

(1642) (*a*) Two.

(*b*) In series. (See Art. **3514.**)

(1643) (*a*) It must allow of continued incandescence while the current is passing, and must last for a reasonable time.

(*b*) Carbonized vegetable substances.

(*c*) It is termed the "filament."

(1644) About 3.44 watts per candle.

(1645) Rise in temperature due to the current.

(1646) Formula **547** may be used here. $E = 9$; $N = 200$; and $F = 700$. Then, the resistance per foot of the conductor is

$$R_f = \frac{9}{200 \times 700} = .0000643 \text{ ohm,}$$

$$= .0643 \text{ ohm per 1,000 feet.}$$

By reference to Table 98, it will be seen that this corresponds to No. 000, B. & S. Ans.

(1647) (*a*) See Arts. **3508, 3509,** and **3512.**

(*b*) 5 or 6 amperes is the limit, which would correspond to 10 or 12 ordinary 16 c. p. lamps. (See Art. **3539.**)

Dynamo-Electric Machine Design.

(CONTINUOUS-CURRENT.)

(QUESTIONS 1648–1711.)

(1648) A reduction in weight for a given output is secured. On the other hand, high speed requires careful balancing and strong construction. (Art. **3549.**)

(1649) The turns embraced within twice the angle of lead of the brushes multiplied by the current flowing through one of those conductors constitute the back ampere-turns. (Art. **3560.**)

(1650) "Volume of current" is a term used to indicate the total amount of current which the armature carries. It may be defined as the sheet of current flowing across the whole armature, and is equal to the number of conductors on the periphery of the armature multiplied by the current flowing in one conductor. (Art. **3573.**)

(1651) The result obtained is a diminution of sparking at the commutator, because, when the iron is saturated, there is less change in the number of lines of force at the moment of short-circuiting a coil. (Art. **3593.**)

(1652) In a four-pole machine there are only two magnetic circuits to be calculated; therefore the total ampere-turns will be double the amount required for one circuit, or, in this case, 9,600 ampere-turns. (Art. **3691.**)

(1653) In order to avoid any liability of the shaft being caught between the bearings when expanded by heat, and also in order to promote smooth wearing of the commutator by allowing oscillation of the armature when running. (Art. **3741.**)

(1654) The output is limited by the carrying capacity of the armature conductors and by the distortion of the field by armature reaction. (Art. **3541.**)

(1655) Small bipolar dynamos may be run at about 2,300 to 2,900 feet per minute; larger machines range between 2,800 to 3,600 feet per minute. Sometimes a fixed speed is used, for small sizes perhaps 3,100, for large sizes 3,500 feet per minute. (Art. **3549.**)

(1656) The advantages of such design are that a compact magnetic circuit is obtained and the weight of copper is reduced. (Art. **3551.**)

(1657) The cross ampere-turns may be defined as equal to one-half the conductors under the pole-pieces multiplied by the current flowing through each; that is, to the whole number of armature ampere-turns less the back ampere-turns. The cross ampere-turns of a bipolar dynamo are equal to one-half the armature current multiplied by the number of conductors enclosed within an angle equal to 180° less twice the angle of lead. (Art. **3562.**)

(1658) The maximum volume of current which an armature may carry is directly proportional to the field ampere-turns required for the air-gap, and inversely proportional to the polar embrace. Therefore, since the angle covered by the pole is usually fixed, the volume of current is affected only by the length and density of the air-gap, and an increase in volume of current must be accompanied by an increase in one of these factors. (Art. **3585.**)

(1659) In order to keep the current at a constant value, for any increase of armature current above the normal amount will distort and weaken the field. (Art. **3592.**)

(1660) (*a*) The effect will be a decrease of efficiency, because, the number of turns being decreased, the current must necessarily increase in order to obtain the same number of ampere-turns, and the heat losses are proportional to the square of the current, but only directly proportional to the resistance of the winding.

(*b*) No.

(*c*) Because the total number of ampere-turns will be practically constant, as the resistance of the winding is decreased and a larger current flows. (Art. **3622.**)

(1661) The external characteristic is useful in showing the performance of a dynamo under load. (Art. **3659.**)

(1662) Steel, hard brass, or phosphor-bronze, varying in thickness from No. 18 to 21 or 22. (Art. **3713.**)

(1663) The pulleys for small machines run at a high speed, which necessitates very careful balancing. This result is best obtained by casting the pulley with a central web and turning all over. (Art. **3747.**)

(1664) About 170° F. (Art. **3544.**)

(1665) Usually not more than 2½ inches. This depth should not be exceeded, as a rule, as the rise in temperature of the inner turns may otherwise become very high. (Art. **3550.**)

(1666) Distortion of the field is produced, causing an uneven distribution of the lines of force. Under the leading pole tips of a dynamo the density becomes less than normal, and under the following pole tips it becomes greater. (Art. **3554.**)

(1667) When the back ampere-turns are increased, owing to the necessity of shifting the brushes, the cross ampere-turns are correspondingly decreased; so that, while the opposition to the field lines of force is greater, the distribution of the lines is more uniform. For this reason, the commutating fringe at the leading pole tip is still sufficiently strong to reverse the current effectively, as long as the current is not excessive. (Art. **3564.**)

(1668) (*a*) The armature ampere-turns consist of the cross ampere-turns and the back ampere-turns.

(*b*) They are equal to one-half the total number of conductors on the surface of the armature multiplied by the current carried by each. (Art. **3569.**)

(1669) When the air-gap density is about 50,000 lines of force per square inch, the length of air-gap may be equal to one-half the width of slot. (Art. **3588.**)

(1670) (*a*) Yes.

(*b*) A high resistance of shunt provides the requisite number of ampere-turns with a low current strength, and as the heat loss is directly proportional to the resistance, but proportional to the square of the current, the latter should preferably be kept at a low value. Also, when the shunt resistance is great, a smaller amount of the total current output of the machine is used in the field, leaving a larger proportion for the external circuit. (Arts. **3622** and **3623.**)

(1671) Bipolar machines may be made in sizes up to about 15 or 20 horsepower. (Art. **3692.**)

(1672) In order that the armature and shaft may be entered between the pole-pieces. (Art. **3753.**)

(1673) A specified overload should be provided against, as being likely to occur frequently. A limit of 15 or 20 per cent. will usually be found sufficient. (Art. **3542.**)

(1674) From about 10 per cent. for a 1 K. W. machine to about 1.5 per cent. for a 100 K. W. machine. (Art. **3552.**)

(1675) (*a*) No.

(*b*) The cross ampere-turns produce a different distribution of lines of force in the field, by distortion; but the total number of lines of force is not changed. (Art. **3554.**)

(1676) The electrical efficiency of a dynamo is obtained by multiplying 100 times the E. M. F. at the brushes, in volts, by the current in amperes available for the external circuit, and dividing the result by the product obtained by multiplying together the total E. M. F., in volts, and the total current in amperes. (Art. **3668.**)

(1677) (*a*) The maximum volume of current which a smooth-core drum armature may carry is equal to 240 times

the field ampere-turns required for the air-gaps, divided by the angle embraced by one pole-piece.

(*b*) For a ring armature, the maximum volume of current is equal to 180 times, instead of 240 times, the above value. (Arts. **3574** and **3575.**)

(1678) Smooth-core armatures are especially suitable for electric-light dynamos, as the load comes on gradually and the brushes may be moved to the proper position for sparkless running. When the load fluctuates considerably and suddenly, the brushes must be kept in one position, which is possible only with a slotted armature, and this style of machine is therefore used for street-railway work and for installations of similar character. (Art. **3593.**)

(1679) The values of the ampere-turns per magnetic circuit are found for no load and for full load, the required E. M. F. and speed being specified for each case. The difference between these two values of the ampere-turns gives the ampere-turns for the series coils. (Art. **3642.**)

(1680) Higher insulation is obtained by this means, as a larger surface of insulating material is introduced between the segments and the shell of the commutator. (Art. **3718.**)

(1681) Allow about 1.11 square inches of surface per watt lost in heating. (Art. **3546.**)

(1682) In the air-gap. (Art. **3553.**)

(1683) (*a*) The field would be neutralized, and the excessive sparking due to this cause would necessitate reduction of load or shutting down the machine. (Art. **3563.**)

(*b*) The M. M. F. in the gap due to the field ampere-turns should be 1½ to 2 times the cross M. M. F. due to the armature current. (Art. **3564.**)

(1684) From 1.1 to 1.5 or 2 square inches of surface should be allowed per watt expended in heating. (Art. **3550.**)

(1685) It becomes difficult to regulate the E. M. F., because there is a wide variation in the temperature of the coils, which heat up continually after the machine is started. The consequent change in resistance causes the E. M. F. to fall steadily, and a constant difference of potential is obtained only after running for some time. (Art. **3623.**)

(1686) Not more than 15 or 16 volts between segments; but a lower difference of potential is rather better. (Art. **3723.**)

(1687) Laminated pole-pieces are used in order to prevent the generation of eddy currents in those parts. (Art. **3762.**)

(1688) The rise in temperature of an armature is equal to 99 times the watts lost, divided by the exposed surface in square inches multiplied by 1 plus 0.00018 times the velocity in feet per second. (See also Art. **3546.**)

(1689) At the pole tips, at nearly diametrically opposite points. (Art. **3553.**)

(1690) A high density is used to counteract the effect of self-induction of the armature coils, as there is only a slight change in the total number of lines of force in the core when those due to the armature current collapse. (Art. **3567.**)

(1691) (*a*) Smooth-core machines.

(*b*) Because this style of machine has low self-induction, which is an important requirement when heavy currents are carried. (Art. **3594.**)

(1692) In order to make provision for changing within certain limits the amount of current passing through the series coils, and thereby regulating the amount of compounding. (Art. **3654.**)

(1693) It would not be possible to remove a brush while the machine was running, if there were no other brush provided to collect the current in the meantime. (Art. **3727.**)

(1694) (*a*) It is usually necessary to consider the back ampere-turns, (*b*) because a lead must be given to the brushes in order to bring the armature coils under the pole tip for commutation of the current. (Art. **3560.**)

(1695) (*a*) 100,000 lines of force per square inch, and sometimes as high as 120,000.

(*b*) For drum armatures, 85,000 to 90,000 lines of force, and for ring armatures 100,000 to 110,000 or more lines of force, per square inch.

(*c*) Cast-iron yokes, about 30,000; for cast steel, about 75,000; and for wrought iron, about 85,000 lines of force per square inch.

(*d*) Cast iron, about 40,000 to 50,000; and cast-steel or wrought-iron forgings, 95,000 to 105,000 lines of force per square inch. (Arts. **3598** to **3601.**)

(1696) The energy loss, in watts, of the shunt and series windings of a compound dynamo should be proportioned to the respective ampere-turns carried by each coil. (Art. **3646.**)

(1697) A leakage coefficient of 1.25 indicates that $\frac{.25}{1.25} = 20$ per cent. of the magnetizing force is wasted. The magnetizing force is 3 per cent. of the total output; therefore 20 per cent. of 3 per cent., or .6 of 1 per cent., of the total output is lost in leakage. (Art. **3693.**)

(1698) The maximum number of ampere-turns which an armature may carry is equal to the volume of current flowing through the conductors divided by the number of field-magnet poles. (Art. **3579.**)

(1699) Copper-gauze brushes collect heavy currents without much heating and interpose but little resistance in the circuit, which is desirable in the case of low-potential lighting dynamos. Since the load comes on gradually, an attendant is always able to shift the brushes by hand to the point of least sparking. (Art. **3727.**)

(1700) $$t = \frac{80\,w}{s},$$

where t = temperature rise of spool in degrees Fahrenheit;
w = watts expended in coils;
s = surface of spools (wire only) in square inches.
(Art. **3550.**)

(1701) When the air-gap is long as compared with the width of the slot, the lines of force spread out, and the density at the pole face becomes more uniform than would be the case with a shorter gap. (Art. **3588.**)

(1702) When the current and E. M. F. on a characteristic curve are laid out to the same scale (as is usual), and a line is drawn from the origin to any point on the curve, the tangent of the angle of slope will indicate the value of the external resistance. The current is equal to the E. M. F. divided by the resistance; the vertical dimension on the curve sheet represents the E. M. F., and the horizontal dimension represents the current. But the former divided by the latter is equal to the tangent of the angle of slope, so that this tangent always represents the value of the resistance. (Art. **3660.**)

(1703) In the inverted bipolar form of dynamo the pole-pieces approach close to the iron bed-plate, and the zinc block is introduced in order to preserve sufficient distance between to prevent undue leakage. (Art. **3759.**)

(1704) (*a*) The back ampere-turns must be neutralized by an equal number of ampere-turns on the field-magnets.

(*b*) The back ampere-turns are equal to $\frac{C\,r\,c}{360}$, where C = total armature current; r = angle of lead of brushes; c = total number of conductors. (Arts. **3560** and **3561.**)

(1705) Considerable self-induction results at commutation on account of the coils being wound over iron teeth, and sparking results. (Art. **3593.**)

(1706) If, on the dynamo characteristic, the resistance of the external circuit when laid out should be of such a value

as to correspond with the initial portion of the characteristic, this constitutes the critical resistance of the circuit, and the instability of operation is extreme. (Art. **3661.**)

(1707) Ring armatures possess greater self-induction than drum armatures, so that a greater angle of lead is required to overcome it, as well as a more powerful field for reversal. In order to obtain this condition, the field ampere-turns must be relatively stronger in the case of a ring armature than of a drum armature, for the same cross ampere-turns. (Art. **3567.**)

(1708) The following current densities per square inch may be used: (*a*) for copper, 150 amperes; (*b*) for brass, 100 amperes; (*c*) for carbon, 30 amperes. (Art. **3727.**)

(1709) (*a*) and (*b*) The change in magnetic density at points close together becomes abrupt, so that if there is any uneven spacing of the conductors, they will be short-circuited at different points relative to the pole-piece, and thus cause sparking. (Art. **3565.**)

(1710) Due to drop of potential through the armature, and to weakening and demagnetizing action of the armature current. (Art. **3662.**)

(1711) We may begin by selecting the armature diameter, and, if necessary, change the figure subsequently. We will take a diameter of 8 inches. Then, since the peripheral velocity is 2,600 feet per minute, the speed will be $\frac{2,600 \times 12}{8 \times 3.1416} = 1,240$ R. P. M. The density in the air-gap may be about 35,000 lines of force per square inch. In determining the size of armature conductor we must allow 600 circ. mils per ampere. 5 H. P. $= 5 \times 746 = 3,730$ watts, and $\frac{3,730}{230} = 16.22$ amperes output. The shunt current will be about .48 ampere; total armature current $= 16.7$ amperes. The cross-sectional area of the conductor is then $\frac{600 \times 16.7}{2} = 5,010$ circ. mils. No. 13 wire $= 5,178$ circ. mils, and will therefore be suitable.

If we allow for an air-gap of $\frac{1}{8}$ in. each side, the bore of the fields will be $8\frac{1}{4}$ in. We will suppose that each pole face covers 125°; then the length of pole face will be $\frac{8.25 \times 3.1416 \times 125}{360} = 9$ in. If the width of the pole face (length of armature) is 6 inches, the area of the pole face at the air-gap will be 54 sq. in. At 35,000 lines per square inch, this will give 1,890,000 lines of force in the air-gap.

By modification of formula **480** we have

$$c = \frac{E \times 60 \times 10^8}{NS} = \frac{230 \times 60 \times 10^8}{1{,}890{,}000 \times 1{,}240} = 588.$$

By selecting 32 slots and 18 conductors per slot we obtain $c = 576$. With this number of conductors the lines of force N must be $\frac{230 \times 10^8 \times 60}{576 \times 1{,}240} = 1{,}932{,}000$. The density in the air-gap will then be $\frac{1{,}932{,}000}{54} = 35{,}780$ lines per square inch.

The diameter of No. 13 wire, double cotton covered, is .081 in. We must allow for 2 coils per slot, as this will allow us 32 bars in the commutator. Each coil will be wound, then, in 9 turns, forming a square in cross-section, being shaped to the armature, bound over with tape, and shellaced. The thickness of the coil when wound will be about $.081 \times 3 + 2 \times .03 = .303$, allowing .03 for tape each side and for irregularities. If, then, we insert .04 in. mica insulation on each side of the coil in the slot, the width of slot will be .383 in. Between two coils in a slot we will place a strip of fiber .1 in. thick, .04 in. of mica at the bottom of the slot, the same above the upper coil, and a piece of hard wood .15 in. thick over this. The depth of the slot will then be $2 \times .303 + .1 + 2 \times .04 + .15 = .936$ in., or, say, $\frac{15}{16}$ in.

As there are 32 slots and 2 coils per slot, the number of winding spaces $w = 64$. By formula **483,** $w = 2y \pm 2$. whence the pitch $y = \frac{64 \pm 2}{2} = 31$ or 33. We will adopt the smaller number 31.

The armature being 8 in. in diameter, the circumference is 25.13 in., and the pitch of the teeth is $\frac{25.13}{32} = .785$ in. at the top. The slots are $\frac{15}{16}$ in. deep, and the diameter at the root of the teeth is therefore $8 - 1\frac{7}{8} = 6\frac{1}{8}$ in., which gives a circumference of 19.24 in. The pitch at the root of the teeth is, then, $\frac{19.24}{32} = .601$ in. The average pitch $= (.785 + .601) \div 2 = .693$ in. The width of slot $= .383$ in.; therefore the average thickness of tooth $= .693 - .383 = .31$ in.

The angle embraced by one pole-piece is 125°. $\frac{125}{360} \times 32 = 11$ teeth under the pole. The effective area $= 11 \times .31 \times$ length of armature. The effective length of the armature is $.88 \times 6 = 5.28$ in. Therefore the effective area of teeth under the pole $= 11 \times .31 \times 5.28 = 18$ square inches. The density in the teeth will be $\frac{1,932,000}{18} = 107,300$. The leakage coefficient is 1.25; then the induction in the magnet core will be $1.25 \times 1,932,000 = 2,415,000$ lines of force. If we take a diameter for the core of $5\frac{3}{8}$ in. corresponding to 24.85 sq. in. area, the density will be $\frac{2,415,000}{24.85} = 97,250$ lines per square inch.

The diameter of the armature core at root of the teeth is $6\frac{1}{8}$ in. Allowing for a shaft $1\frac{3}{8}$ in. in diameter, the radial depth of the core is $2\frac{1}{4}$ in. Then the density will be $\frac{966,000}{2.25 \times 5.28} =$ 81,400 lines per sq. in.

The current in the armature at full load is 16.7 amperes and the back ampere-turns $= (16 - 11) \times 18 \times \frac{16.7}{2} = 751$.

The yoke may be cast iron, say 10 in. average length, with a density of 30,000 lines of force per sq. in. For the pole-pieces we will allow 4 inches each, or a total of 8 in., with a density of 50,000.

For the magnet cores we may allow a length of 4 inches each, or 8 inches total. Then the total ampere-turns will be:

	Length. In.	Density.	Amp.-turns per In.	Ampere-turns.
Air-gap	$\frac{1}{4}$	35,780		2,800
Magnet core	10	97,250	110	1,100
Cast-iron yoke, flanged	10	30,000	43	430
Pole-pieces	8	50,000	10	80
Armature core	5	81,400	30	150
Armature teeth	$1\frac{7}{8}$	107,300	100	187
Back ampere-turns				751
				5,498

The total ampere-turns we may call 5,500.

The diameter inside of magnet spool = $5\frac{7}{8}$ in., and depth of winding may be, say, $1\frac{1}{8}$ in. The mean diameter will be 7 inches, corresponding to a circumference of 21.99 in., or 1.83 feet. By formula **592** the area of the wire is $\frac{12 \times 1.83 \times 5,500}{230} =$ 525 circ. mils. The nearest size of wire is No. 23, B. & S., having a cross-sectional area of 509.5 circ. mils. Winding volume = $4 \times 1.125 \times 2 = 9$ sq. in. Turns per sq. in. = 1,307, making a total of 11,763 turns. Total length in feet = $1.83 \times 11,763 = 21,520$, and the resistance will be, cold, about $20.37 \times 21.52 = 438$ ohms. Allowing for a temperature rise of about 40° F., the resistance, hot, will be about 477 ohms. The shunt current = $\frac{230}{477} = .483$ ampere. Watts loss = $CE =$ $.483 \times 230 = 111$, and the loss in the shunt = $\frac{111 \times 100}{3,730} =$ 2.98 per cent.

The cylindrical surface of the armature = $8 \times \pi \times 6 =$ 150.78 sq. in., and at the ends about 50 sq. in. additional, for radiation of heat. There are 288 complete turns of wire, each turn being about 3 feet long. Total length = $288 \times 3 = 864$ feet.

No. 13 wire has a resistance per 1,000 feet of 2 ohms; then two lengths in parallel, each 432 ft., have a resistance of .432 ohm. Allowing for heating and effect of joints, we may call this resistance .5 ohm. The C^2R loss will then be

$16.7^2 \times .5 = 139.45$ watts. The temperature rise, by formula **559,** is

$$t = \frac{99 \times 139.45}{200.78 \times (1 + .00018 \times 45)} = \frac{13,805}{202.4} = 68^\circ \text{ F.}$$

Outside diameter of field spools $= 8\frac{1}{2}$ in. Length of each $= 4$ in. Total surface $= 8\frac{1}{2} \times \pi \times 8 = 204$ sq. in. Watts lost (as already found) $= 111$. The temperature rise of the spools is, by formula **560,**

$$t = \frac{80 \times 111}{204} = 43^\circ \text{ F.}$$

MOTOR DESIGN.

(CONTINUOUS-CURRENT.)

(QUESTIONS 1712–1747.)

(1712) (*a*) The counter E. M. F. is the E. M. F. generated by the armature conductors cutting across the magnetic field when the motor is in motion. (Art. **3782.**)

(*b*) The counter E. M. F. may be calculated from the expression

$$E_m = \frac{n C N}{10^8},$$

which in this case gives

$$E_m = \frac{1,000 \times 400 \times 3,000,000}{60 \times 10^8} = 200 \text{ volts.} \quad \text{Ans.}$$

(Art. **3787.**)

(1713) A shunt motor runs at nearly constant speed because the field strength is nearly constant, and also because the counter E. M. F. which the machine has to generate drops only a slight amount when the motor is loaded. (Arts. **3792** and **3793.**)

(1714) (*a*) We have the relation between the line E. M. F. and the counter E. M. F. given in formula **601.** Hence we have

$$E = E_m + C R_a.$$

$$220 = E_m + 100 \times .05.$$

$$E_m = 215 \text{ volts.} \quad \text{Ans.} \quad (\text{Art. } \mathbf{3783.})$$

(*b*) If the armature were held from turning, the counter

E. M. F., E_m, would be zero, the current would be limited simply by the armature resistance, and we would have

$$C = \frac{E}{R_a} = \frac{220}{.05} = 4{,}500 \text{ amperes.} \quad \text{Ans.}$$

(1715) Series motors are better adapted for street-railway work because they can be made to give a very strong effort for short intervals; also because they are less expensive and their field windings are less easily damaged. (Art. **3799.**)

(1716) (*a*) If the balance gives a reading of 10 lb. at a radius of 2 feet, the useful torque is 20 ft.-lb., and the torque acting on the armature is this plus the 1 ft.-lb. necessary to make up for friction. The total torque acting on the armature is, therefore, 21 ft.-lb. From formula **605** we have

$$T' = .00117 \frac{N C C_m}{10^6},$$

where N is the magnetic flux, C the number of armature conductors, and C_m the current flowing through the armature.

Hence $$21 = .00117 \times \frac{N \times 300 \times 25}{10^6},$$

or $$N = \frac{21 \times 10^6}{.00117 \times 300 \times 25} = 2{,}393{,}100 \text{ lines, nearly.} \quad \text{Ans.}$$

(*b*) The total torque acting on the armature is 21 foot-pounds, and since the diameter of the armature is 12 in., or 1 foot, the pull at the circumference of the armature must be 42 lb. This pull will be exerted upon the conductors under the poles, so that the average pull for conductor will be

$$\frac{42}{\frac{2}{3} \times 300} = .21 \text{ lb.} \quad \text{Ans.} \quad \text{(Arts. } \mathbf{3787} \text{ and } \mathbf{3788.}\text{)}$$

(1717) (*a*) The commercial efficiency is the ratio of the total useful power delivered at the pulley to the total power supplied from the mains.

(*b*) 10 H. P. = 7,460 watts delivered; hence, input must be $\frac{7,460}{.85} = 8,776$ watts, and the current supplied must be $\frac{8,776}{500} = 17.55$ amperes. Ans. (Art. **3785.**)

(1718) (*a*) The speed decreases as the load is increased, because the motor, being operated from constant-potential mains, has to generate a nearly constant E. M. F. As the load increases the field strength increases, hence the speed must diminish.

(*b*) It is not safe to throw the load entirely off the motor because the speed would become so high that the armature would be liable to injury. (Arts. **3798** and **3799.**)

(1719) Strengthening the field lowers the speed of the motor, because the stronger the field the less the speed at which the armature has to turn to generate the required counter E. M. F. (Art. **3794.**)

(1720) The efficiency of conversion of a motor of this capacity should be about .904 and the electrical efficiency about .916 (see Fig. 1428). The commercial efficiency would therefore be $.904 \times .916 = .828$. If the output at the pulley is $20 \times 746 = 14,920$ watts, the input must be $\frac{14,920}{.828} = 18,020$ watts, nearly. The total current input will then be $\frac{18,020}{220} = 81.9$ amperes; 3% of this current, or 2.46 amp., will flow through the field, so that the armature should be designed to carry about $81.9 - 2.46 = 79.44$ amperes. The efficiency of conversion is .904; hence, the total electrical energy active in the armature is $\frac{14,920}{.904} = 16,500$ watts, nearly. The current flowing through the armature is 79.44 amperes; hence, the counter E. M. F. must be $\frac{16,500}{79.44} = 208$ volts, nearly. We must then design as a dynamo a machine capable of generating 208 volts at a

speed of 800 R. P. M., and have a current-carrying capacity of 79.44 amperes in the armature and 2.46 amperes in the field. Ans.

(1721) (*a*) One method for regulating the speed is the cutting down of the E. M. F. across the armature terminals by the insertion of a resistance in the armature circuit. Another method is by strengthening or weakening the field by means of a field rheostat.

(*b*) The latter method is the more economical. (Arts. **3794** and **3795.**)

(1722) (*a*) A resistance is necessary at starting because the counter E. M. F. is then zero; and since the armature resistance is very low, a very large current would flow if the armature were connected directly across the mains.

(*b*) A burned-out armature would result, unless the circuit was at once broken by some safety device, such as fuses or a circuit-breaker; in any event, the commutator is quite liable to be burned. (Art. **3805.**)

(1723) (*a*) The total torque T' can at once be calculated from formula **605:**

$$T' = .00117 \frac{N C C_m}{10^6} = .00117 \times \frac{2{,}500{,}000 \times 500 \times 20}{1{,}000{,}000} =$$

29.25 foot-pounds. Ans.

(*b*) If the diameter of the armature is 9 in., the radius is $\frac{3}{8}$ ft. Consequently the pull at the circumference of the armature must be $\frac{29.25 \times 8}{3} = 78$ pounds; 60% of the conductors are under the poles, so that the pull per conductor must be $\frac{78}{.60 \times 500} = .26$ lb. (Art. **3787.**)

(1724) (*a*) The connection from field terminal *2* to the line is wrong. This connection should be made to the line on the switch side of the rheostat, as shown in Fig. 1422.

(*b*) If the connections were as shown and the main switch thrown, the field would not be excited, because no connec-

tion would be made to the field until the rheostat arm was moved from the dead point to the first resistance point. Current would then flow through both armature and field, but the magnets would be but very slightly energized, so that very little starting torque would be produced. There would be no danger of short circuit, as the rheostat is properly connected in series with the armature.

(1725) (*a*) The efficiency of conversion of a motor is the ratio of the total energy delivered at the pulley to the active electrical energy in the armature; i. e., to the armature current multiplied by the counter E. M. F. The greater the core losses and friction losses, the lower will be the efficiency of conversion.

(*b*) The total energy active in the armature is $100 \times 10 = 1{,}000$ watts. Total energy delivered is 1 H. P. = 746 watts. Efficiency of conversion $= \frac{746}{1000} = .746$, or 74.6%.

(Art. **3785.**)

(1726) The speed of a series motor may be regulated by the same methods already given for a shunt motor, namely, by cutting down the applied E. M. F. by means of a resistance in series with the motor, or by varying the field strength. The field strength, in the case of a series motor, may be varied by cutting in or out a number of the field turns or by shunting a portion of the current around the field. (Art. **3800.**)

(1727) By reversing the direction of the current either through the armature or through the field. Care must be taken not to reverse both when changing the connections, otherwise the direction of rotation will remain unaltered. (Arts. **3813** and **3814.**)

(1728) A motor may be made to give constant speed regulation by providing the field with a number of series turns which are connected so as to oppose the shunt turns. This secures constant speed, because the field is weakened slightly as the load is applied, thus making the armature

keep up its speed instead of falling off in proportion to the lowering of the counter E. M. F. (Art. **3804.**)

(1729) (*a*) The cross-magnetization distorts the field, shifting it backwards as regards the direction of rotation. (See Fig. 1412.)

(*b*) The shifting of the field has a tendency to move the non-sparking point of the brushes backwards instead of forwards, as in the case of a dynamo. (Art. **3789.**)

(1730) (*a*) The armature back ampere-turns tend to demagnetize the field, as shown in Fig. 1413.

(*b*) In order to keep the speed constant, the field must be weakened slightly as the load is increased. The back ampere-turns help to bring about the necessary reduction in field strength, and consequently aid in keeping the speed constant. (Art. **3790.**)

(1731) By referring to Fig. 1428, we find that the electrical efficiency of a 30 H. P. motor should be about .925, and the efficiency of conversion about .92. The commercial efficiency will therefore be about $.92 \times .925 = .851$. The total input at full load will therefore be $\frac{30 \times 746}{.851} = 26{,}300$ watts, nearly. The total current input will be $\frac{26{,}300}{500} = 52.6$ amperes. Since the field and armature are connected in series, both must be designed for 52.6 amperes. The total electrical energy active in the armature is $\frac{30 \times 746}{.92} =$ 24,326 watts. The E. M. F. which the armature must generate at full load must then be $\frac{24{,}326}{52.6} = 462.5$ volts, nearly. The 37.5 volts difference between this E. M. F. and that of the mains represents the drop through the series field and the armature. The dynamo must then have a current output of 52.6 amperes and generate 462.5 volts when running at a speed of 800 R. P. M. (Arts. **3817** and **3818.**)

(1732) Cast steel is used on account of its good magnetic qualities and mechanical strength. These qualities enable a motor to be made light compared with its output, which is an important consideration in street-railway work. (Art. **3830.**)

(1733) (*a*) If no regulating device were used, the torque acting on the armature would be the same for all loads, because the current is automatically kept constant by the dynamo. The result is that the motor slows down when loaded and races if the load is thrown off.

(*b*) The voltage across the terminals rises as the load is increased. As the output increases the input must also increase, and since the current is constant, it follows that the potential across the terminals must increase. (Arts. **3802** and **3803.**)

(1734) Railway motors are usually of the four-pole type, and by using a two-circuit winding two brushes only are needed. This does away with the necessity of having two brushes on the under side of the commutator, where they would be hard to get at. Two-circuit windings do not become unbalanced electrically like multiple-circuit windings in case the field should become stronger on one side than the other, due to the air-gap becoming shorter on that side. (Art. **3833.**)

(1735) By reversing the connections between the armature and field so as to reverse the direction of the current through either the armature or field, but not through both. Reversing the line connections would not reverse the direction of rotation. (Art. **3815.**)

(1736) To keep down the bad effects of armature reaction and make the motor give good speed regulation. If the field is very strong compared with the armature, the shifting of the brushes from no load to full load will be very slight, and if fairly thick carbon brushes are used, the motor may be operated at all loads, with its brushes fixed in position and without sparking. (Art. **3789.**)

(1737) This will be seen by referring to Fig. 1410. As the conductor moves down in the direction of the arrow, it is readily seen that the E. M. F. induced in it must be directed *up through the paper*. The *current which is causing the wire to move* is flowing down through the paper ; in other words, the E. M. F. is opposed to the current. (Art. **3782.**)

(1738) If the field circuit were broken while the motor was running, the counter E. M. F. would at once become nearly zero, and a very large current would flow through the armature, the effect being much the same as if the armature were thrown across the mains without any resistance at starting. If the field circuit were broken in this way, the armature would be burned out unless protected by fuses, and in any event it is probable that the commutator would be more or less burned. (Art. **3795.**)

(1739) Because when run as motors these machines have their fields excited from the mains, and their field magnetization is therefore practically constant, no matter what the load may be. When they are run as dynamos, they excite their own fields, and every falling off in voltage generated causes a corresponding falling off in the field excitation, thus causing poor regulation for constant potential. (Art. **3816.**)

(1740) In order to secure a good starting effort. If the current is supplied to both at the same instant, a large current will flow through the armature and only a very small current through the high-resistance field, the consequence being that there is no field present for the armature current to react on and the starting torque is very small. (Art. **3807.**)

(1741) By short-circuiting it. Breaking the circuit would cut off the current from the whole circuit, and besides would be a dangerous operation with such high-tension dynamos as are used to operate arc circuits. (Art. **3811.**)

(1742) Because starting rheostats are designed to carry the current only for a short time, and if the current is allowed

to flow through them continuously they will be burned out. (Art. **3806.**)

(1743) (*a*) The electrical efficiency of a motor is the ratio of the total electrical power developed in the armature to the total electrical power supplied from the mains.

(*b*) If the armature resistance is high, the loss in the armature will be large and the electrical efficiency correspondingly low. A high-resistance armature causes bad speed regulation, as the higher the armature resistance the more the counter E. M. F. and speed must drop when the motor is loaded. (Art. **3785.**)

(1744) The commercial efficiency is equal to the product of the electrical efficiency and the efficiency of conversion; hence, if the electrical efficiency is .90 and the efficiency of conversion .85, the commercial efficiency must be .765, or 76.5%. (Art. **3785.**)

(1745) (*a*) The torque is the turning moment or twisting effort exerted on the armature by the reaction of the current in the armature conductors on the field.

(*b*) This is shown by formula **605.** The field excitation N is independent of the speed; hence, for a given current C_m, the torque is given by formula **605,** and the speed does not enter into the equation. (Art. **3787.**)

(1746) (*a*) The principal sources of loss are the same as those in a dynamo; namely, C^2R losses in field and armature, hysteresis and eddy-current losses in the armature core, and frictional losses.

(*b*) The output is limited by the same factors which limit the output of a dynamo, the heating effect in the armature being the principal limiting factor.

(1747) If the motor were short-circuited after being disconnected from the mains, it would be brought to a sudden standstill, because it would act as a dynamo and send current through the switch B. The energy stored up by the momentum of the armature would be rapidly used up,

and the machine would soon stop. It will easily be seen that the motor will act as a dynamo if it is remembered that all the time the motor is working, its E. M. F. is opposed to that of the mains or trying to send a current opposite to that from the mains. The result is that as soon as the main E. M. F. is removed and the switch *B* closed, the motor E. M. F. is able to set up a current, thus making the motor act as a dynamo.

THEORY OF ALTERNATING-CURRENT APPARATUS.

(QUESTIONS 1748–1840.)

(1748) (*a*) The essential difference between an alternating current and a direct current is that the former is constantly reversing the direction of its flow, whereas the latter flows uniformly in the same direction. The alternating current passes through a number of positive and negative values during each complete cycle, and this process is repeated over and over. (Arts. **3838** to **3840.**)

(*b*) The frequency is the number of complete cycles which the current passes through in 1 second. (Art. **3842.**)

(1749) (*a*) Two alternating currents differ in phase, or are out of phase, when both do not pass through similar values at the same instant. If, for instance, one current did not rise to its maximum value till, say, a fraction of a second after the other had reached its maximum, the two currents would be out of phase. (Art. **3854.**)

(*b*) Two currents are in quadrature when one lags behind the other by just $\frac{1}{4}$ of a period; that is, when one current is at its maximum value, the other is just passing through zero. In this case the angle of lag is 90°. (Art. **3860.**)

(*c*) When two currents are in opposition they are directly opposed to each other; i. e., when one is at a positive maximum, the other is at a negative maximum. It follows, therefore, that when two currents are in opposition, the angle of phase difference must be 180°. (Art. **3860.**)

(1750) (*a*) The inductance of a coil or circuit is that quantity which multiplied by the current and 10^8 and divided

by the number of turns in the circuit will give the total number of magnetic lines threading the circuit when the given current is flowing. The inductance L, or coefficient of self-induction, is a measure of the ability which a coil has of setting up lines through itself and, hence, of the ability of the coil to produce a counter E. M. F. of self-induction.

(*b*) Anything which makes it easier for a coil to set up lines around itself increases the inductance L. Surrounding a coil with iron provides a very good path for the magnetic lines, hence the number of lines which the coil sets up is greatly increased, thus making L large. (Arts. **3874** to **3876.**)

(1751) (*a*) A transformer is a device for changing an alternating current at one voltage to a current at another voltage. They are used extensively in connection with lighting and power transmission plants for transforming down from high voltages used on the line. They are also used in connection with some power plants for increasing the pressure supplied by the alternators to that desired for transmission. (Art. **4000.**)

(*b*) No; a transformer can not be used in a direct-current circuit. The E. M. F. induced in the secondary of a transformer is produced by the alternating magnetic flux which passes through it. In order to produce an alternating magnetic flux it is necessary to have an alternating current in the primary, consequently an ordinary static transformer would not operate on a direct-current circuit.

(*c*) A transformer always consists of three essential parts, primary coil, secondary coil, and laminated iron core, the arrangement of these parts being shown in Figs. 1548 to 1551. (Art. **4019.**)

(1752) A three-phase alternating-current system is one in which three alternating currents are employed which differ in phase by $\frac{1}{3}$ of a complete period, or 120°. A three-phase circuit is simply three single-phase circuits, the currents in which differ in phase as described. (Art. **3862.**)

(1753) (*a*) A rotary transformer is a machine for changing alternating current to direct or the reverse.

(*b*) The principle of such machines is explained in Arts. **4050** and **4051**, and the connections for a two-pole two-phase machine would be as shown in Fig. 1577.

(1754) No. In a single-phase rotary transformer with a single armature winding, the ratio of transformation is fixed, the relation being

$$\bar{E} = .707\ V,$$

where V is the direct-current voltage and $\bar{E}$ the alternating voltage. If $\bar{E}$ were 50 volts, V would be $\frac{50}{.707} = 70.7$ volts instead of 100. **(Art. 4051.)**

(1755) (*a*) By the capacity of a line or circuit is meant that property which enables it to hold a certain charge or quantity of electricity. (See Art. **3894.**)

(*b*) The unit of capacity is the farad, and it may be defined as that capacity which will take up a charge of 1 coulomb (1 ampere for 1 second) with a rise of pressure of 1 volt. (Art. **3899.**) The microfarad is the unit usually employed for expressing capacities.

(*c*) If the two capacities are connected in parallel, the combined capacity is simply the sum of the two, or 30 microfarads. **(Art. 3896.)**

(1756) Since the alternator has 12 poles, the angular distance from the center of one pole-piece to the center of the next will be $\frac{360°}{12} = 30°$. The distance from the center of one pole to the center of the next of the same polarity corresponds to 360°, and since the coils are to be displaced 120°, the actual angular displacement will be one-third of the angular distance between the centers of two poles of the same polarity. The coils must then be displaced $\frac{1}{3} \times 30 \times 2$ or 20°, since $30 \times 2 = 60°$ is the distance from the center of one pole to the center of the next of the *same polarity*. **(Art. 3950.)**

(1757) (*a*) The *reactance* of a circuit is that quantity which multiplied by the current gives the E. M. F. necessary to overcome the inductance of the circuit. The reactance is equal to $2\pi n L$, where n is the frequency and L the inductance.

(*b*) Yes; the reactance changes with the frequency, and the E. M. F. of self-induction will be greater the higher the frequency.

(*c*) Reactance is a quantity of the same nature as resistance and is expressed in *ohms*. (Arts. **3884** and **3885.**)

(1758) In a balanced three-phase system, the voltage between each of the three pairs of lines should be the same. If, however, the circuits become unevenly loaded, the three pressures might vary.

(1759) The transformer should be connected as shown in Fig. 106.

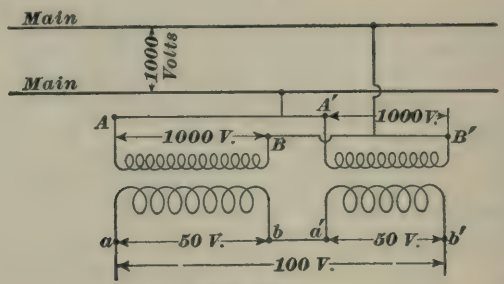

FIG. 106.

Since the primary coils are wound for 1,000 volts and are to be connected across 1,000-volt mains, they must be connected in parallel, A to A' and B to B'. The secondaries each furnish 50 volts, so that if 100-volt lamps are to be operated, the two secondary coils must be connected in series. Therefore, a is connected to the line, b is connected to A', and the other terminal b' is carried to the other line.

(1760) (*a*) The *impedance* of a circuit may be defined as that quantity which multiplied by the current $\overline{C}$ gives the impressed E. M. F. necessary to maintain the cur-

rent in the circuit. The impedance is always equal to $\sqrt{\text{resistance}^2 + \text{reactance}^2}$. In other words, the impedance is the *apparent* resistance which the circuit offers to the flow of the current. (Arts. **3889** and **3890.**)

(*b*) Ohm's law for alternating-current circuits takes the form:

$$\text{Current} = \frac{\text{electromotive force}}{\text{impedance}}. \qquad \text{(Art. 3891.)}$$

(1761) The resistance of the circuit is negligible, hence all the pressure supplied must be used up in overcoming the reactance. The reactance is equal to $2 \pi n L = 2 \times 3.14 \times 60 \times .5 = 188.4$ ohms, and the E. M. F. necessary to overcome the reactance is equal to the product of the reactance and the current. The E. M. F. $\bar{E}$, therefore, which must be supplied by the alternator is

$\bar{E} = 2 \pi n L \bar{C} = 188.4 \times 5 = 942.0$ volts. (Art. **3886.**)

(1762) (*a*) In the **Y** winding, one end of each of the windings is brought to a common connection, and the other ends are carried to the three collector rings. In the Δ method, the three windings are connected up into a closed circuit and the collector rings are attached to the points where the different phases join. (Arts. **3986** and **3987.**)

(*b*) No; the output is not affected by changing from one scheme of connection to the other, because if the current output is increased by changing from **Y** to Δ, the voltage is decreased in the same ratio, and, hence, the watts output is not altered. (Art. **3995.**)

(1763) (*a*) The effective value of an alternating current is the expression for that value of the current which would produce the same power or heating effect in a circuit as a continuous current of the same amount. (Art. **3866.**)

(*b*) The effective value is equal to .707 times the maximum value, or is equal to the maximum value divided by $\sqrt{2}$; hence, if 150 amperes is the maximum which the current reaches, the effective value of the same current would

be $\frac{150}{\sqrt{2}} = 106$ amperes. It follows, therefore, that if a continuous current of 106 amperes were sent through the resistance, the same amount of heat would be produced as by an alternating current having a maximum value of 150 amperes. (Art. **3867.**)

(1764) Each transformer has 400 turns on its primary and 50 on its secondary, therefore the ratio of transformation is 8. Since the coils are connected up Δ to the primary, the pressure across each primary coil must be 1,000 volts; consequently, the pressure induced in each secondary will be $\frac{1000}{8} = 125$ volts. In other words, the voltage per phase in the secondary is 125, and since the three phases are connected up **Y**, the voltage between the secondary lines will be $125 \times \sqrt{3} = 125 \times 1.732 = 216.5$ volts. Ans.

(Art. **4026.**)

(1765) (*a*) The efficiency of a transformer is the ratio of the useful watts obtained from the secondary to the watts supplied to the primary; or, efficiency $= \frac{\text{output}}{\text{input}}$. The efficiency is always less than 1.

(*b*) 2.2 H. P. $= 2.2 \times 746 = 1{,}641$ watts $=$ useful output. Input $= 1{,}800$ watts; hence, efficiency $= \frac{1{,}641}{1{,}800} = .912$, or the transformer gives out 91.2% of the energy supplied to it. (Art. **4000.**)

(1766) In order to induce an E. M. F. such as that in the secondary of a transformer, all that it is necessary to have is a coil so arranged as to thread a varying magnetic flux. If the flux were fixed, as it is in dynamos, it would be necessary to move the coils in order to set up an E. M. F. In a transformer, however, the flux is made to vary by the alternating current in the primary, hence it is not necessary to move the secondary in order to make it cut lines of force. In other words, the magnetic flux is constantly changing,

and for this reason it is not necessary that any of the transformer coils should move.

(1767) (*a*) If an alternating-current circuit contains inductance, the current flowing will not vary, or rise up and down, in unison with the E. M. F., but will lag behind by a certain fraction of a period. The part of a complete cycle by which the current lags is usually denoted by an angle which represents the amount of lag. For example, one complete cycle is equivalent to 360°; hence, if the angle of lag were said to be 90°, it would mean that the current came to its maximum just $\frac{1}{4}$ of a period behind the E. M. F. If the angle of lag were 60°, the current would be behind the E. M. F. by $\frac{1}{6}$ of a period. (Arts. **3854** and **3855.**)

(*b*) The angle of lag may be obtained from the expression

$$\tan \Phi = \frac{2 \pi n L}{R},$$

as explained in Art. **3892,** so that if the frequency, inductance, and resistance for any given circuit are known, the angle of lag can at once be estimated. No commercial instrument has yet been made which will measure the angle of lag, so it always has to be calculated.

(1768) (*a*) If the frequency is 60 and the inductance is .2 henry, the reactance $= 2 \pi n L = 2 \times 3.14 \times 60 \times .2 =$ 75.36 ohms.

The resistance = 12 ohms; hence,

$$\text{impedance} = \sqrt{R^2 + (2 \pi n L)^2} = \sqrt{144 + 5{,}679} = 76.3 \text{ ohms.}$$

$$\text{Current} = \frac{\text{E. M. F.}}{\text{impedance}} = \frac{500}{76.3} = 6.55 \text{ amperes.} \quad \text{Ans.}$$

(*b*) Tangent of angle of lag $= \frac{2 \pi n L}{R}$.

$$\text{Tan } \Phi = \frac{75.3}{12} = 6.28 \text{ and } \Phi = 80° 57'. \quad \text{Ans.}$$

(1769) (*a*) If the resistance had been negligible, we would have

$$\text{current} = \frac{\text{E. M. F.}}{\sqrt{0^2 + (2\pi n L)^2}} \text{ or current} = \frac{500}{75.36} = 6.63 \text{ amperes}$$

Ans

(*b*) $\text{Tan } \Phi = \dfrac{2\pi n L}{R} = \dfrac{75.3}{0} = \infty;$

hence, Φ = angle of lag = 90°. Ans.

In this problem and the preceding one the reactance is large compared with the resistance; hence, removing the resistance does not affect the value of the current very much.

(1770) (*a*) An alternator may be compounded by winding the field spools with a second set of coils which are connected in series with the line. The current in these coils therefore increases with the load and thus strengthens the field-magnets. The initial magnetization is supplied by means of a set of coils excited from a separate direct-current dynamo. (Art. **3966.**)

(*b*) The use and action of the rectifier is explained in Art. **3968.** Its action is to keep changing the connections between the compound field winding and the circuit so that the current in the field coils will always be in the same direction.

(*c*) The number of sections in the rectifier must be the same as the number of poles on the alternator.

(1771) (*a*) A non-inductive resistance is one which is incapable of setting up lines of force around itself, and hence has no inductance.

(*b*) No; the current is in phase with the E. M. F. when flowing in a non-inductive resistance. A non-inductive resistance simply retards the flow of the current in the same way that a resistance prevents the flow of a continuous current. If the resistance has no self-induction, there can be no induced E. M. F. set up to prevent the current rising and falling in step with the E. M. F.; hence, in such a resistance the current does not lag behind the E. M. F.

(*c*) Water resistances and incandescent lamps are practically non-inductive. (Art. **3872.**)

(1772) (*a*) Iron cores are always used in transformers to provide a path of low magnetic resistance for the lines threading the primary and secondary coils.

(*b*) If no core were used, the lines would have to be set up through the air, which offers a high resistance to the flow of the lines; consequently it would require a very large magnetizing current to set up the flux. The magnetizing current would, therefore, be enormously increased by removing the core and the transformer would be practically inoperative. (Art. **4003.**)

(1773) Every time the armature of an alternator turns through an angle equal to that from one pole to the next of the same polarity, the E. M. F. passes through one complete cycle. Hence, in one revolution the E. M. F. will pass through as many cycles as there are pairs of poles, and if $s =$ rev. per second and $P =$ number of poles, we have

$$\text{frequency} = \frac{P}{2} s. \quad \text{(Art. 3950.)}$$

(1774) The current coil of a wattmeter is of very low resistance, and if connected across the mains instead of in series with them, a very large current would result, which would be almost certain to burn out the instrument unless protected by fuses. (Art. **3945.**)

(1775) In such a winding the E. M. F.'s induced in the coils under the N poles will be opposite in direction to the E. M. F.'s in the coils under the S poles. In order, then, that the E. M. F.'s may add up properly, the connections of every other coil must be reversed, as shown in Figs. 1512 and 1513. The student should read Art. **3954** in connection with this question. Fig. 1513 is the winding diagram for an armature of this type, and by following the arrowheads it will easily be seen that the connections of every other coil must be reversed.

(1776) The capacity $J = 10$ microfarads $= .000010$ farad. Frequency $= 125$; resistance $= 100$ ohms; capacity reactance $= \frac{1}{2\pi n J} = \frac{1{,}000{,}000}{2 \times 3.14 \times 125 \times 10} = 127.4$ ohms, nearly.

$$\text{Impedance} = \sqrt{R^2 + \left(\frac{1}{2\pi n J}\right)^2} = \sqrt{100^2 + (127.4)^2} =$$

$$\sqrt{26{,}230} = 162, \text{ nearly.}$$

$$C = \frac{E}{\text{impedance}} = \frac{1{,}000}{162} = 6.17 \text{ amperes.} \quad \text{Ans.}$$

In substituting in the expression $\frac{1}{2\pi n J}$, care must be taken to have J expressed in farads, not microfarads.

(1777) (*a*) If the transformers were connected in the manner indicated, they would be burned out, unless protected by fuses. The figure shows the two secondaries connected up in series into a closed circuit, so that the combined E. M. F. of the two secondaries would set up a large current and thus burn out the coils.

(*b*) The trouble could be remedied by connecting the transformers as shown in Fig. 107, *a* being connected to *a'*, and *b* to *b'*.

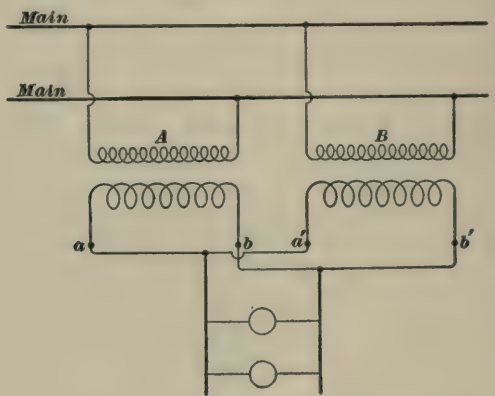

FIG. 107.

(1778) (*a*) It has been shown in Art. **3862**, and it can also be seen by referring to Fig. 1448, that, in a three-phase

system, when the current in one line is at its maximum value, the currents in the other two lines are one-half as great and in the opposite direction. Hence, if the current in one line is at its maximum value of 150 amperes, the currents in the other two lines at the same instant must each be 75 amperes.

(*b*) The current of 75 amperes in these two lines will be flowing in a direction opposite to that of the current in the first-mentioned line.

(1779) (*a*) If the alternator runs at a speed of 300 R. P. M., the rev. per second = 5. There are 20 poles, or 10 pairs of poles; hence, by formula **646,**

$$\text{frequency} = p \times s = 10 \times 5 = 50 \text{ cycles per second.}$$

(*b*) Since the frequency is 50, it follows that the current from such a machine will pass through one complete cycle of values in $\frac{1}{50}$ second.

(1780) If the inductance is very large as compared with the resistance, nearly all the E. M. F. required to set up the current through the circuit will be used up in overcoming the inductance, and will, therefore, be almost 90° ahead of the current in phase. Consequently, in such a circuit the current will lag very nearly 90° behind the E. M. F. (Art. **3922.**)

(1781) If the armature of an alternator has high self-induction, a large part of the E. M. F. generated may be used up in forcing the current through the armature itself, leaving only a small terminal E. M. F. available at the collector rings. If the inductance were very high, there might be such a falling off in terminal pressure that the maximum output which could be taken from the machine might be less than that which the armature could safely deliver without overheating, and the machine would not deliver the power which it was really capable of supplying. (Art. **3930.**)

(1782) (*a*) The resistance of the circuit is considered to be zero; hence the impedance $\sqrt{R^2 + \left(\frac{1}{2\pi n J}\right)^2}$ becomes $\frac{1}{2\pi n J}$, and

$$\bar{C} = \frac{\bar{E}}{\frac{1}{2\pi n J}} = 2\pi n J \bar{E}.$$

Frequency $= n = 125$ cycles per sec.

Current $= \bar{C} = 2$ amperes.

$$J = \frac{\bar{C}}{2\pi n \bar{E}} \text{ farads} = \frac{2 \times 1{,}000{,}000}{2 \times 3.14 \times 125 \times 1{,}000} \text{ microfarads} =$$

2.54 microfarads. Ans.

(*b*) Since there is no resistance in the circuit, all the E. M. F. is used in overcoming that of the condenser, and the current is at right angles to the E. M. F. and 90° ahead of it. The current is therefore wattless, and no energy is being expended in the circuit. The number of watts expended is therefore zero.

(1783) (*a*) A synchronous motor is one which runs in step or synchronism with the alternator driving it. (Art. **4033.**)

(*b*) A single-phase synchronous motor will not start up of its own accord because, when it is standing still and a rapidly reversing current is sent through its armature, there is an effort exerted tending to make it turn first in one direction and then in the other, the consequence being that the armature does not start up.

(*c*) If the motor be run up to speed, the coils move past the poles just as the current in the coils reverses; consequently the torque acting on the armature is always in the same direction and the motor continues running. (Art. **4034.**)

(1784) The counter E. M. F. set up in a coil through which a flux N is made to vary is given by formula **617,** Art. **3882.**

$$\bar{E} = \frac{4.44\,N\,T\,n}{10^8}.$$

The magnetic flux N is in this case 200,000 lines; the frequency $n = 125$; the required counter E. M. F. $\bar{E} =$ 50 volts.

Hence we have

$$T = \frac{\bar{E} \times 10^8}{4.44\,n\,N} = \frac{50 \times 10^8}{4.44 \times 125 \times 200{,}000} = 45 \text{ turns.} \quad \text{Ans.}$$

(1785) (*a*) The effect of resistance in the primary and secondary coils of a transformer is to cause a loss in them due to the heating, thus making the efficiency lower than it would otherwise be.

(*b*) The resistance in the coils also causes a falling off in the secondary voltage when the transformer is loaded, thereby making the regulation poor. (Art. **4014.**)

(1786) (*a*) The henry is the unit of self-induction, and a coil of T turns would have an inductance of 1 henry if it set up a flux of $\frac{10^8}{T}$ lines through itself when the current in it was 1 ampere. Or it might be defined as that quantity which multiplied by the current and 10^8 gives the product of the flux and the number of turns.

(*b*) For example, if a coil of 50 turns had an inductance of 5 henrys it would mean that, with a current of 1 ampere flowing through the coil, there would be 10,000,000 lines passing through the center of the coil, since

$$\text{flux} \times 50 = 5 \times \text{current} \times 10^8,$$

or $$\text{flux} = \frac{5 \times 1 \times 100{,}000{,}000}{50} = 10{,}000{,}000 \text{ lines.}$$

(Art. **3875.**)

(1787) If the armature has a high inductance, the greater part of the E. M. F. generated will be used up in

forcing the current through the armature itself. The result is that the E. M. F. is not able to set up a very large current when the machine is short-circuited, and, consequently, no damage results, as would be the case with a direct-current machine. (Art. **3963.**)

(1788) The great advantage of polyphase over single-phase synchronous motors is that the former will start of their own accord and run up to synchronism, whereas the latter have to be started up from some outside source of power. (Art. **4034.**)

(1789) The chief advantage which induction motors possess over synchronous motors is that they will start with a strong torque, and will therefore start up under load, whereas synchronous motors will not start on anything but a light load. Induction motors also have the advantage of not requiring, under ordinary circumstances, any brushes or collector rings. (Art. **4038.**)

(1790) The essential parts of a polyphase induction motor are the field, or primary, into which currents from the line are led, and the armature, or secondary, in which currents are induced by the moving field set up by the primary. (Art. **4039.**) The method of constructing and winding the field and armature is such as that described in Arts. **4040** to **4042.**

(1791) (*a*) Line pressure or impressed E. M. F. = 1,000 volts. Frequency = 60 cycles per sec. Self-induction = .3 henry. Resistance = 150 ohms. Capacity = 15 microfarads = .000015 farad.

In such a circuit as this the current will of course be equal to $\frac{\text{impressed E. M. F.}}{\text{impedance}}$ This circuit contains resistance, self-induction, and capacity; hence, the

$$\text{impedance} = \sqrt{R^2 + \left(2\pi n L - \frac{1}{2\pi n f}\right)^2}.$$ See Art. **3910.**

The reactance due to self-induction $= 2\pi n L = 2\pi \times 60 \times .3 = 113$ ohms.

The reactance due to capacity $=$

$$\frac{1}{2\pi n f} = \frac{1{,}000{,}000}{2 \times 3.14 \times 60 \times 15} = 176.9 \text{ ohms.}$$

$$\left(2\pi n L - \frac{1}{2\pi n f}\right) = 113 - 176.9 = -63.9 \text{ ohms.}$$

Then, the impedance $=$

$$\sqrt{150^2 + (-63.9)^2} = \sqrt{22{,}500 + 4{,}083} = \sqrt{26{,}583} = 163 \text{ ohms.}$$

Hence, current $= \frac{1{,}000}{163} = 6.13$ amperes. Ans.

(*b*) Since the capacity reactance is greater than the reactance due to self-induction, it follows that the component of the E. M. F. at right angles to the current is 90° *behind* the current, as shown in Fig. 108. The impressed

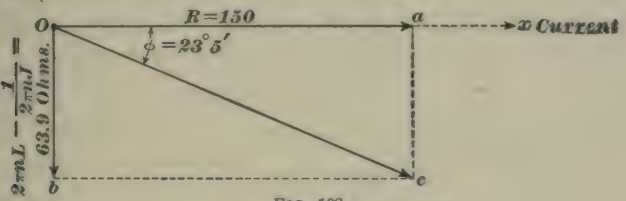

FIG. 108.

E. M. F. is therefore in the direction of $O\,c$, and the angle by which the current leads the E. M. F. is such that

$$\tan \Phi = \frac{2\pi n L - \frac{1}{2\pi n f}}{R} = \frac{63.9}{150} = .426,$$

and $\Phi = 23° 5'$. Ans.

(*c*) The watts expended $= C E \cos \Phi = 1{,}000 \times 6.13 \times .919 = 5{,}633$ watts. Ans.

(1792) (*a*) The power factor is that quantity by which the apparent watts expended in an alternating-current circuit must be multiplied to give the true watts. It is equal numerically to the cosine of the angle of lag between

the current and E. M. F., and the more nearly the current and E. M. F. are in phase with each other the larger is the value of the power factor. (Art. **3926.**)

(*b*) When the current and E. M. F. are exactly in phase, the power factor has its greatest value, $\Phi = 0$ and $\cos \Phi = 1$. In this case the apparent watts, or volts × amperes, are equal to the true watts. (Art. **3927.**)

(1793) (*a*) The best method of measuring power in an alternating-current circuit is by the use of a wattmeter, the most reliable type of which is the Siemens dynamometer.

(*b*) A description of this instrument will be found by referring to Art. **3944.**

(*c*) A wattmeter should be connected with its current coil in series with the circuit in which the power is to be

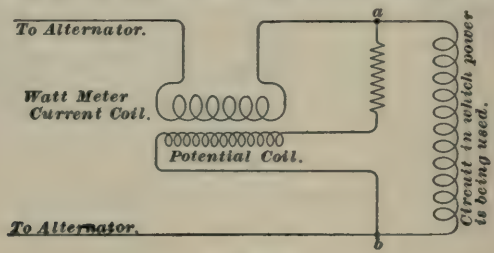

FIG. 109.

measured. The fine wire coil should be connected across the terminals of the circuit *a b*, as shown in Fig. 109.

(1794) If a transformer were connected with its primary across continuous-current mains, no counter E. M. F. would be set up in the primary, because the current would not be changing. The primary would, therefore, not be able to choke back the current, and a rush of current would take place which would be limited only by the resistance of the primary coil; consequently a current would flow which would probably be great enough to burn out the coil unless it were protected by fuses.

(1795) The armature of an induction motor does not revolve quite so fast as the magnetic field set up by the currents in the field winding; consequently there is a cutting

of magnetic lines by the armature conductors, which sets up an electromotive force in them. Since the armature coils, or conductors, are connected up into closed circuits, this induced E. M. F. sets up currents which react on the field and so produce the motion. (Art. **4042.**)

(1796) If the secondary were to become short-circuited, it would be equivalent to connecting it to a circuit of very low resistance. The consequence would be a heavy rush of current through the secondary coil, which would allow a correspondingly large current to flow in the primary.

(1797) The speed of an induction motor may be regulated by inserting an adjustable resistance in series with the secondary winding. The resistance limits the amount of current which flows in the secondary coils, and the torque may be increased or diminished, as desired. This arrangement necessitates the use of collector rings and brushes, as a resistance for regulating purposes to be used continually could not very well be placed within the armature itself on account of the heat which it would develop. (Art. **4044.**)

(1798) (*a*) Magnetic leakage is caused by the flux which the secondary tends to set up being opposed to that set up by the primary. The two opposing magnetic fluxes cause lines to leak between the coils, as shown in Art. **4015,** and this leakage effect increases as the load on the secondary increases.

(*b*) The effect of magnetic leakage is to cause a drop in the secondary voltage, because all the magnetic lines do not get through the secondary.

(*c*) The effect of magnetic leakage may be largely reduced by so arranging the coils that there is little chance for magnetic lines to leak between them. This may be accomplished by placing the coils one over the other, or by winding the primary and secondary in sections and interleaving them. (Art. **4016.**)

(1799) (*a*) The current in the armature *increases* as the load is applied in very much the same way that the

current in the armature of an ordinary shunt-wound dynamo increases with increase of load.

(*b*) The change in the secondary current is brought about by the slight slowing down in speed of the armature. This slowing down causes the relative motion between the revolving armature and field to become greater, thus making the conductors cut more lines of force per second and increasing the voltage induced in them. The increase of voltage is accompanied by a corresponding increase of current, which causes the motor to develop a greater torque and thus adjust itself to the new condition of load. (Art. **4042.**)

(1800) The winding suitable for the field of a three-phase induction motor would be similar to that used on a three-phase alternator armature. The windings used on polyphase induction-motor fields are usually distributed windings consisting of 2 or 3 coils per pole per phase. (Art. **4045.**)

(1801) If the motor be loaded too heavily, the armature will be retarded through so great an angle that the motor will slip out of step with the alternator driving it, and will rapidly come to a standstill. (Art. **4035.**)

(1802) (*a*) If 25 amperes is the effective value, the maximum value will be $25\sqrt{2}$, because the effective value $= \frac{\text{max. value}}{\sqrt{2}}$.

$$25\sqrt{2} = 35.36 \text{ amperes.} \quad \text{Ans.}$$

(*b*) Effective value = 1.11 average value; hence, average value $= \frac{25}{1.11} = 22.52$ amperes. (Art. **3867.**)

(1803) If the effective voltage of the machine is 2,000, the maximum pressure which the insulation of the armature is subjected to will be the maximum pressure corresponding to 2,000 effective volts; that is, $2{,}000\sqrt{2}$ or $2{,}000 \times 1.414 =$ 2,828 volts. (Art. **3867.**)

(1804) (*a*) If the alternator has 10 poles, or 5 pairs of poles, and runs at the rate of 1,500 R. P. M., its frequency will be, by formula **646,** $n = 5 \times \frac{1500}{60} = 125$ cycles per second. (Art. **3956.**) Ans.

(*b*) The voltage between the collector rings at no load is given by formula **617,**

$$\overline{E} = \frac{4.44\, N\, T\, n}{10^8}.$$

N is the magnetic flux from one pole and equals 900,000; T is the number of *turns* connected in series and is equal to $\frac{800}{2} = 400$. Hence,

$$\overline{E} = \frac{4.44 \times 900{,}000 \times 125 \times 400}{100{,}000{,}000} = 1{,}998 \text{ volts.}$$

(Art. **3960.**) Ans.

(1805) If an E. M. F. E be applied to a condenser of J farads, the quantity of charge which the condenser will take up will be $Q = J\, E$ coulombs. This follows from the definition of a farad. In this case, since $J = 20$ microfarads, and $E = 1{,}000$ volts, $Q = .000020 \times 1{,}000 = .020$ coulomb. But coulombs = amperes × seconds, or $Q = C\, t$, where C is the average current.

Then $Q = .1 \times t$, and $t = \frac{.020}{.1} = .2$ second.

The current of $\frac{1}{10}$ of an ampere would, therefore, have to flow for $\frac{1}{5}$ second to charge the condenser up to a potential of 1,000 volts. (Arts. **3900** and **3905.**)

(1806) If the motor has 6 poles, or 3 pairs of poles, we must have, by formula **647,** the relation $s = \frac{n}{p} = \frac{60}{3} =$ 20 rev. per second. The speed is therefore 20 revolutions per second, or 1,200 rev. per minute, when the machine is not loaded, because at no load an induction motor should run almost exactly in synchronism with the alternator driving it.

(1807) The current flowing through a synchronous motor armature varies in proportion to the load which the machine is carrying by virtue of the change in phase relation between the counter E. M. F. of the motor and the E. M. F. supplied by the alternator driving it. When a load is thrown on the machine, the angular position of the motor armature is displaced a little from the angular position occupied at the same instant by the armature of the alternator. The E. M. F. of the motor, therefore, lags a little behind that of the alternator and thus allows a current to pass between the two machines. (Art. **4035.**)

(1808) The general effect of inductance is to increase the size of wire necessary to transmit a given amount of power beyond what would be required if no inductance were present. In other words, it requires some E. M. F. to overcome the inductance as well as the resistance, and if the total amount of drop in the line is fixed, it is evident that the wire has to be made just so much larger to keep the total drop within the specified limits. If the load on the line is inductive, the current for a given amount of power is also increased, and if the drop were to be kept the same, the wire would have to be larger than would be necessary if the load were non-inductive. (Art. **3934.**)

(1809) (*a*) The reactance due to self-induction is greater than that due to capacity; hence the effect of the self-induction will predominate and the current will lag behind the E. M. F.

(*b*) The resultant reactance is $20 - 15 = 5$ ohms, and the resistance being 10 ohms, the tangent of the angle of lag is $\frac{5}{10} = .5$;

or, $\tan \Phi = .5$,

and the angle of lag $\Phi = 26° 34'$.

(1810) The field of an alternator which is not provided with compound coils may be adjusted by having an adjustable rheostat either in the field circuit of the alternator or in the shunt field circuit of the exciter, or in both, as shown

in Fig. 1521. It is customary to have both rheostats, because a number of alternators can then be excited from one exciter and their field currents adjusted independently. This could not be done if the rheostat in the alternator field circuit were not used. By varying these two rheostats the strength of field and, hence, the voltage of the alternator may be easily adjusted. (Art. **3966.**)

(1811) If the secondary of the motor were of high resistance, there would have to be a considerable E. M. F. induced in the conductors to set up the current to produce the required torque. This means that a motor with such an armature would require a large slip, and there would consequently be a considerable falling off in speed from no load to full load. Such a motor, therefore, would give poor speed regulation. (Art. **4043.**)

(1812) (*a*) The machine generates 1,100 volts with a Y-connected winding; hence the voltage generated in each phase must be, by formula **650,**

$$\bar{\varepsilon} = \frac{1{,}100}{\sqrt{3}} = 635 \text{ volts.}$$

Consequently, if the armature be connected up Δ, the line voltage will be 635, because in this case the line voltage is the same as the voltage per phase. (Arts. **3992** and **3993.**) Ans.

(*b*) If the current output were 50 amperes with the Y winding, it means that the wire on the armature must be capable of carrying 50 amperes, because the current per phase is the same as the line current in such a winding. (Art. **3990.**) If the winding were connected Δ, the allowable line current would therefore be, by formula **654,**

$$\bar{C} = 50 \times \sqrt{3} = 86.6 \text{ amp.} \quad \text{Ans.}$$

(1813) (*a*) Eddy-current losses act in the same way as an additional current in the secondary; that is, they simply increase the primary current over and above what it should be for the given secondary current. This means that the eddy-current loss has to be made up for by taking additional

current from the line, thus lowering the efficiency of the transformer.

(*b*) Eddy-current losses may be reduced by laminating the core or building it up out of thin sheets of iron lightly insulated from each other. This breaks up the circuits in which the eddy currents tend to flow, and so prevents their generation. (Art. **4017.**)

(1814) The automatic adjustment of a transformer to different conditions of load is brought about by the reaction of the primary and secondary coils. The magnetism produced by the secondary is opposed to that of the primary, and, consequently, the resulting magnetizing effect is zero. The result is that the magnetic flux remains practically constant at the value which is set up by the magnetizing current. (Art. **4009.**) Since the flux remains constant, the E. M. F. induced in the secondary must also be constant. (Art. **4010.**)

(1815) The current which a transformer takes from the line on open circuits has to set up the magnetic flux N and supply the energy necessary to make up for the core losses. The no-load current consists, therefore, of two components, one of which represents the magnetizing current (i. e., the current which is necessary to set up the flux N against the reluctance of the magnetic circuit) at right angles to the E. M. F., and therefore wattless, while the other component represents the current necessary to make up for the core losses, and is in phase with the E. M. F. The no-load current is the resultant of these components. If the magnetic reluctance and core losses are large, the no-load current will be large, and if the transformer is so designed as to make these quantities as small as possible without interfering with the design in other respects, the no-load current will be reduced to the smallest practical amount for any given case. (Art. **4011** and following.)

(1816) It is necessary that an induction motor should exert a strong torque when starting up under load, and in

order to do this, the reaction between the armature currents and the magnetic field must be strong. If no resistance were inserted in the secondary circuit, the armature currents would be very heavy and a strong torque would result, were it not for the fact that these strong secondary currents react on the primary so as to greatly weaken the field. In order, then, to have a strong starting torque, these secondary currents must be limited to a certain extent, and this is done by inserting a starting resistance. It would not do to leave this resistance in while the motor was running, as it would make the speed regulation bad and also lower the efficiency. The resistance is therefore arranged so that it may be cut out when the motor gets up to speed. (Art. **4043.**)

(1817) (*a*) The slip of an induction motor is the difference in speed between the revolving armature and the revolving field set up by the primary windings. It is usually expressed in per cent. of the synchronous speed. (Art. **4042.**)

(*b*) If the armature were standing still, the slip would be the same as the speed of the revolving field or 100%.

(1818) If $\overline{E}$ be the effective alternating voltage and V the direct-current voltage, we have, for a three-phase rotary transformer,

$$\overline{E} = .612\ V;$$

hence, $\overline{E} = .612 \times 220 = 134.6$ volts. (Art. **4053.**)

(1819) The E. M. F. *to overcome* self-induction is the equal and opposite of the E. M. F. of self-induction. The E. M. F. of self-induction is at right angles to the magnetization because it is equal to the rate of change of the magnetization. The magnetism is *changing* most rapidly when it is passing through its zero value; hence the induced E. M. F. is a maximum when the magnetization and current are zero. When the current is flowing *into the coil*, or in the *positive direction*, the *induced* E. M. F. is opposed to the current, or is in the negative direction; hence, the curves representing current and induced E. M. F. are as shown in

Fig. 1460. The E. M. F. to overcome self-induction is the equal and opposite of the induced E. M. F., and is therefore 90° ahead of the current in phase. (Arts. **3879** and **3880.**)

(1820) (*a*) The hysteresis loss, like the eddy-current loss, causes an increase in the primary current over and above what would be called for by the current which the secondary is delivering. The hysteresis loss therefore lowers the efficiency, and also causes an increase in the power which the transformer consumes both on open circuit and when loaded.

(*b*) The hysteresis loss is caused by a kind of molecular friction in the iron, which causes a resistance to changes of magnetism. This resistance per unit volume of iron depends upon the quality of the iron and the density at which it is worked. It follows, therefore, that in order to keep down the hysteresis loss, iron of very good quality should be used and the core should be worked at a low magnetic density.

(*c*) In most lighting systems the line pressure is maintained all day, so that the iron losses are continual, whereas the copper loss is limited to the time that current is used. On this account iron losses should be kept very small. (Arts. **4017** and **4018.**)

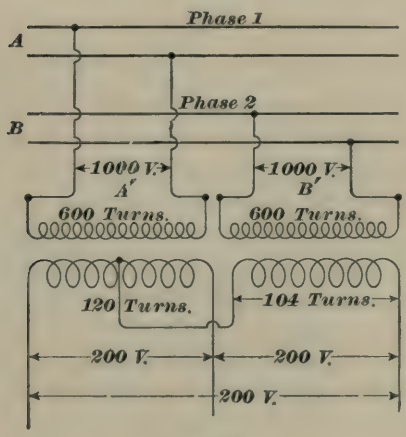

FIG. 110.

(1821) (*a*) The two transformers would be connected up as shown in Fig. 110.

(*b*) The primaries of the two transformers are each wound with 600 turns of wire, and are connected across the two-phase mains *A* and *B*, as shown. The secondary of *A'* must furnish 200 volts, and since the primary voltage is 1,000, the secondary turns must be $\frac{200}{1000} \times 600 = 120$.

The secondary turns on B' are $\frac{\sqrt{3}}{2}$ times as many as on A', as shown in Art. **4027**; hence, the turns on the secondary of $B' = \frac{\sqrt{3}}{2} \times 120 = 104$ turns, nearly.

(1822) Open slots are used because they do not give rise to such a high inductance in the armature and field. (Art. **4046.**)

(1823) (*a*) The secondary of the transformer has to generate 30 volts when the primary pressure is 2,000 volts. There are 800 turns on the primary; hence there must be $\frac{30}{2{,}000} \times 800 = 12$ turns on the secondary.

(*b*) The cross-section of the wire must be $20 \times 1{,}000$, or 20,000 circular mils, which is about the cross-section of a No. 7 B. & S. wire.

(*c*) The output of the transformer would be 20 amperes at 30 volts = 600 watts. (Art. **4028.**)

(1824) The pressure between each of the pairs of alternating-current mains is 400 volts. Hence the direct-current pressure will be $V = \frac{400}{.707} = 565.7$ volts. (Art. **4052.**)

(1825) (*a*) The pressure furnished by the main winding will be, by formula **617,**

$$E = \frac{4.44\,N\,T\,n}{10^8}.$$

450 R. P. M. $= \frac{450}{60}$ rev. per sec. There are 8 pairs of poles; hence, by formula **646,** the frequency $n = 8 \times \frac{450}{60} = 60$ cycles per sec. There are 800 conductors, or 400 turns, on the armature, and $N = 1{,}250{,}000$.

Then, $E = \frac{4.44 \times 1{,}250{,}000 \times 400 \times 60}{100{,}000{,}000} = 1{,}332$ volts. (Art. **3960.**) Ans.

(*b*) The voltage generated in the teazer winding will be $\frac{1}{4}$ of that in the main winding, since the winding has $\frac{1}{4}$ the

number of turns on the main winding. The teazer voltage will therefore be $\frac{1332}{4} = 333$ volts. The pressure between the middle and outer rings will be the resultant of $\frac{1332}{2}$ volts and 333 volts at right angles to the main voltage. Hence, the voltage between outer and middle rings is

$$\sqrt{\left(\frac{1,332}{2}\right)^2 + 333^2} = 745 \text{ volts, nearly.} \quad \text{(Art. 3998.)}$$

(1826.) (*a*) The primary coil must generate a back pressure of 2,000 volts; hence we have

$$\bar{E} = \frac{4.44\,N\,T\,n}{10^8},$$

$$2{,}000 = \frac{4.44 \times 500{,}000 \times T \times 125}{10^8},$$

or $$T = \frac{10^8 \times 2{,}000}{4.44 \times 500{,}000 \times 125} = 720 \text{ turns, nearly.} \quad \text{Ans.}$$

(*b*) If the secondary voltage is to be 100, the ratio of transformation is 20, and the secondary turns, $\frac{720}{20} = 36$. (Art. **4003.**) Ans.

(1827) (*a*) 20 H. P. delivered at the pulley is equal to 20×746 watts. The watts delivered are, then, 14,920. The commercial efficiency is .8; hence the *actual* watts which must be delivered to the motor will be $\frac{14{,}920}{.8} = 18{,}650$. The power factor is .85; hence the apparent watts which will be delivered will be $\frac{18{,}650}{.85} = 21{,}941$. This motor is supplied from three-phase mains at a voltage of 220; hence, since watts $= \bar{C}\,\bar{E}\,\sqrt{3}$ (Art. **3994**), we have

$$21{,}941 = \bar{C}\,220 \times \sqrt{3},$$

or $$\bar{C} = \frac{21{,}941}{220 \times \sqrt{3}} = 57.6 \text{ amp.} \quad \text{Ans.}$$

(*b*) If the field windings are connected up △, the current in them will be, by formula **655,**

$$\frac{\text{line current}}{\sqrt{3}} = \frac{57.6}{\sqrt{3}} = 33.2 \text{ amp.} \quad \text{Ans.}$$

(1828) In order to obtain a pressure of 10,000 volts at the end of the line, the pressure at the station would have to be 11,000 volts when the full load was being transmitted. 200 H. P. = 200 × 746 = 149,200 watts.

Hence, $$\text{current} = \frac{149,200}{10,000} = 14.92 \text{ amperes.}$$

$$\text{Current} \times \text{impedance} = 1,000 \text{ volts,}$$

or $$\text{impedance} = \frac{1,000}{14.92}.$$

The line is 25 miles long, and the total length of wire is 50 miles; then,

$$\text{Impedance per mile} = \frac{1,000}{14.92} \times \frac{1}{50} = 1.34 \text{ ohms.}$$

The wires are strung 18 inches apart, and the frequency is 60. By referring to Table 114 (Art. **3934**) it is seen that the impedance per mile of No. 3 B. & S., 18 in. apart, frequency 60, is 1.223 ohms per mile, while that of No. 4 is 1.467. No. 3 wire would, therefore, probably be used. Ans.

(1829) (*a*) No; the magnetic flux remains practically constant, no matter what the load is, because the total magnetizing effect of the coils does not change with the load. (Art. **4009.**)

(*b*) The core losses also remain practically constant, as these depend upon the intensity of the magnetic flux, and must be constant as long as the flux remains the same. (Art. **4017.**)

(1830) If the current in the system is lagging, the power factor is less than 1, and the volt-amperes, or apparent watts, are greater than the true watts. For a given amount of power supplied and a fixed line voltage, it follows that the current required will be greater than if the power

factor were 1. The alternator *current* output is therefore reached before the machine is delivering the actual *power* or *watts* which it is capable of, and the useful output is thereby limited. (Art. **3930.**)

(1831) The Cardew voltmeter, or, in fact, any alternating-current voltmeter, may be used to measure the current in a circuit by causing the current to flow through a known non-inductive resistance and measuring the drop through the resistance by means of the voltmeter. Since the current and E. M. F. in the non-inductive resistance are in phase, $\bar{C} = \frac{\bar{E}}{R}$, where $\bar{C}$ is the current, and $\bar{E}$ is the drop through the resistance R. That is, $\bar{E}$ is the reading given by the voltmeter. (Art. **3937.**)

(1832) Draw a line $O\,b$, Fig. 111, to scale, representing the current of 50 amperes. Since this is to be resolved into two equal currents differing 60° in phase, draw from O the lines $O\,c$ and $O\,d$, making angles of 30° with $O\,b$. From b draw the lines $b\,e$ and $b\,f$ parallel to $O\,d$ and $O\,c$ respectively. Then will $O\,e$ and $O\,f$ represent the two required currents to scale. By scaling off the figure, it is found that $O\,e = O\,f = 28.8$ amp. The value of $O\,e$ may also be found by trigonometry as follows: Drop the perpendicular $e\,g$. Then, $O\,g = \frac{50}{2} = 25$; $\cos 30° = \frac{O\,g}{O\,e}$;

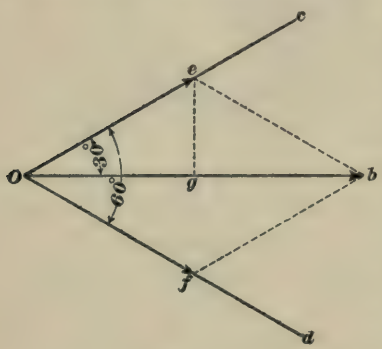

FIG. 111.

and $$O\,e = \frac{25}{\cos 30°} = \frac{25}{.866} = 28.8 \text{ amp.} \quad \text{Ans.}$$

(1833) (*a*) Frequency = 60, number of pairs of poles = 5. By formula **647,** rev. per second = $\frac{60}{5} = 12$, or rev. per minute at no load = 720. Ans.

(*b*) The slip is 5%; therefore the armature runs $720 \times .05 = 36$ rev. per minute less than the revolving field. The speed at full load will, then, be $720 - 36 = 684$ rev. per minute.

(1834) (*a*) The method of constructing a sine curve is explained in Art. **3846** and is illustrated in Fig. 1438.

(*b*) A sine curve is changing most rapidly at the points where it crosses the horizontal, that is, when it is passing through zero. The slant of the curve is greatest where the curve crosses the axis, hence the change in the ordinate corresponding to a change in time is greater at this point than anywhere else.

(1835) (*a*) The output is 150 K. W. Then, by formula **652,**

$$150{,}000 = \bar{C}\,\bar{E}\sqrt{3};$$

or, line current $\bar{C} = \dfrac{150{,}000}{1{,}000 \times \sqrt{3}} = 86.6$ amperes. Ans.

(*b*) The current in the armature winding will be, by formula **655,**

$$\bar{c} = \frac{86.6}{\sqrt{3}} = 50 \text{ amperes.} \quad \text{Ans.} \quad \text{(Art. } \mathbf{3993.}\text{)}$$

(1836) A non-inductive resistance of known value is connected in series with the circuit, the power in which is to be measured. The drop across the non-inductive resistance, across the circuit in which the power is expended, and also the drop across both combined, is measured, thus giving three voltmeter readings. If $\bar{E}_1$ is the drop across the resistance, $\bar{E}_2$ the drop across the circuit in which the power is being measured, and $\bar{E}$ the drop across both combined, then, by formula **641,**

$$W = \frac{1}{2R}(\bar{E}^2 - \bar{E}_1^2 - \bar{E}_2^2),$$

where W is the watts expended and R the value of the non-inductive resistance. (Art. **3948.**)

(1837) (*a*) Open-circuit windings are preferred because they are better adapted for the production of high voltages than closed-circuit windings. They allow more conductors to be connected up in series, thus giving the high voltage which is usually required for alternating-current work. (Art. **3954.**)

(*b*) The kind of winding used does not affect the frequency. The only quantities which affect the frequency of an alternator are the number of poles and the speed. The type of winding affects only the voltage and current output of the machine. (Art. **3951.**)

(1838) (*a*) The load on the secondary is $100 \times 50 = 5{,}000$ watts.

The apparent input in watts is $1{,}000 \times 6.5 = 6{,}500$ watts.

Hence, the apparent efficiency is $\frac{5{,}000}{6{,}500} = .769$ or 76.9%. Ans.

(*b*) The real watts supplied to the primary $= 6{,}500 \times .85 = 5{,}525$. Ans.

(*c*) The true efficiency is, therefore, $\frac{5{,}000}{5{,}525} = 90.5\%$. Ans.

(1839) Three-phase Y-connected windings can not be used for rotary transformers because the winding of a rotary transformer has to be at the same time both a direct-current and an alternating-current winding. Windings for continuous-current machines are of the closed-circuit type, hence the alternating-current winding must also form a closed circuit. This means that the Δ winding has to be used in such machines. (Art. **4053.**)

(1840) If the opening in the coil is much narrower than the pole-piece, there is danger of some of the E. M. F.'s in the different turns opposing each other, because they will be cutting lines in the same direction at once. The result of this would be to diminish the total E. M. F. generated by the alternator. (Art. **3959.**)

DESIGN OF ALTERNATING-CURRENT APPARATUS.

(QUESTIONS 1841–1918.)

(1841) It has been shown that, other things being equal, the hysteresis loss is proportional to the frequency. The eddy-current loss is proportional to the square of the frequency. It follows, therefore, that a machine of given size and output which runs at a high frequency will usually have a larger core loss than one running at a low frequency. The number of poles on continuous-current machines is usually small compared with the number on alternators, hence the frequency of the magnetic cycles in direct-current armature cores is not generally as high as it is in the cores of alternator armatures. For this reason alternators are liable to have larger core losses than continuous-current machines of corresponding speed and output. (Arts. **4059, 4068** to **4070.**)

(1842) If the inductance of an armature is very high and the current sufficiently large, all the E. M. F. generated may be used to drive the current through the armature against this inductance, and the useful terminal E. M. F. will therefore fall off to zero. If the inductance were very high and the field excitation not raised as the machine was being loaded, this limiting current might be less than the rated full-load current of the machine. (Art. **4060.**)

(1843) If an armature is well ventilated, it should be capable of radiating from .04 to .06 watt per square inch per degree rise, which, with 40° C. rise, would mean a total radiation of from 1.6 to 2.4 watts. The student will understand

that these are only average values, because this quantity depends very largely on the construction of the armature and the peripheral speed at which it runs. The radiation constant in individual cases might, therefore, vary considerably from the above values. (Art. **4071.**)

(**1844**) It has been shown that when the armature has self-induction, as is nearly always the case, the effect of the armature reaction is both to distort and to weaken the field. It follows, then, that if the field is not correspondingly strengthened by increasing the exciting current, the voltage at the terminals of the machine will fall off. (Arts. **4073, 4074.**)

(**1845**) If the armature reaction is made excessive, the field may be so weakened as the load is applied that the E. M. F. will decrease when the current tends to increase, thus keeping the current constant. (Art. **4081.**)

(**1846**) If the alternator runs at a speed of 450 R. P. M. and has 16 poles, the frequency will be $\frac{450}{60} \times 8 = 60$ cycles per second.

(*a*) In this case there are 16 slots, and since there are 16 poles, we have only one group of conductors per pole; consequently we may use formula **617,**

$$\bar{E} = \frac{4.44\,N\,T\,n}{10^8},$$

because all the conductors will be active at the same instant. There are 16 slots, with 30 conductors in each, making in all 480 conductors, or 240 turns. We have, then,

$$\bar{E} = \frac{4.44 \times 3{,}000{,}000 \times 240 \times 60}{10^8} = 1{,}918 \text{ volts.} \quad \text{Ans.}$$

(Art. **4086.**)

(*b*) If we used 32 slots, with 15 conductors per slot, we would have the same total number of conductors as before, but only half of them would be active at the same instant. The E. M. F. in this case would therefore be .707 times that which was obtained with the 16-slot arrangement, or, by formula **664,**

$\bar{E} = 1{,}918 \times .707 = 1{,}356$ volts. Ans. (Art. **4087.**)

(*c*) If 48 slots with 10 conductors per slot are used, the E. M. F.'s in the three sets will be displaced, as shown in Fig. 1591, and the E. M. F. obtained would be .667 times that given by the concentrated winding. We would have, then, by formula **666,**

$\bar{E} = 1{,}918 \times .667 = 1{,}279$ volts. Ans. (Art. **4088.**)

(1847) (*a*) Bars are generally used for the windings of machines of which the current output is large, or, in general, in any armature where a very large wire or a large number of wires in multiple would be required to carry the current. Low-voltage machines are usually bar-wound, because the lower the voltage the greater is the current for a given output.

(*b*) Wire conductors are preferred for machines where the current output is comparatively small and the voltage high. A conductor made up of a number of wires in multiple is often used where it is desired to make the conductor flexible. (Arts. **4103** to **4105.**)

(1848) Two-layer windings are largely used because they enable the ends of the coils to pass each other without interfering, and utilize all the available space at the end of the armature. They also have the advantage of accommodating one coil for each slot; for with such windings there are as many slots as coils, whereas with a single-layer winding there would have to be twice as many slots as coils (one slot for each side of each coil). (Art. **4093.**)

(1849) (*a*) If the alternator runs at the rate of 300 R. P. M. and has 12 poles, the frequency will be $\frac{300}{60} \times 6 = 30$ cycles per second. If the armature has 12 slots, there will be only one group of conductors per pole, and we may write

$$T = \frac{\bar{E} \times 10^8}{4.44 \times N \times n},$$

or $$T = \frac{1{,}500 \times 10^8}{4.44 \times 3{,}000{,}000 \times 30} = 375 \text{ turns.}$$ Ans.

(Art. **4086.**)

(*b*) If the winding is placed in 24 slots, one half the conductors will be idle while the other half is active, so that we will have the relation

$$T = \frac{\bar{E} \times 10^8}{4.44 \times N \times n \times .707},$$

and since $\bar{E}$ is to be 1,500 volts in both cases, the value of T must be

$$T = \frac{1{,}500 \times 10^8}{4.44 \times 3{,}000{,}000 \times n \times .707} = 530 \text{ turns.}$$

In order, then, to obtain the same voltage, 155 turns more have to be used with the subdivided winding than with the concentrated winding. Ans. (Art. **4087.**)

(1850) (*a*) A wire having a diameter of .001 inch would have a cross-sectional area of 1 circular mil. One foot of copper wire having this cross-section has a resistance at ordinary temperatures of about 10.4 ohms.

(*b*) At the temperatures usually met with in well-designed electrical machinery the resistance per mil-foot is about 11.5 ohms. (Art. **4139.**)

(1851) When the coils are heavy, they generally contain a large number of turns, thus causing the voltage between adjacent layers to be considerable, and making it necessary to provide insulation between the layers, as explained in Arts. **4110** and **4214.**

(1852) If the winding is to be arranged in two layers, there will be as many slots as there are coils. In this case we are to provide for a two-coil winding, i. e., two coils per pole per phase. The machine has 16 poles and is to be wound for three phases; hence there will be $16 \times 3 \times 2 = 96$ slots and 96 coils. (Arts. **4087, 4088.**)

(1853) When the field is excited by a separately excited winding only, the coils are subjected to no higher pressure than that generated by the exciter. In case the machine is provided with compound coils which are in connection with the armature winding, the full pressure of the

machine is tending to break down the insulation and set up a short circuit through the frame and armature core. When this method of field excitation is used, it follows that the compound coils should be thoroughly insulated from the separately excited coils, and from the frame of the machine as well. (Arts. **4161, 4162.**)

(1854) In large machines it is not necessary to rectify all the current in order to supply the compound coils. A shunt across the rectifier terminals is therefore mounted on the armature, which revolves with it. By this means a considerable portion of the current is allowed to flow around the rectifier, which cuts down the amount of current to be handled by the rectifier brushes and reduces the sparking. (Arts. **4182** and **4202.**)

(1855) The principal factor limiting the output of an alternator is the heating effect due to the core losses and copper losses, as explained in Arts. **4059** to **4062.** The output of an alternator is not usually a strictly definite quantity, because the same machine may be capable of a greater output under one set of conditions than it would under another. For example, a machine could give a larger output without overheating in a large airy dynamo room than it could if it were placed in the hold of a ship where the ventilation was poor and the temperature high.

(1856) The effects of armature reaction in alternators are not of such importance as in direct-current machines because the reaction does not give rise to any sparking. The only bad effect it can have is to cause a falling off in the terminal voltage, and this can be compensated for by strengthening the field. (Art. **4061.**)

(1857) (*a*) Eddy currents may be avoided to a large extent by laminating the core and insulating the laminations from each other sufficiently to prevent the currents from flowing between the disks.

(*b*) The eddy-current losses in alternators are apt to be high if the core is not carefully built up, because of the high frequency.

(*c*) The eddy-current loss increases with the square of the frequency; thus, doubling the frequency makes this loss four times as great. (Art. **4070.**)

(**1858**) Armatures may be wound to have low inductance by using a small number of turns per coil or using a large number of slots with few conductors in each slot. (Art. **4079.**)

(**1859**) The machine has 8 poles and runs at a speed of 900 R. P. M.; its frequency is therefore $\frac{900}{60} \times 4 = 60$ cycles per second.

(*a*) The armature has a total of 24 slots, hence there must be 8 slots per phase, or 1 slot per pole-piece per phase. There will then be 480 conductors, or 240 turns in each phase connected in series, and since there is only one group per phase, we may write as the E. M. F. generated in each phase

$$\bar{E} = \frac{4.44\,N\,T\,n}{10^8} = \qquad \text{(See formula } \mathbf{617.}\text{)}$$

$$\frac{4.44 \times 2{,}500{,}000 \times 240 \times 60}{10^8} = 1{,}598 \text{ volts.} \quad \text{Ans.}$$

(Art. **4086.**)

(*b*) If 48 slots are used, with 30 conductors in each slot, we shall have the same total number of conductors as before, but we shall have 16 slots to each phase, or two groups of conductors per pole-piece. Then, by formula **669,**

$$\bar{E} = \frac{4.44\,N\,T\,n}{10^8} \times .965 =$$

$1{,}598 \times .965 = 1{,}542$ volts. Ans. (Art. **4090.**)

(*c*) If the whole 1,440 conductors were evenly distributed, we would have

$$\bar{E} = \frac{4.44\,N\,T\,n}{10^8} \times .95 =$$

$1{,}598 \times .95 = 1{,}518$ volts. Ans. (Art. **4090.**)

(**1860**) About 4,500 feet per minute would be a fair value for a machine of this size and output. (Arts. **4082, 4083.**)

(1861) (*a*) From 14 to 18 mils, the thicker iron being used for low-frequency machines.

(*b*) The cores are usually light, because of the small magnetic flux which they have to carry as compared with direct-current machines. This is a consequence of the use of a large number of poles, the flux being so split up that the cross-section of iron in the core does not need to be very large to carry it. (Arts. **4095, 4096.**)

(1862) A distributed winding generally results in a large number of crossings of the coils at the ends of the armature, thus making them difficult to insulate for high voltages. If high pressure is needed from such a machine, it can be generated at a low pressure and stepped up by means of transformers. (Art. **4089.**)

(1863) (*a*) If the output is 200 K. W. and the full-load voltage 2,300, the full-load current will be $\frac{200,000}{2,300} =$ 86.9 amperes. If we allow 500 circular mils per ampere, we would then require a cross-section of $86.9 \times 500 =$ 43,450 circular mils. Ans. (Art. **4125.**)

(*b*) Two No. 7 wires in multiple would give 41,640 circular mils, or four No. 10 wires would give 41,520 circular mils. One No. 4 wire would give 41,740 circular mils, but this would be too stiff to wind up easily into coils.

(1864) (*a*) If k represents the resistance of 1 foot of the wire 1 mil in diameter or 1 circular mil in area, then $L \times k$ would be the resistance of the given length of wire *if it had a cross-section of only 1 circular mil*, L being the length expressed in feet. The resistance of any wire of given length is inversely proportional to the area of its cross-section; for example, doubling the cross-section halves the resistance, etc. It follows, then, that if the conductor has a cross-section of m circular mils, its resistance will be

$$R = \frac{L \times k}{m}.$$ (See formula **680,** Art. **4139.**)

(*b*) The hot resistance of 1 mil-foot of copper is about

11.5 ohms, hence we have for the given length and cross-section, by formula **682,**

$$R = \frac{1{,}600 \times 11.5}{11{,}500} = 1.6 \text{ ohms.} \quad \text{Ans.} \quad \text{(Art. } \mathbf{4139.}\text{)}$$

(1865) The output would not be changed, because, in changing from a Y to a Δ winding, the voltage is reduced by the same ratio that the current output is increased. The size of slot required will be about the same in either case, and the size of the armature required will be the same. (Art. **4151.**)

(1866) Shafts for induction motors should be specially stiff on account of the short air-gap used. (Art. **4269.**)

(1867) (*a*) The density in the core of a 125-cycle transformer may be taken as 20,000 lines per square inch for a fair average value. This would make the value of the flux $N = 200{,}000$ lines. (Art. **4211.**)

(*b*) We have, by formula **697,**

$$E_p = \frac{4.44\, N\, T_p\, n}{10^8},$$

or

$$1{,}000 = \frac{4.44 \times 200{,}000 \times T_p \times 125}{10^8};$$

hence, $T = \dfrac{1{,}000 \times 10^8}{4.44 \times 200{,}000 \times 125} = 900$ turns, nearly. Ans. (Art. **4222.**)

(*c*) If the secondary voltage is to be 50, the number of secondary turns required will be, by formula **698,**

$$T_s = T_p \times \frac{E_s}{E_p} = 900 \times \frac{50}{1{,}000} = 45 \text{ turns.} \quad \text{Ans.} \quad \text{(Art. } \mathbf{4223.}\text{)}$$

(1868) Alternators with armatures having high inductance will show a considerable falling off in terminal pressure from no load to full load, on account of the armature reaction and the induction drop in the armature. In order to keep the voltage constant, an additional field excitation must be supplied, and the greater the armature reaction and inductance the larger must be the additional excitation.

This additional excitation is supplied by means of the series coils; hence they must be more powerful for armatures with high inductance than for those with low inductance. (Art. **4184.**)

(1869) 30,000 lines per square inch may be taken as an average value of the density in cast-iron yokes. In some machines the density is lower than this, in order to make the yoke have enough cross-section to give sufficient mechanical strength. The density may be run somewhat higher than 30,000 without danger of saturation, but the above may be taken as an average value. (Art. **4169.**)

(1870) If the magnetic circuit of a transformer is broken by an air-gap, the magnetic reluctance is greatly increased. A large magnetic reluctance requires a large magnetizing current to set up the flux necessary to produce the counter E. M. F. The effect of an air-gap would then be to considerably increase the no-load current. (Art. **4213.**)

(1871) The series coils may be supplied with a current which varies in proportion to the load by connecting them to the secondary coil of a transformer, the primary of which is in series with the armature. This transformer revolves with the armature, and as the primary and secondary windings are not electrically connected in any way, the insulation of the series coils is not subjected to a high potential. (Art. **4201.**)

(1872) If the motor is to run at 1,500 revolutions per minute, we must have (see formula **646**) $n = p \times s$, or $125 = p \times 1\frac{5}{6}\frac{0}{0}$, whence $p = 5$, and the number of poles will therefore be 10. The coils used in winding the field will bridge over $\frac{1}{10}$ of the circumference, thus giving rise to a 10-pole field when current is sent through them. (Art. **4253.**)

(1873) When an armature is run on open circuit in a fully excited field, there will be a rise in temperature due to the losses which occur in the core. We have a mass of iron

in which the magnetism is being constantly changed, hence the conditions necessary for setting up hysteresis and eddy-current losses are present, and these core losses will occur whether the armature is delivering current or not. (Art. **4063.**)

(1874) (*a*) The safe working temperature of an armature is determined by the temperature which the insulating materials are capable of standing continuously without deterioration.

(*b*) About 40° C. (Arts. **4064, 4065.**)

(1875) A low-resistance armature requires a small amount of slip, consequently the falling off in speed from no load to full load is small compared with what it would be if the armature resistance were high and the slip correspondingly large. (Art. **4247.**)

(1876) When the armature has no self-induction, the current in the armature coils is in phase with the E. M. F. In such a case the armature exerts no demagnetizing action on the field, but tends only to distort it, crowding the lines of force to one side of the pole-pieces. (Art. **4073.**)

(1877) Armatures with concentrated windings consisting of a small number of heavy coils generally have high inductance. Coils such as are used on armatures of this class usually have a large number of turns, and the self-induction is consequently high. (Art. **4078.**)

(1878) For a machine of this size we may take 5,000 feet per minute as a fair value for the belt speed. This will give, by formula **691,**

$$\text{Diameter of pulley} = \frac{12 \times 5{,}000}{3.14 \times 450} = 42\tfrac{1}{2} \text{ in., nearly.}$$

By formula **692,**

$$\text{Width of belt} = .7 \times \frac{250{,}000}{5{,}000} = 35 \text{ in.}$$

Allowing 1 inch clearance on each side of the belt would give 37 inches as the width of the pulley face. Ans. (Art. **4200.**)

(1879) The secondary-core losses are small because the frequency in the secondary is very low. The hysteresis loss is proportional to the frequency, and the eddy-current loss to the square of the frequency; hence a low frequency will result in small core losses. (Art. **4241.**)

(1880) If the current in the armature lags behind the E. M. F., a current is flowing in the coils when they are directly under the poles, and this current tends to demagnetize the field. The current in the conductors also tends to distort the field, so that when the current lags behind the E. M. F., the effect is not only to distort the field, but also to demagnetize it. (Arts. **4074, 4075.**)

(1881) (*a*) Inductances in series are added like resistances; hence, if we have 10 coils, each having an inductance of .01 henry, the total inductance will be .1 henry.

(*b*) If each coil consists of one-half the number of turns, its inductance will be one-quarter of what it was before, because the inductance is proportional to the square of the number of turns. The inductance of each coil in the second case will therefore be $\frac{1}{4} \times .01 = .0025$ henry. There are 20 coils connected in series, hence the total inductance in the second case will be $20 \times .0025 = .05$ henry. The total number of turns has not been changed, but the inductance has been reduced to one-half of what it was in the former case. (Art. **4079.**)

(1882) (*a*) Windings for induction-motor fields are usually almost identical with the winding which would be used for the corresponding polyphase armature. Each phase consists of a number of coils grouped together and spaced around the periphery of the field in the same way that they would be arranged on a polyphase armature.

(*b*) Windings placed in slots give rise to less magnetic leakage, thus making the inductance of the motor less when loaded, and thereby increasing the power factor. (Art. **4244.**)

(1883) The C^2R loss in small alternators is a considerably larger percentage of the output than in the case of large machines. The percentage rapidly decreases, however, until a point is soon reached beyond which it decreases very little. To decrease the C^2R loss much below 1.5 or 2% requires too large an expenditure in the shape of copper. (Art. **4066.**)

(1884) (*a*) It was pointed out in Art. **4068** that the hysteresis loss in a given mass of iron depends upon the quality of the iron, and is proportional to the volume, to the frequency, and to the 1.6 power of the magnetic density. The loss then in a given mass of iron worked at a given frequency and density is a quantity which can at once be calculated when the magnetic quality of the iron is known.

(*b*) Laminating the core will have no effect upon the core, provided the volume and quality of the iron are not changed in any way. Usually, laminating the core will decrease the actual cross-section of iron, and this might cause a slight increase in the hysteresis loss due to the increased density. The object in laminating a core is to keep down the eddy-current loss, and it has little or no effect on the hysteresis loss, which is a loss in the iron itself, not a loss due to currents eddying around in the iron.

(1885) (*a*) The core loss in the first transformer is 120 watts; hence, watt-hours lost per day $= 120 \times 24 = 2{,}880$, or 2.88 kilowatt-hours. The cost of this per year will then be $2\ 88 \times .10 \times 365 = \$105.12$. The first cost of the transformer is \$30, so that the cost of the core losses plus interest and depreciation will be $\$105.12 + \$3 = \$108.12$.

The core loss in the second transformer is 90 watts. The loss in watt-hours for 1 day will be $90 \times 24 = 2{,}160$. This core loss will cost per year $2{,}160 \times .10 \times 365 = \$78.84$. The transformer costs \$50, so that the interest and depreciation will amount to \$5 per year, making the total cost \$83.84.

(*b*) The second transformer; for, although the first cost of this transformer is greater than that of the other, it will be much the cheaper to operate, the first cost being less in

either case than the expense of operating. (Arts. **4230** to **4232.**)

(1886) The speed is 900 revolutions per minute and the field has 8 poles; consequently the frequency will be 60 cycles per second. (See formula **646.**) Taking curve *B*, Fig. 1584, we find the watts lost per cubic inch per 100 cycles at a density of 25,000 to be about .275. The total loss will therefore be

$$W = \frac{2{,}200 \times .275 \times 60}{100} = 363 \text{ watts.} \quad \text{Ans.} \quad \text{(Art. } \mathbf{4068.}\text{)}$$

(1887) The higher the peripheral speed the greater will be the amount of heat which can be radiated with a given rise in temperature. (Art. **4071.**)

(1888) The strength of the series coils may be varied by placing a non-inductive resistance or shunt across their terminals. By adjusting this resistance, the proportion of the total current which passes through the coils may be varied as desired. For example, if the shunt had a resistance equal to that of the coils, the current would divide about equally between the two. (Art. **4182.**)

(1889) The eddy-current loss increases as the square of the frequency; doubling the speed would double the frequency; hence the eddy-current loss would be made four times as great. It might also be noted here that the hysteresis loss would be made only twice as large, as this loss increases directly with the frequency. (Art. **4070.**)

(1890) If the resistance is inserted in the field, the field current is cut down and the field strength thereby reduced. The strong armature currents react on this weak field, making it still weaker, and resulting in a small starting torque. When the resistance is inserted in the armature, the whole line voltage is applied to the field and is effective in setting up a strong magnetic flux. The resistance in the armature circuit limits the secondary currents and prevents them from weakening the field set up by the primary, so that a good starting torque is the result. (Arts. **4246, 4247.**)

(1891) (*a*) The quantities which determine the size of wire to be used for the field winding are: the ampere-turns to be set up, the mean length of a turn in feet, the resistance of a mil-foot of the wire; and the voltage applied to the coils.

(*b*) Let l_m be the mean length of a turn, p the number of pairs of poles on the machine, E the exciter voltage, e the drop in the regulating rheostat, Ct the ampere-turns supplied by one pair of coils, and 11.5 the resistance of 1 mil-foot of copper at the working temperature of the field. Then,

$$\text{Circular mils cross-section of field wire} = \frac{p \times l_m \times Ct \times 11.5}{E - e}.$$

(Arts. **4176, 4177.**)

(1892) (*a*) The angular distance between the arms of the rocker depends upon the number of poles on the machine. The arms must be so arranged that when one brush presses on one segment of the rectifier, the other presses on the other segment. There are as many segments as poles, so that the angular distance between the arms and brushes will depend upon the angular distance between the poles.

(*b*) If the machine has 16 poles, there will be 16 segments in the rectifier, and the angular distance from center to center of segments will be $22\frac{1}{2}°$. The brushes would be set so as to bring them as far apart as possible, and still have them in contact with the two different segments. This would make the angle between the tips $7 \times 22.5 = 157.5°$. (Art. **4197.**)

(1893) (*a*) The air-gap should be made small in order to keep down the magnetizing current and increase the power factor of the machine by decreasing the cross-section of the leakage path. A small increase in the length of the air-gap would make quite a large increase in the current necessary to set up the magnetic flux in the field.

(*b*) The air-gap density should generally be low, otherwise a large magnetizing current will be required. It will usually be found to lie somewhere between 20,000 and 30,000 lines per square inch. (Art. **4249.**)

(1894) (*a*) The flux through the yoke is one-half that through the pole-pieces in the case of a multipolar field with circular yoke and radial pole-pieces.

(*b*) The flux through the pole-pieces is equal to the flux through the air-gap multiplied by the coefficient of magnetic leakage; that is, the flux through the poles has to be greater than that through the air-gap by an amount sufficient to make up for the lines which leak across without going through the air-gap. (Arts. **4167** to **4169.**)

(1895) The yoke is usually made of cast iron rather than steel in the case of alternators because there is no need for making special efforts to make the yoke cross-section small. The required cross-section is usually small anyway, on account of the small magnetic flux, so that a steel yoke is not necessary, besides being more expensive and difficult to cast. (Art. **4156.**)

(1896) (*a*) There are 72 slots, with 12 conductors per slot, in the primary winding, making a total of 864 conductors, or 288 conductors per phase. In the secondary winding there are 48 slots, with 2 bars per slot, making a total of 96 bars, or 32 bars per phase. In the primary we have 288 conductors per phase connected in series, while in the secondary there are 32 connected in series. If the primary current in each phase is 50 amperes and the power factor of the secondary is 1, the secondary current will be

$$\tfrac{288}{32} \times 50 = 450 \text{ amperes.} \quad \text{Ans.}$$

(*b*) If we allow 800 circular mils per ampere, the cross-section of the secondary winding bar in *circular mils* will be

$$450 \times 800 = 360{,}000.$$

To reduce this to square mils, we must multiply by $\frac{\pi}{4}$; hence the cross-section in square mils will be

$$360{,}000 \times .785 = 282{,}600 \text{ sq. mils.} \quad \text{Ans.} \quad \text{(Art. } \mathbf{4261.}\text{)}$$

(1897) (*a*) A unicoil or concentrated winding is one in which there is only one group of conductors per pole, so

that all the conductors in a given phase are active at the same instant.

(*b*) The chief advantages of this style of winding are: (1) Ease of arrangement on the armature. (2) Ease with which high insulation can be secured. (3) A high E. M. F. generated for a given length of active conductor.

(*c*) The disadvantages of concentrated windings are: (1) High armature reaction. (2) High armature self-induction. Both these tend to produce bad regulation for constant pressure. Concentrated windings also localize the heat generated in the coils, so that the coils are more apt to get hot than in armatures where they are spread out more. (Arts. **4085, 4086.**)

(1898) For a density of 40,000 lines per square inch, we find the loss per cubic inch per 100 cycles to be .60 watt. (See curve *B*, Fig. 1584.) The total loss in the teeth will then be

$$\frac{200 \times .60 \times 125}{100} = 150 \text{ watts.}$$

The density in the core under the teeth is 20,000, and the loss per cubic inch per 100 cycles corresponding to this density is about .19 watt. The total loss in the core proper will therefore be

$$\frac{500 \times .19 \times 125}{100} = 118.75 \text{ watts.}$$

The total loss would then be

$$150 + 118.75 = 268.75 \text{ watts.} \quad \text{Ans.} \quad \text{(Art. 4068.)}$$

(1899) (*a*) Since the alternator runs at 600 revolutions per minute and has 12 poles, its frequency will be 60. There are 24 slots in all, or 12 for each phase; consequently, there is only one group of conductors per pole-piece per phase. There are $12 \times 50 = 600$ conductors, or 300 turns in each phase.

Then,
$$E = \frac{4.44\, N\, T\, n}{10^8} =$$

$$\frac{4.44 \times 2{,}000{,}000 \times 300 \times 60}{10^8} = 1{,}598.4 \text{ volts.} \quad \text{Ans.}$$

(Art. **4087.**)

(*b*) If the winding were spread evenly over the surface, the voltage obtained would be reduced 10%, or we would have to multiply the expression above by .90. Hence we have

$E = 1{,}598.4 \times .9 = 1{,}438.56$ volts. Ans. (Art. **4090.**)

(1900) Bolts passing through the cores of armatures or transformers are liable to give rise to eddy currents by short-circuiting the disks. Whenever it is necessary to pass bolts through such cores, they should be insulated lightly by means of a paper tube, but it is best when possible to make the construction such that the bolts do not need to pass through the core. (Art. **4070.**)

(1901) Stationary armatures are best adapted for high voltages because there is usually more room on them for insulation, and, not being subjected to vibration of any kind, the insulation is not liable to be injured. The use of the stationary armature also does away with the necessity of insulating collector rings, brush-holders, etc., for high potentials. (Art. **4111.**)

(1902) (*a*) The useful output at full load will be 5,000 watts. The input will be $5{,}000 + 60 + 50 + 95 = 5{,}205$ watts. The full-load efficiency will therefore be $\frac{5{,}000}{5{,}205} = 96.06\%$. Ans.

(*b*) At $\frac{1}{2}$ load the core loss will be the same as before, but the copper losses will be only $\frac{1}{4}$ as great, because the current will be one-half as great in the primary and secondary (loss varies with the square of the current). The copper loss in the primary at $\frac{1}{2}$ load will therefore be 15 watts, and in the secondary 12.5 watts. The total losses at $\frac{1}{2}$ load will then be 122.5 watts. The useful output will be 2,500 watts, so that the efficiency is $\frac{2{,}500}{2{,}622.5} = 95.3\%$. Ans.

(*c*) The total useful output during the day is $5{,}000 \times 6 = 30{,}000$ watt-hours.

The total hysteresis loss during the day is $95 \times 24 = 2{,}280$ watt-hours.
The total C^2R loss during the day is $(60 + 50) \times 6 = 660$ watt-hours.

Total loss in 24 hours 2,940 watt-hours.

Hence,

$$\text{All-day efficiency} = \frac{30{,}000}{32{,}940} = 91.07\%. \quad \text{Ans.}$$

(Arts. **4230** to **4232.**)

(1903) No; there can be no hysteresis loss when the magnetism is steady. If the field exciting current were fluctuating there might be some loss, but as soon as the current becomes steady the frequency is 0; consequently the hysteresis loss is 0. (Art. **4068.**)

(1904) (*a*) Oiled linen tape and mica. Coils should also be well treated with insulating varnish after all moisture has been driven off by baking. Tape is usually sufficient by itself, except for coils intended to stand high pressures, in which case it is best to use mica in addition to the tape.

(*b*) Slots are usually lined with a trough made up of alternate layers of paper and mica. Other materials, such as micanite and leatheroid, are also used. (Arts. **4110** to **4117.**)

(1905) Highly saturated teeth in a direct-current machine tend to keep the armature from reacting strongly on the field, and thus reduce sparking. In the case of an alternator or induction motor, the highly saturated teeth could do no good in this respect, and would only tend to make the hysteresis and eddy-current loss large. (Art. **4119.**)

(1906) (*a*) The use of oil ensures largely against breakdowns in insulation, because if the insulation is punctured at any point the oil at once fills up the gap. Oil also serves to conduct the heat away from the coils to the case of the transformer and thus get rid of it to the surrounding air. (Art. **4210.**)

(*b*) It is important that the insulation between the primary and secondary coils should be high because there is always a tendency for the high-tension primary current to jump across, especially if there should happen to be a ground on the secondary circuit. Any connection between the primary and secondary would bring the high primary pressure into connection with it and be liable to injure the apparatus connected to the circuit. (Art. **4214.**)

(1907) About 30,000 lines per square inch. The value might vary slightly either side of this amount without making any appreciable change in the armature-core loss or in the ampere-turns required for the magnetic circuit. (Art. **4120.**)

(1908) If the total output is 75 K. W. at 1,100 volts, the current in each phase will be

$$\frac{37.5 \times 1{,}000}{1{,}100} = 34.09 \text{ amperes.}$$

Each phase has a resistance of .75 ohm.; hence the $C^2 R$ loss in each phase at full load will be

$$(34.09)^2 \times .75 = 871.6 \text{ watts.}$$

The total loss in the armature will therefore be

$$871.6 \times 2 = 1{,}743.2 \text{ watts.} \quad \text{Ans.} \quad \text{(Art. } \mathbf{4145.}\text{)}$$

(1909) (*a*) For cast-iron pole-pieces the air-gap density should not run much over 30,000 lines per square inch, as this will bring the density in the cast-iron pole-pieces quite high enough when allowance is made for magnetic leakage.

(*b*) With wrought-iron pole-pieces the density in the air-gap may be as high as 40,000 or 45,000 lines per square inch. (Art. **4121.**)

(1910) The method of obtaining the length of the face of iron parallel to the shaft, in order to limit the density to the desired amount, has been explained in Art. **4257.** This assumes that the intensity of the magnetization decreases as we move either side of the center of the pole face according to the sine law. This is nearly the case in motors with

distributed wound fields. The formula giving the required relation then is

$$l = \frac{N \times 2p}{2 \times d_f \times \mathbf{B}}.$$

(1911) (*a*) A distributed winding is one which is split up into a large number of coils and spaced around the armature, as distinguished from one which is provided with a few heavy coils only.

(*b*) These windings have the advantages of low self-induction and consequent good regulation for constant potential. They also radiate the heat generated in the windings more easily than armatures having heavy coils. They have the disadvantages that they give a lower E. M. F. for a given length of active conductor and are more difficult to insulate and construct. (Arts. **4084** to **4086.**)

(1912) (*a*) If the line voltage is 500 and the winding is △-connected, the pressure generated in each phase must be 500; hence we have

$$\bar{E} = \frac{4.44\,N\,T\,n}{10^8} \times .95.$$

$$500 = \frac{4.44 \times 500{,}000 \times T \times 60 \times .95}{10^8}$$

$$T = \frac{500 \times 10^8}{4.44 \times 500{,}000 \times 60 \times .95} = 396 \text{ turns, nearly.} \quad \text{Ans.}$$

(Art. **4257.**)

(*b*) If the power factor is .85 when 10,000 true watts are being delivered, the apparent watts will be $\frac{10{,}000}{.85} = 11{,}765$, nearly.

We have $$W = \bar{C}\,\bar{E}\sqrt{3},$$

or $$11{,}765 = \bar{C}\,500\sqrt{3}.$$

$$\bar{C} = \frac{11{,}765}{500\sqrt{3}} = 13.59 \text{ amp.} \quad \text{Ans.} \quad \text{(Art. } \mathbf{4251.}\text{)}$$

(*c*) Each phase would take $\frac{1}{3}$ the input, or $\frac{11{,}765}{3}$ apparent

watts. The voltage per phase is 500; hence the current per phase will be

$$\frac{11{,}765}{3 \times 500} = 7.84 \text{ amp.} \quad \text{Ans.}$$

The same result might also be obtained by dividing the line current by $\sqrt{3}$, because in a Δ-connected armature the current in each phase is equal to the line current divided by $\sqrt{3}$.

(1913) (*a*) If the flux from one pole is 2,500,000 lines, the flux through a section of the armature under the slots will be $\frac{1}{2}N$, or 1,250,000 lines. The density is to be 30,000 lines per square inch; hence the area of cross-section of the iron will be

$$\frac{1{,}250{,}000}{30{,}000} = 41.66 \text{ sq. in.} \quad \text{Ans.}$$

(*b*) Since the length of the iron part of the core parallel to the shaft is 15 in., the depth of iron under the slots must be

$$\frac{41.66}{15} = 2.77 \text{ in., say } 2\tfrac{3}{4} \text{ in.}$$

The inside diameter will be $28 - (4 + 5\frac{1}{2}) = 18\frac{1}{2}$ in. Ans. (Art. **4136.**)

(1914) (*a*) Magnetic leakage may, to a large extent, be prevented by splitting the coils up into a number of sections and sandwiching the primary and secondary sections between each other. This reduces the magnetomotive force of any one coil, and thus reduces the tendency to leakage. If the coils are arranged one over the other, with very little space between them, and made such that the leakage path will be long and of small cross-section, the magnetic reluctance of the leakage path will be high, and the leakage consequently small. (Art. **4213.**)

(*b*) If the secondary current is 50 amperes and the resistance of the secondary .02 ohm, the drop due to the secondary resistance will be $50 \times .02 = 1$ volt.

The current in the primary corresponding to 50 amperes in the secondary will be $\frac{50}{20} = 2.5$ amperes, neglecting the magnetizing current. The drop in the primary at full load will then be $2.5 \times 5 = 12.5$ volts. This 12.5 volts drop in the primary will cause a drop in the secondary of $\frac{12.5}{20} =$ .625 volt, and the total drop due to the resistance of both coils will be 1.625 volts. Ans.

(1915) The mean length of a turn is 40 inches, or $\frac{40}{12}$ feet. The ampere-turns supplied by 1 pair of coils is 4,000. The voltage of the exciter is 220 volts and the drop in the regulator 25 volts, so that the voltage actually applied to the field is 195. Applying formula **690,** we have

$$m = \frac{4 \times \frac{40}{12} \times 4{,}000 \times 11.5}{195} = 3{,}145 \text{ cir. mils.}$$

No. 15 B. & S. has a cross-section of 3,257 circular mils, so that this size would be suitable. (Art. **4177.**)

(1916) (*a*) The machine has to deliver a total of 50 K.W. or $\frac{50}{3}$ K.W. in each phase. The voltage generated per phase will be 1,100, because the armature is to be Δ wound. The full-load current in each phase will therefore be

$$\frac{50{,}000}{3 \times 1{,}100} = 15.15 \text{ amp.}$$

The loss in each phase will then be

$$(15.15)^2 \times 1.5 = 344.4 \text{ watts.}$$

The total C^2R loss will be $344.4 \times 3 = 1{,}033.2$ watts. Ans.

(*b*) The drop in each phase will be $CR = 15.15 \times 1.5 =$ 22.7 volts, and since the phases in a Δ winding are connected directly across the line, the drop in the line pressure due to the armature resistance will also be 22.7 volts. (Art. **4151.**)

(1917) If the poles are not laminated, eddy currents will be set up in them by the teeth of the armature sweeping past the faces. The coarse armature teeth set up tufts

of magnetism which sweep across the pole-pieces, and thus set up eddy currents if the pole tip is not laminated or broken up in some way by grooves or wrought-iron pieces cast in, as explained in Art. **4155.**

(1918) See Art. **4143.** The total output is 175 K.W.; hence the output in each phase will be $\frac{175,000}{2}$ watts. The voltage is 3,000; hence the current in each phase is

$$\frac{175,000}{2 \times 3,000} = 29.16 \text{ amp.}$$

Allowing 500 circular mils per ampere, we get the cross-section necessary:

$$29.16 \times 500 = 14,580 \text{ cir. mils.}$$

This is between a No. 8 and No. 9 B. & S. The former would probably be used, as it gives a little over the required section rather than under it.

A TEXTBOOK

ON

MECHANICAL AND ELECTRICAL ENGINEERING

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

TABLES AND FORMULAS

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TABLES AND FORMULAS.

This volume contains all the principal Tables and Formulas which are likely to be used by the student in practice. They have been collected and placed in this volume in order to make them convenient for ready reference, so that the student will not be obliged to hunt them out in the preceding volumes. The number after each formula is the same as the number following the same formula in the text.

TABLE

OF

COMMON LOGARITHMS

OF NUMBERS

From 1 to 10,000.

N	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
0	— ∞	**20**	30 103	**40**	60 206	**60**	77 815	**80**	90 309
1	00 000	21	32 222	41	61 278	61	78 533	81	90 849
2	30 103	22	34 242	42	62 325	62	79 239	82	91 381
3	47 712	23	36 173	43	63 347	63	79 934	83	91 908
4	60 206	24	38 021	44	64 345	64	80 618	84	92 428
5	69 897	25	39 794	45	65 321	65	81 291	85	92 942
6	77 815	26	41 497	46	66 276	66	81 954	86	93 450
7	84 510	27	43 136	47	67 210	67	82 607	87	93 952
8	90 309	28	44 716	48	68 124	68	83 251	88	94 448
9	95 424	29	46 240	49	69 020	69	83 885	89	94 939
10	00 000	**30**	47 712	**50**	69 897	**70**	84 510	**90**	95 424
11	04 139	31	49 136	51	70 757	71	85 126	91	95 904
12	07 918	32	50 515	52	71 600	72	85 733	92	96 379
13	11 394	33	51 851	53	72 428	73	86 332	93	96 848
14	14 613	34	53 148	54	73 239	74	86 923	94	97 313
15	17 609	35	54 407	55	74 036	75	87 506	95	97 772
16	20 412	36	55 630	56	74 819	76	88 081	96	98 227
17	23 045	37	56 820	57	75 587	77	88 649	97	98 677
18	25 527	38	57 978	58	76 343	78	89 209	98	99 123
19	27 875	39	59 106	59	77 085	79	89 763	99	99 564
20	30 103	**40**	60 206	**60**	77 815	**80**	90 309	**100**	00 000

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.		
100	00 000	043	087	130	173	217	260	303	346	389			
101	432	475	518	561	604	647	689	732	775	817			
102	860	903	945	988	*030	*072	*115	*157	*199	*242			
103	01 284	326	368	410	452	494	536	578	620	662		**44**	**43**	**42**
104	703	745	787	828	870	912	953	995	*036	*078	1	4.4	4.3	4.2
105	02 119	160	202	243	284	325	366	407	449	490	2	8.8	8.6	8.4
106	531	572	612	653	694	735	776	816	857	898	3	13.2	12.9	12.6
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302	4	17.6	17.2	16.8
108	03 342	383	423	463	503	543	583	623	663	703	5	22.0	21.5	21.0
109	743	782	822	862	902	941	981	*021	*060	*100	6	26.4	25.8	25.2
110	04 139	179	218	258	297	336	376	415	454	493	7	30.8	30.1	29.4
111	532	571	610	650	689	727	766	805	844	883	8	35.2	34.4	33.6
112	922	961	999	*038	*077	*115	*154	*192	*231	*269	9	39.6	38.7	37.8
113	05 308	346	385	423	461	500	538	576	614	652		**41**	**40**	**39**
114	690	729	767	805	843	881	918	956	994	*032	1	4.1	4.0	3.9
115	06 070	108	145	183	221	258	296	333	371	408	2	8.2	8.0	7.8
116	446	483	521	558	595	633	670	707	744	781	3	12.3	12.0	11.7
117	819	856	893	930	967	*004	*041	*078	*115	*151	4	16.4	16.0	15.6
118	07 188	225	262	298	335	372	408	445	482	518	5	20.5	20.0	19.5
119	555	591	628	664	700	737	773	809	846	882	6	24.6	24.0	23.4
120	918	954	990	*027	*063	*099	*135	*171	*207	*243	7	28.7	28.0	27.3
121	08 279	314	350	386	422	458	493	529	565	600	8	32.8	32.0	31.2
122	636	672	707	743	778	814	849	884	920	955	9	36.9	36.0	35.1
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307		**38**	**37**	**36**
124	09 342	377	412	447	482	517	552	587	621	656	1	3.8	3.7	3.6
125	691	726	760	795	830	864	899	934	968	*003	2	7.6	7.4	7.2
126	10 037	072	106	140	175	209	243	278	312	346	3	11.4	11.1	10.8
127	380	415	449	483	517	551	585	619	653	687	4	15.2	14.8	14.4
128	721	755	789	823	857	890	924	958	992	*025	5	19.0	18.5	18.0
129	11 059	093	126	160	193	227	261	294	327	361	6	22.8	22.2	21.6
130	394	428	461	494	528	561	594	628	661	694	7	26.6	25.9	25.2
131	727	760	793	826	860	893	926	959	992	*024	8	30.4	29.6	28.8
132	12 057	090	123	156	189	222	254	287	320	352	9	34.2	33.3	32.4
133	385	418	450	483	516	548	581	613	646	678		**35**	**34**	**33**
134	710	743	775	808	840	872	905	937	969	*001	1	3.5	3.4	3.3
135	13 033	066	098	130	162	194	226	258	290	322	2	7.0	6.8	6.6
136	354	386	418	450	481	513	545	577	609	640	3	10.5	10.2	9.9
137	672	704	735	767	799	830	862	893	925	956	4	14.0	13.6	13.2
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270	5	17.5	17.0	16.5
139	14 301	333	364	395	426	457	489	520	551	582	6	21.0	20.4	19.8
140	613	644	675	706	737	768	799	829	860	891	7	24.5	23.8	23.1
141	922	953	983	*014	*045	*076	*106	*137	*168	*198	8	28.0	27.2	26.4
142	15 229	259	290	320	351	381	412	442	473	503	9	31.5	30.6	29.7
143	534	564	594	625	655	685	715	746	776	806		**32**	**31**	**30**
144	836	866	897	927	957	987	*017	*047	*077	*107	1	3.2	3.1	3.0
145	16 137	167	197	227	256	286	316	346	376	406	2	6.4	6.2	6.0
146	435	465	495	524	554	584	613	643	673	702	3	9.6	9.3	9.0
147	732	761	791	820	850	879	909	938	967	997	4	12.8	12.4	12.0
148	17 026	056	085	114	143	173	202	231	260	289	5	16.0	15.5	15.0
149	319	348	377	406	435	464	493	522	551	580	6	19.2	18.6	18.0
150	609	638	667	696	725	754	782	811	840	869	7	22.4	21.7	21.0
											8	25.6	24.8	24.0
											9	28.8	27.9	27.0
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.		

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
150	17 609	638	667	696	725	754	782	811	840	869	
151	898	926	955	984	*013	*041	*070	*099	*127	*156	
152	18 184	213	241	270	298	327	355	384	412	441	29 | 28
153	469	498	526	554	583	611	639	667	696	724	1 | 2.9 | 2.8
154	752	780	808	837	865	893	921	949	977	*005	II | 5.8 | 5.6
155	19 033	061	089	117	145	173	201	229	257	285	3 | 8.7 | 8.4
156	312	340	368	396	424	451	479	507	535	562	4 | 11.6 | 11.2
157	590	618	645	673	700	728	756	783	811	838	5 | 14.5 | 14.0
158	866	893	921	948	976	*003	*030	*058	*085	*112	6 | 17.4 | 16.8
159	20 140	167	194	222	249	276	303	330	358	385	7 | 20.3 | 19.6
160	412	439	466	493	520	548	575	602	629	656	III | 23.2 | 22.4
161	683	710	737	763	790	817	844	871	898	925	9 | 26.1 | 25.2
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192	
163	21 219	245	272	299	325	352	378	405	431	458	27 | 26
164	484	511	537	564	590	617	643	669	696	722	1 | 2.7 | 2.6
165	748	775	801	827	854	880	906	932	958	985	2 | 5.4 | 5.2
166	22 011	037	063	089	115	141	167	194	220	246	3 | 8.1 | 7.8
167	272	298	324	350	376	401	427	453	479	505	4 | 10.8 | 10.4
168	531	557	583	608	634	660	686	712	737	763	5 | 13.5 | 13.0
169	789	814	840	866	891	917	943	968	994	*019	6 | 16.2 | 15.6
170	23 045	070	096	121	147	172	198	223	249	274	7 | 18.9 | 18.2
171	300	325	350	376	401	426	452	477	502	528	8 | 21.6 | 20.8
172	553	578	603	629	654	679	704	729	754	779	9 | 24.3 | 23.4
173	805	830	855	880	905	930	955	980	*005	*030	
174	24 055	080	105	130	155	180	204	229	254	279	25
175	304	329	353	378	403	428	452	477	502	527	1 | 2.5
176	551	576	601	625	650	674	699	724	748	773	2 | 5.0
177	797	822	846	871	895	920	944	969	993	*018	II | 7.5
178	25 042	066	091	115	139	164	188	212	237	261	4 | 10.0
179	285	310	334	358	382	406	431	455	479	503	5 | 12.5
180	527	551	575	600	624	648	672	696	720	744	6 | 15.0
181	768	792	816	840	864	888	912	935	959	983	7 | 17.5
182	26 007	031	055	079	102	126	150	174	198	221	8 | 20.0
183	245	269	293	316	340	364	387	411	435	458	III | 22.5
184	482	505	529	553	576	600	623	647	670	694	
185	717	741	764	788	811	834	858	881	905	928	24 | 23
186	951	975	998	*021	*045	*068	*091	*114	*138	*161	1 | 2.4 | 2.3
187	27 184	207	231	254	277	300	323	346	370	393	2 | 4.8 | 4.6
188	416	439	462	485	508	531	554	577	600	623	3 | 7.2 | 6.9
189	646	669	692	715	738	761	784	807	830	852	4 | 9.6 | 9.2
190	875	898	921	944	967	989	*012	*035	*058	*081	5 | 12.0 | 11.5
191	28 103	126	149	171	194	217	240	262	285	307	6 | 14.4 | 13.8
192	330	353	375	398	421	443	466	488	511	533	7 | 16.8 | 16.1
193	556	578	601	623	646	668	691	713	735	758	8 | 19.2 | 18.4
194	780	803	825	847	870	892	914	937	959	981	9 | 21.6 | 20.7
195	29 003	026	048	070	092	115	137	159	181	203	
196	226	248	270	292	314	336	358	380	403	425	22 | 21
197	447	469	491	513	535	557	579	601	623	645	1 | 2.2 | 2.1
198	667	688	710	732	754	776	798	820	842	863	2 | 4.4 | 4.2
199	885	907	929	951	973	994	*016	*038	*060	*081	3 | 6.6 | 6.3
200	30 103	125	146	168	190	211	233	255	276	298	4 | 8.8 | 8.4
											5 | 11.0 | 10.5
											6 | 13.2 | 12.6
											7 | 15.4 | 14.7
											8 | 17.6 | 16.8
											9 | 19.8 | 18.9
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
200	30 103	125	146	168	190	211	233	255	276	298	
201	320	341	363	384	406	428	449	471	492	514	
202	535	557	578	600	621	643	664	685	707	728	
203	750	771	792	814	835	856	878	899	920	942	
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154	
205	31 175	197	218	239	260	281	302	323	345	366	
206	387	408	429	450	471	492	513	534	555	576	
207	597	618	639	660	681	702	723	744	765	785	
208	806	827	848	869	890	911	931	952	973	994	
209	32 015	035	056	077	098	118	139	160	181	201	
210	222	243	263	284	305	325	346	366	387	408	
211	428	449	469	490	510	531	552	572	593	613	
212	634	654	675	695	715	736	756	777	797	818	
213	838	858	879	899	919	940	960	980	*001	*021	
214	33 041	062	082	102	122	143	163	183	203	224	
215	244	264	284	304	325	345	365	385	405	425	
216	445	465	486	506	526	546	566	586	606	626	
217	646	666	686	706	726	746	766	786	806	826	
218	846	866	885	905	925	945	965	985	*005	*025	
219	34 044	064	084	104	124	143	163	183	203	223	
220	242	262	282	301	321	341	361	380	400	420	
221	439	459	479	498	518	537	557	577	596	616	
222	635	655	674	694	713	733	753	772	792	811	
223	830	850	869	889	908	928	947	967	986	*005	
224	35 025	044	064	083	102	122	141	160	180	199	
225	218	238	257	276	295	315	334	353	372	392	
226	411	430	449	468	488	507	526	545	564	583	
227	603	622	641	660	679	698	717	736	755	774	
228	793	813	832	851	870	889	908	927	946	965	
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154	
230	36 173	192	211	229	248	267	286	305	324	342	
231	361	380	399	418	436	455	474	493	511	530	
232	549	568	586	605	624	642	661	680	698	717	
233	736	754	773	791	810	829	847	866	884	903	
234	922	940	959	977	996	*014	*033	*051	*070	*088	
235	37 107	125	144	162	181	199	218	236	254	273	
236	291	310	328	346	365	383	401	420	438	457	
237	475	493	511	530	548	566	585	603	621	639	
238	658	676	694	712	731	749	767	785	803	822	
239	840	858	876	894	912	931	949	967	985	*003	
240	38 021	039	057	075	093	112	130	148	166	184	
241	202	220	238	256	274	292	310	328	346	364	
242	382	399	417	435	453	471	489	507	525	543	
243	561	578	596	614	632	650	668	686	703	721	
244	739	757	775	792	810	828	846	863	881	899	
245	917	934	952	970	987	*005	*023	*041	*058	*076	
246	39 094	111	129	146	164	182	199	217	235	252	
247	270	287	305	322	340	358	375	393	410	428	
248	445	463	480	498	515	533	550	568	585	602	
249	620	637	655	672	690	707	724	742	759	777	
250	794	811	829	846	863	881	898	915	933	950	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
250	39 794	811	829	846	863	881	898	915	933	950	
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123	
252	40 140	157	175	192	209	226	243	261	278	295	18
253	312	329	346	364	381	398	415	432	449	466	1 | 1.8
254	483	500	518	535	552	569	586	603	620	637	2 | 3.6
255	654	671	688	705	722	739	756	773	790	807	3 | 5.4
256	824	841	858	875	892	909	926	943	960	976	4 | 7.2
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	5 | 9.0
258	41 162	179	196	212	229	246	263	280	296	313	6 | 10.8
259	330	347	363	380	397	414	430	447	464	481	7 | 12.6
											8 | 14.4
260	497	514	531	547	564	581	597	614	631	647	9 | 16.2
261	664	681	697	714	731	747	764	780	797	814	
262	830	847	863	880	896	913	929	946	963	979	
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	17
264	42 160	177	193	210	226	243	259	275	292	308	1 | 1.7
265	325	341	357	374	390	406	423	439	455	472	2 | 3.4
266	488	504	521	537	553	570	586	602	619	635	3 | 5.1
267	651	667	684	700	716	732	749	765	781	797	4 | 6.8
268	813	830	846	862	878	894	911	927	943	959	5 | 8.5
269	975	991	*008	*024	*040	*056	072	*088	*104	*120	6 | 10.2
											7 | 11.9
270	43 136	152	169	185	201	217	233	249	265	281	8 | 13.6
271	297	313	329	345	361	377	393	409	425	441	9 | 15.3
272	457	473	489	505	521	537	553	569	584	600	
273	616	632	648	664	680	696	712	727	743	759	16
274	775	791	807	823	838	854	870	886	902	917	1 | 1.6
275	933	949	965	981	996	*012	*028	*044	*059	*075	2 | 3.2
276	44 091	107	122	138	154	170	185	201	217	232	3 | 4.8
277	248	264	279	295	311	326	342	358	373	389	4 | 6.4
278	404	420	436	451	467	483	498	514	529	545	5 | 8.0
279	560	576	592	607	623	638	654	669	685	700	6 | 9.6
											7 | 11.2
280	716	731	747	762	778	793	809	824	840	855	8 | 12.8
281	871	886	902	917	932	948	963	979	994	*010	9 | 14.4
282	45 025	040	056	071	086	102	117	133	148	163	
283	179	194	209	225	240	255	271	286	301	317	15
284	332	347	362	378	393	408	423	439	454	469	1 | 1.5
285	484	500	515	530	545	561	576	591	606	621	2 | 3.0
286	637	652	667	682	697	712	728	743	758	773	3 | 4.5
287	788	803	818	834	849	864	879	894	909	924	4 | 6.0
288	939	954	969	984	*000	*015	*030	*045	*060	*075	5 | 7.5
289	46 090	105	120	135	150	165	180	195	210	225	6 | 9.0
											7 | 10.5
290	240	255	270	285	300	315	330	345	359	374	8 | 12.0
291	389	404	419	434	449	464	479	494	509	523	9 | 13.5
292	538	553	568	583	598	613	627	642	657	672	
293	687	702	716	731	746	761	776	790	805	820	14
294	835	850	864	879	894	909	923	938	953	967	1 | 1.4
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114	2 | 2.8
296	47 129	144	159	173	188	202	217	232	246	261	3 | 4.2
297	276	290	305	319	334	349	363	378	392	407	4 | 5.6
298	422	436	451	465	480	494	509	524	538	553	5 | 7.0
299	567	582	596	611	625	640	654	669	683	698	6 | 8.4
											7 | 9.8
300	712	727	741	756	770	784	799	813	828	842	8 | 11.2
											9 | 12.6
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
300	47 712	727	741	756	770	784	799	813	828	842	
301	857	871	885	900	914	929	943	958	972	986	
302	48 001	015	029	044	058	073	087	101	116	130	
303	144	159	173	187	202	216	230	244	259	273	
304	287	302	316	330	344	359	373	387	401	416	
305	430	444	458	473	487	501	515	530	544	558	**15**
306	572	586	601	615	629	643	657	671	686	700	1 | 1.5 2 | 3.0
307	714	728	742	756	770	785	799	813	827	841	3 | 4.5
308	855	869	883	897	911	926	940	954	968	982	4 | 6.0
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	5 | 7.5 6 | 9.0
310	49 136	150	164	178	192	206	220	234	248	262	7 | 10.5 8 | 12.0
311	276	290	304	318	332	346	360	374	388	402	9 | 13.5
312	415	429	443	457	471	485	499	513	527	541	
313	554	568	582	596	610	624	638	651	665	679	
314	693	707	721	734	748	762	776	790	803	817	
315	831	845	859	872	886	900	914	927	941	955	
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	**14**
317	50 106	120	133	147	161	174	188	202	215	229	1 | 1.4 2 | 2.8
318	243	256	270	284	297	311	325	338	352	365	3 | 4.2
319	379	393	406	420	433	447	461	474	488	501	4 | 5.6 5 | 7.0
320	515	529	542	556	569	583	596	610	623	637	6 | 8.4
321	651	664	678	691	705	718	732	745	759	772	7 | 9.8 8 | 11.2
322	786	799	813	826	840	853	866	880	893	907	9 | 12.6
323	920	934	947	961	974	987	*001	*014	*028	*041	
324	51 055	068	081	095	108	121	135	148	162	175	
325	188	202	215	228	242	255	268	282	295	308	
326	322	335	348	362	375	388	402	415	428	441	
327	455	468	481	495	508	521	534	548	561	574	**13**
328	587	601	614	627	640	654	667	680	693	706	1 | 1.3 2 | 2.6
329	720	733	746	759	772	786	799	812	825	838	3 | 3.9
330	851	865	878	891	904	917	930	943	957	970	4 | 5.2 5 | 6.5
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	6 | 7.8
332	52 114	127	140	153	166	179	192	205	218	231	7 | 9.1 8 | 10.4
333	244	257	270	284	297	310	323	336	349	362	9 | 11.7
334	375	388	401	414	427	440	453	466	479	492	
335	504	517	530	543	556	569	582	595	608	621	
336	634	647	660	673	686	699	711	724	737	750	
337	763	776	789	802	815	827	840	853	866	879	
338	892	905	917	930	943	956	969	982	994	*007	**12**
339	53 020	033	046	058	071	084	097	110	122	135	1 | 1.2 2 | 2.4
340	148	161	173	186	199	212	224	237	250	263	3 | 3.6
341	275	288	301	314	326	339	352	364	377	390	4 | 4.8 5 | 6.0
342	403	415	428	441	453	466	479	491	504	517	6 | 7.2
343	529	542	555	567	580	593	605	618	631	643	7 | 8.4 8 | 9.6
344	656	668	681	694	706	719	732	744	757	769	9 | 10.8
345	782	794	807	820	832	845	857	870	882	895	
346	908	920	933	945	958	970	983	995	*008	*020	
347	54 033	045	058	070	083	095	108	120	133	145	
348	158	170	183	195	208	220	233	245	258	270	
349	283	295	307	320	332	345	357	370	382	394	
350	407	419	432	444	456	469	481	494	506	518	
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350	54 407	419	432	444	456	469	481	494	506	518	
351	531	543	555	568	580	593	605	617	630	642	
352	654	667	679	691	704	716	728	741	753	765	
353	777	790	802	814	827	839	851	864	876	888	
354	900	913	925	937	949	962	974	986	998	*011	13
355	55 023	035	047	060	072	084	096	108	121	133	1 1.3
356	145	157	169	182	194	206	218	230	242	255	2 2.6
357	267	279	291	303	315	328	340	352	364	376	3 3.9
358	388	400	413	425	437	449	461	473	485	497	4 5.2
359	509	522	534	546	558	570	582	594	606	618	5 6.5
360	630	642	654	666	678	691	703	715	727	739	6 7.8
361	751	763	775	787	799	811	823	835	847	859	7 9.1
362	871	883	895	907	919	931	943	955	967	979	8 10.4
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	9 11.7
364	56 110	122	134	146	158	170	182	194	205	217	
365	229	241	253	265	277	289	301	312	324	336	
366	348	360	372	384	396	407	419	431	443	455	12
367	467	478	490	502	514	526	538	549	561	573	1 1.2
368	585	597	608	620	632	644	656	667	679	691	2 2.4
369	703	714	726	738	750	761	773	785	797	808	3 3.6
370	820	832	844	855	867	879	891	902	914	926	4 4.8
371	937	949	961	972	984	996	*008	*019	*031	*043	5 6.0
372	57 054	066	078	089	101	113	124	136	148	159	6 7.2
373	171	183	194	206	217	229	241	252	264	276	7 8.4
374	287	299	310	322	334	345	357	368	380	392	8 9.6
375	403	415	426	438	449	461	473	484	496	507	9 10.8
376	519	530	542	553	565	576	588	600	611	623	
377	634	646	657	669	680	692	703	715	726	738	
378	749	761	772	784	795	807	818	830	841	852	
379	864	875	887	898	910	921	933	944	955	967	11
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	1 1.1
381	58 092	104	115	127	138	149	161	172	184	195	2 2.2
382	206	218	229	240	252	263	274	286	297	309	3 3.3
383	320	331	343	354	365	377	388	399	410	422	4 4.4
384	433	444	456	467	478	490	501	512	524	535	5 5.5
385	546	557	569	580	591	602	614	625	636	647	6 6.6
386	659	670	681	692	704	715	726	737	749	760	7 7.7
387	771	782	794	805	816	827	838	850	861	872	8 8.8
388	883	894	906	917	928	939	950	961	973	984	9 9.9
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59 106	118	129	140	151	162	173	184	195	207	10
391	218	229	240	251	262	273	284	295	306	318	1 1.0
392	329	340	351	362	373	384	395	406	417	428	2 2.0
393	439	450	461	472	483	494	506	517	528	539	3 3.0
394	550	561	572	583	594	605	616	627	638	649	4 4.0
395	660	671	682	693	704	715	726	737	748	759	5 5.0
396	770	780	791	802	813	824	835	846	857	868	6 6.0
397	879	890	901	912	923	934	945	956	966	977	7 7.0
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086	8 8.0
399	60 097	108	119	130	141	152	163	173	184	195	9 9.0
400	206	217	228	239	249	260	271	282	293	304	
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400	60 206	217	228	239	249	260	271	282	293	304	
401	314	325	336	347	358	369	379	390	401	412	
402	423	433	444	455	466	477	487	498	509	520	
403	531	541	552	563	574	584	595	606	617	627	
404	638	649	660	670	681	692	703	713	724	735	
405	746	756	767	778	788	799	810	821	831	842	
406	853	863	874	885	895	906	917	927	938	949	**11**
407	959	970	981	991	*002	*013	*023	*034	*045	*055	1 1.1
408	61 066	077	087	098	109	119	130	140	151	162	2 2.2
409	172	183	194	204	215	225	236	247	257	268	3 3.3
410	278	289	300	310	321	331	342	352	363	374	4 4.4
411	384	395	405	416	426	437	448	458	469	479	5 5.5
412	490	500	511	521	532	542	553	563	574	584	6 6.6
413	595	606	616	627	637	648	658	669	679	690	7 7.7
414	700	711	721	731	742	752	763	773	784	794	8 8.8
415	805	815	826	836	847	857	868	878	888	899	9 9.9
416	909	920	930	941	951	962	972	982	993	*003	
417	62 014	024	034	045	055	066	076	086	097	107	
418	118	128	138	149	159	170	180	190	201	211	
419	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
421	428	439	449	459	469	480	490	500	511	521	
422	531	542	552	562	572	583	593	603	613	624	**10**
423	634	644	655	665	675	685	696	706	716	726	1 1.0
424	737	747	757	767	778	788	798	808	818	829	2 2.0
425	839	849	859	870	880	890	900	910	921	931	3 3.0
426	941	951	961	972	982	992	*002	*012	*022	*033	4 4.0
427	63 043	053	063	073	083	094	104	114	124	134	5 5.0
428	144	155	165	175	185	195	205	215	225	236	6 6.0
429	246	256	266	276	286	296	306	317	327	337	7 7.0
430	347	357	367	377	387	397	407	417	428	438	8 8.0
431	448	458	468	478	488	498	508	518	528	538	9 9.0
432	548	558	568	579	589	599	609	619	629	639	
433	649	659	669	679	689	699	709	719	729	739	
434	749	759	769	779	789	799	809	819	829	839	
435	849	859	869	879	889	899	909	919	929	939	
436	949	959	969	979	988	998	*008	*018	*028	*038	
437	64 048	058	068	078	088	098	108	118	128	137	**9**
438	147	157	167	177	187	197	207	217	227	237	1 0.9
439	246	256	266	276	286	296	306	316	326	335	2 1.8
440	345	355	365	375	385	395	404	414	424	434	3 2.7
441	444	454	464	473	483	493	503	513	523	532	4 3.6
442	542	552	562	572	582	591	601	611	621	631	5 4.5
443	640	650	660	670	680	689	699	709	719	729	6 5.4
444	738	748	758	768	777	787	797	807	816	826	7 6.3
445	836	846	856	865	875	885	895	904	914	924	8 7.2
446	933	943	953	963	972	982	992	*002	*011	*021	9 8.1
447	65 031	040	050	060	070	079	089	099	108	118	
448	128	137	147	157	167	176	186	196	205	215	
449	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
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450	65 321	331	341	350	360	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	706	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	10
458	66 087	096	106	115	124	134	143	153	162	172	1 1.0
459	181	191	200	210	219	229	238	247	257	266	2 2.0
460	276	285	295	304	314	323	332	342	351	361	3 3.0
461	370	380	389	398	408	417	427	436	445	455	4 4.0
462	464	474	483	492	502	511	521	530	539	549	5 5.0
463	558	567	577	586	596	605	614	624	633	642	6 6.0
464	652	661	671	680	689	699	708	717	727	736	7 7.0
465	745	755	764	773	783	792	801	811	820	829	8 8.0
466	839	848	857	867	876	885	894	904	913	922	9 9.0
467	932	941	950	960	969	978	987	997	*006	*015	
468	67 025	034	043	052	062	071	080	089	099	108	
469	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
471	302	311	321	330	339	348	357	367	376	385	
472	394	403	413	422	431	440	449	459	468	477	9
473	486	495	504	514	523	532	541	550	560	569	1 0.9
474	578	587	596	605	614	624	633	642	651	660	2 1.8
475	669	679	688	697	706	715	724	733	742	752	3 2.7
476	761	770	779	788	797	806	815	825	834	843	4 3.6
477	852	861	870	879	888	897	906	916	925	934	5 4.5
478	943	952	961	970	979	988	997	*006	*015	*024	6 5.4
479	68 034	043	052	061	070	079	088	097	106	115	7 6.3
480	124	133	142	151	160	169	178	187	196	205	8 7.2
481	215	224	233	242	251	260	269	278	287	296	9 8.1
482	305	314	323	332	341	350	359	368	377	386	
483	395	404	413	422	431	440	449	458	467	476	
484	485	494	502	511	520	529	538	547	556	565	
485	574	583	592	601	610	619	628	637	646	655	
486	664	673	681	690	699	708	717	726	735	744	
487	753	762	771	780	789	797	806	815	824	833	8
488	842	851	860	869	878	886	895	904	913	922	1 0.8
489	931	940	949	958	966	975	984	993	*002	*011	2 1.6
490	69 020	028	037	046	055	064	073	082	090	099	3 2.4
491	108	117	126	135	144	152	161	170	179	188	4 3.2
492	197	205	214	223	232	241	249	258	267	276	5 4.0
493	285	294	302	311	320	329	338	346	355	364	6 4.8
494	373	381	390	399	408	417	425	434	443	452	7 5.6
495	461	469	478	487	496	504	513	522	531	539	8 6.4
496	548	557	566	574	583	592	601	609	618	627	9 7.2
497	636	644	653	662	671	679	688	697	705	714	
498	723	732	740	749	758	767	775	784	793	801	
499	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
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500	69 897	906	914	923	932	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	157	165	174	183	191	200	209	217	226	234	
504	243	252	260	269	278	286	295	303	312	321	
505	329	338	346	355	364	372	381	389	398	406	
506	415	424	432	441	449	458	467	475	484	492	
507	501	509	518	526	535	544	552	561	569	578	
508	586	595	603	612	621	629	638	646	655	663	
509	672	680	689	697	706	714	723	731	740	749	
510	757	766	774	783	791	800	808	817	825	834	
511	842	851	859	868	876	885	893	902	910	919	
512	927	935	944	952	961	969	978	986	995	*003	
513	71 012	020	029	037	046	054	063	071	079	088	
514	096	105	113	122	130	139	147	155	164	172	
515	181	189	198	206	214	223	231	240	248	257	
516	265	273	282	290	299	307	315	324	332	341	
517	349	357	366	374	383	391	399	408	416	425	
518	433	441	450	458	466	475	483	492	500	508	
519	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
521	684	692	700	709	717	725	734	742	750	759	
522	767	775	784	792	800	809	817	825	834	842	
523	850	858	867	875	883	892	900	908	917	925	
524	933	941	950	958	966	975	983	991	999	*008	
525	72 016	024	032	041	049	057	066	074	082	090	
526	099	107	115	123	132	140	148	156	165	173	
527	181	189	198	206	214	222	230	239	247	255	
528	263	272	280	288	296	304	313	321	329	337	
529	346	354	362	370	378	387	395	403	411	419	
530	428	436	444	452	460	469	477	485	493	501	
531	509	518	526	534	542	550	558	567	575	583	
532	591	599	607	616	624	632	640	648	656	665	
533	673	681	689	697	705	713	722	730	738	746	
534	754	762	770	779	787	795	803	811	819	827	
535	835	843	852	860	868	876	884	892	900	908	
536	916	925	933	941	949	957	965	973	981	989	
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
538	73 078	086	094	102	111	119	127	135	143	151	
539	159	167	175	183	191	199	207	215	223	231	
540	239	247	255	263	272	280	288	296	304	312	
541	320	328	336	344	352	360	368	376	384	392	
542	400	408	416	424	432	440	448	456	464	472	
543	480	488	496	504	512	520	528	536	544	552	
544	560	568	576	584	592	600	608	616	624	632	
545	640	648	656	664	672	679	687	695	703	711	
546	719	727	735	743	751	759	767	775	783	791	
547	799	807	815	823	830	838	846	854	862	870	
548	878	886	894	902	910	918	926	933	941	949	
549	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
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8	7.2
9	8.1

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4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

7	
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3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

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551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
561	896	904	912	920	927	935	943	950	958	966	
562	974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75 051	059	066	074	082	089	097	105	113	120	
564	128	136	143	151	159	166	174	182	189	197	
565	205	213	220	228	236	243	251	259	266	274	
566	282	289	297	305	312	320	328	335	343	351	
567	358	366	374	381	389	397	404	412	420	427	
568	435	442	450	458	465	473	481	488	496	504	
569	511	519	526	534	542	549	557	565	572	580	
570	587	595	603	610	618	626	633	641	648	656	
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
581	418	425	433	440	448	455	462	470	477	485	
582	492	500	507	515	522	530	537	545	552	559	
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585	716	723	730	738	745	753	760	768	775	782	
586	790	797	805	812	819	827	834	842	849	856	
587	864	871	879	886	893	901	908	916	923	930	
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592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
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605	176	183	190	197	204	211	219	226	233	240	
606	247	254	262	269	276	283	290	297	305	312	
607	319	326	333	340	347	355	362	369	376	383	8
608	390	398	405	412	419	426	433	440	447	455	1 0.8
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636	346	353	359	366	373	380	387	393	400	407	
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653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
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671	672	679	685	692	698	705	711	718	724	730	
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673	802	808	814	821	827	834	840	847	853	860	
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676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
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681	315	321	327	334	340	347	353	359	366	372	
682	378	385	391	398	404	410	417	423	429	436	
683	442	448	455	461	467	474	480	487	493	499	
684	506	512	518	525	531	537	544	550	556	563	
685	569	575	582	588	594	601	607	613	620	626	6
686	632	639	645	651	658	664	670	677	683	689	1 0.6
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697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
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704	757	763	770	776	782	788	794	800	807	813	
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737	747	753	759	764	770	776	782	788	794	800	**5**
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787	597	603	609	614	620	625	631	636	642	647	
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843	583	588	593	598	603	609	614	619	624	629	
844	634	639	645	650	655	660	665	670	675	681	
845	686	691	696	701	706	711	716	722	727	732	
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847	788	793	799	804	809	814	819	824	829	834	
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9	4.5

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
850	92 942	947	952	957	962	967	973	978	983	988	
851	993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93 044	049	054	059	064	069	075	080	085	090	
853	095	100	105	110	115	120	125	131	136	141	
854	146	151	156	161	166	171	176	181	186	192	
855	197	202	207	212	217	222	227	232	237	242	
856	247	252	258	263	268	273	278	283	288	293	
857	298	303	308	313	318	323	328	334	339	344	6
858	349	354	359	364	369	374	379	384	389	394	1 0.6
859	399	404	409	414	420	425	430	435	440	445	2 1.2
860	450	455	460	465	470	475	480	485	490	495	3 1.8
861	500	505	510	515	520	526	531	536	541	546	4 2.4
862	551	556	561	566	571	576	581	586	591	596	5 3.0
863	601	606	611	616	621	626	631	636	641	646	6 3.6
864	651	656	661	666	671	676	682	687	692	697	7 4.2
865	702	707	712	717	722	727	732	737	742	747	8 4.8
866	752	757	762	767	772	777	782	787	792	797	9 5.4
867	802	807	812	817	822	827	832	837	842	847	
868	852	857	862	867	872	877	882	887	892	897	
869	902	907	912	917	922	927	932	937	942	947	
870	952	957	962	967	972	977	982	987	992	997	
871	94 002	007	012	017	022	027	032	037	042	047	
872	052	057	062	067	072	077	082	086	091	096	
873	101	106	111	116	121	126	131	136	141	146	5
874	151	156	161	166	171	176	181	186	191	196	1 0.5
875	201	206	211	216	221	226	231	236	240	245	2 1.0
876	250	255	260	265	270	275	280	285	290	295	3 1.5
877	300	305	310	315	320	325	330	335	340	345	4 2.0
878	349	354	359	364	369	374	379	384	389	394	5 2.5
879	399	404	409	414	419	424	429	433	438	443	6 3.0
880	448	453	458	463	468	473	478	483	488	493	7 3.5
881	498	503	507	512	517	522	527	532	537	542	8 4.0
882	547	552	557	562	567	571	576	581	586	591	9 4.5
883	596	601	606	611	616	621	626	630	635	640	
884	645	650	655	660	665	670	675	680	685	689	
885	694	699	704	709	714	719	724	729	734	738	
886	743	748	753	758	763	768	773	778	783	787	
887	792	797	802	807	812	817	822	827	832	836	
888	841	846	851	856	861	866	871	876	880	885	4
889	890	895	900	905	910	915	919	924	929	934	1 0.4
890	939	944	949	954	959	963	968	973	978	983	2 0.8
891	988	993	998	*002	*007	*012	*017	*022	*027	*032	3 1.2
892	95 036	041	046	051	056	061	066	071	075	080	4 1.6
893	085	090	095	100	105	109	114	119	124	129	5 2.0
894	134	139	143	148	153	158	163	168	173	177	6 2.4
895	182	187	192	197	202	207	211	216	221	226	7 2.8
896	231	236	240	245	250	255	260	265	270	274	8 3.2
897	279	284	289	294	299	303	308	313	318	323	9 3.6
898	328	332	337	342	347	352	357	361	366	371	
899	376	381	386	390	395	400	405	410	415	419	
900	424	429	434	439	444	448	453	458	463	468	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
900	95 424	429	434	439	444	448	453	458	463	468	
901	472	477	482	487	492	497	501	506	511	516	
902	521	525	530	535	540	545	550	554	559	564	
903	569	574	578	583	588	593	598	602	607	612	
904	617	622	626	631	636	641	646	650	655	660	
905	665	670	674	679	684	689	694	698	703	708	
906	713	718	722	727	732	737	742	746	751	756	
907	761	766	770	775	780	785	789	794	799	804	
908	809	813	818	823	828	832	837	842	847	852	
909	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	
911	952	957	961	966	971	976	980	985	990	995	5
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	1 0.5
913	96 047	052	057	061	066	071	076	080	085	090	2 1.0
914	095	099	104	109	114	118	123	128	133	137	3 1.5
915	142	147	152	156	161	166	171	175	180	185	4 2.0
916	190	194	199	204	209	213	218	223	227	232	5 2.5
917	237	242	246	251	256	261	265	270	275	280	6 3.0
918	284	289	294	298	303	308	313	317	322	327	7 3.5
919	332	336	341	346	350	355	360	365	369	374	8 4.0
920	379	384	388	393	398	402	407	412	417	421	9 4.5
921	426	431	435	440	445	450	454	459	464	468	
922	473	478	483	487	492	497	501	506	511	515	
923	520	525	530	534	539	544	548	553	558	562	
924	567	572	577	581	586	591	595	600	605	609	
925	614	619	624	628	633	638	642	647	652	656	
926	661	666	670	675	680	685	689	694	699	703	
927	708	713	717	722	727	731	736	741	745	750	
928	755	759	764	769	774	778	783	788	792	797	
929	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	
931	895	900	904	909	914	918	923	928	932	937	
932	942	946	951	956	960	965	970	974	979	984	
933	988	993	997	*002	*007	*011	*016	*021	*025	*030	
934	97 035	039	044	049	053	058	063	067	072	077	4
935	081	086	090	095	100	104	109	114	118	123	1 0.4
936	128	132	137	142	146	151	155	160	165	169	2 0.8
937	174	179	183	188	192	197	202	206	211	216	3 1.2
938	220	225	230	234	239	243	248	253	257	262	4 1.6
939	267	271	276	280	285	290	294	299	304	308	5 2.0
940	313	317	322	327	331	336	340	345	350	354	6 2.4
941	359	364	368	373	377	382	387	391	396	400	7 2.8
942	405	410	414	419	424	428	433	437	442	447	8 3.2
943	451	456	460	465	470	474	479	483	488	493	9 3.6
944	497	502	506	511	516	520	525	529	534	539	
945	543	548	552	557	562	566	571	575	580	585	
946	589	594	598	603	607	612	617	621	626	630	
947	635	640	644	649	653	658	663	667	672	676	
948	681	685	690	695	699	704	708	713	717	722	
949	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
950	97 772	777	782	786	791	795	800	804	809	813	
951	818	823	827	832	836	841	845	850	855	859	
952	864	868	873	877	882	886	891	896	900	905	
953	909	914	918	923	928	932	937	941	946	950	
954	955	959	964	968	973	978	982	987	991	996	
955	98 000	005	009	014	019	023	028	032	037	041	
956	046	050	055	059	064	068	073	078	082	087	
957	091	096	100	105	109	114	118	123	127	132	
958	137	141	146	150	155	159	164	168	173	177	
959	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	
961	272	277	281	286	290	295	299	304	308	313	
962	318	322	327	331	336	340	345	349	354	358	**5**
963	363	367	372	376	381	385	390	394	399	403	1 | 0.5
964	408	412	417	421	426	430	435	439	444	448	2 | 1.0
965	453	457	462	466	471	475	480	484	489	493	3 | 1.5
966	498	502	507	511	516	520	525	529	534	538	4 | 2.0
967	543	547	552	556	561	565	570	574	579	583	5 | 2.5
968	588	592	597	601	605	610	614	619	623	628	6 | 3.0
969	632	637	641	646	650	655	659	664	668	673	7 | 3.5
970	677	682	686	691	695	700	704	709	713	717	8 | 4.0
971	722	726	731	735	740	744	749	753	758	762	9 | 4.5
972	767	771	776	780	784	789	793	798	802	807	
973	811	816	820	825	829	834	838	843	847	851	
974	856	860	865	869	874	878	883	887	892	896	
975	900	905	909	914	918	923	927	932	936	941	
976	945	949	954	958	963	967	972	976	981	985	
977	989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99 034	038	043	047	052	056	061	065	069	074	
979	078	083	087	092	096	100	105	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	
981	167	171	176	180	185	189	193	198	202	207	**4**
982	211	216	220	224	229	233	238	242	247	251	1 | 0.4
983	255	260	264	269	273	277	282	286	291	295	2 | 0.8
984	300	304	308	313	317	322	326	330	335	339	3 | 1.2
985	344	348	352	357	361	366	370	374	379	383	4 | 1.6
986	388	392	396	401	405	410	414	419	423	427	5 | 2.0
987	432	436	441	445	449	454	458	463	467	471	6 | 2.4
988	476	480	484	489	493	498	502	506	511	515	7 | 2.8
989	520	524	528	533	537	542	546	550	555	559	8 | 3.2
990	564	568	572	577	581	585	590	594	599	603	9 | 3.6
991	607	612	616	621	625	629	634	638	642	647	
992	651	656	660	664	669	673	677	682	686	691	
993	695	699	704	708	712	717	721	726	730	734	
994	739	743	747	752	756	760	765	769	774	778	
995	782	787	791	795	800	804	808	813	817	822	
996	826	830	835	839	843	848	852	856	861	865	
997	870	874	878	883	887	891	896	900	904	909	
998	913	917	922	926	930	935	939	944	948	952	
999	957	961	965	970	974	978	983	987	991	996	
1000	00 000	004	009	013	017	022	026	030	035	039	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

TABLES

OF

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS

GIVING THE VALUES OF THE FUNCTIONS FOR ALL DEGREES AND MINUTES FROM 0° TO 90°

′	0°		1°		2°		3°		4°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.00000	1.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	1.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	1.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	1.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	1.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	1.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	1.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	1.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	1.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	1.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	1.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99967	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99781	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17865	.98481	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.17365	.98481	.19081	.98163	.20791	[illegible]	.22495	.97437	.24192	.97030	60
1	[illegible]	.98476	[illegible]	[illegible]	[illegible]	.97809	.22523	.97430	.24220	.97023	59
2	.17422	[illegible]	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	[illegible]	.98146	.20877	[illegible]	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	[illegible]	[illegible]	[illegible]	.24305	.97001	56
5	.17508	.98455	.19224	.98135	[illegible]	[illegible]	.22637	[illegible]	.24333	.96994	55
6	.17537	.98450	.19252	[illegible]	.20962	[illegible]	.22665	.97398	[illegible]	.96987	54
7	.17565	[illegible]	[illegible]	.98124	[illegible]	[illegible]	.22693	.97391	.24390	.96980	53
8	[illegible]	[illegible]	.19309	.98118	[illegible]	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	[illegible]	.22750	.97378	.24446	.96966	51
10	.17651	.98430	[illegible]	[illegible]	.21076	.97754	[illegible]	[illegible]	.24474	[illegible]	50
11	.17680	[illegible]	.19395	.98101	[illegible]	[illegible]	.22807	[illegible]	.24503	.96952	49
12	.17708	.98420	[illegible]	.98096	[illegible]	[illegible]	.22835	[illegible]	.24531	.96945	48
13	.17737	[illegible]	.19452	[illegible]	[illegible]	.97735	.22863	.97351	[illegible]	.96937	47
14	.17766	.98409	.19481	.98084	.21189	[illegible]	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	[illegible]	.21218	[illegible]	[illegible]	[illegible]	.24615	[illegible]	45
16	.17823	[illegible]	[illegible]	.98073	.21246	.97717	[illegible]	.97331	.24644	.96916	44
17	.17852	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.22977	[illegible]	.24672	[illegible]	43
18	[illegible]	[illegible]	[illegible]	.98061	[illegible]	[illegible]	.23005	.97318	.24700	.96902	42
19	[illegible]	[illegible]	[illegible]	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	[illegible]	.24756	.96887	40
21	[illegible]	.98373	.19680	[illegible]	.21388	.97686	.23090	[illegible]	.24784	[illegible]	39
22	.17995	.98368	[illegible]	.98039	.21417	.97680	.23118	[illegible]	.24813	.96873	38
23	[illegible]	.98362	.19737	.98033	[illegible]	[illegible]	.23146	[illegible]	.24841	.96866	37
24	[illegible]	[illegible]	[illegible]	[illegible]	.21474	.97667	[illegible]	[illegible]	.24869	.96858	36
25	[illegible]	.98352	.19794	.98021	[illegible]	.97661	[illegible]	.97271	.24897	.96851	35
26	[illegible]	[illegible]	.19823	.98016	[illegible]	[illegible]	.23231	[illegible]	[illegible]	.96844	34
27	[illegible]	[illegible]	.19851	.98010	.21559	[illegible]	.23260	[illegible]	[illegible]	.96837	33
28	[illegible]	.98336	.19880	.98004	.21587	.97642	.23288	.97251	[illegible]	.96829	32
29	[illegible]	[illegible]	[illegible]	[illegible]	.21616	.97636	.23316	.97244	.25010	[illegible]	31
30	.18224	.98325	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.96815	30
31	.18252	[illegible]	.19965	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.25066	.96807	29
32	[illegible]	.98315	.19994	[illegible]	.21701	[illegible]	.23401	[illegible]	[illegible]	[illegible]	28
33	[illegible]	[illegible]	.20022	[illegible]	.21729	.97611	.23429	[illegible]	.25122	[illegible]	27
34	.18338	[illegible]	.20051	[illegible]	[illegible]	.97604	[illegible]	[illegible]	.25151	.96786	26
35	.18367	.98299	[illegible]	[illegible]	.21786	[illegible]	[illegible]	.97203	.25179	.96778	25
36	.18395	.98294	[illegible]	.97958	.21814	[illegible]	[illegible]	.97196	.25207	[illegible]	24
37	[illegible]	[illegible]	[illegible]	[illegible]	.21843	[illegible]	.23542	.97189	[illegible]	[illegible]	23
38	.18452	[illegible]	.20165	[illegible]	[illegible]	.97579	[illegible]	[illegible]	.25263	.96756	22
39	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.97573	[illegible]	.97176	.25291	.96749	21
40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.23627	[illegible]	.25320	.96742	20
41	.18538	[illegible]	[illegible]	[illegible]	.21956	[illegible]	.23656	.97162	.25348	[illegible]	19
42	.18567	[illegible]	[illegible]	[illegible]	.21985	[illegible]	[illegible]	[illegible]	.25376	.96727	18
43	.18595	[illegible]	[illegible]	.97916	[illegible]	.97547	[illegible]	[illegible]	.25404	[illegible]	17
44	[illegible]	[illegible]	.20336	.97910	.22041	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	16
45	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.23769	.97134	[illegible]	.96705	15
46	.18681	[illegible]	[illegible]	[illegible]	.22098	[illegible]	.23797	[illegible]	.25488	.96697	14
47	.18710	[illegible]	[illegible]	[illegible]	.22126	.97521	.23825	[illegible]	.25516	[illegible]	13
48	.18738	[illegible]	[illegible]	.97887	.22155	[illegible]	.23853	[illegible]	[illegible]	[illegible]	12
49	[illegible]	[illegible]	[illegible]	.97881	[illegible]	[illegible]	.23882	[illegible]	.25573	[illegible]	11
50	.18795	[illegible]	.20507	[illegible]	.22212	[illegible]	.23910	[illegible]	.25601	[illegible]	10
51	[illegible]	.98212	.20535	.97869	[illegible]	[illegible]	.23938	[illegible]	[illegible]	[illegible]	9
52	[illegible]	[illegible]	.20563	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8
53	[illegible]	[illegible]	.20592	[illegible]	[illegible]	[illegible]	.23995	[illegible]	[illegible]	[illegible]	7
54	.18910	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.24023	[illegible]	[illegible]	[illegible]	6
55	.18938	[illegible]	[illegible]	.97845	[illegible]	[illegible]	.24051	[illegible]	.25741	.96630	5
56	.18967	[illegible]	[illegible]	[illegible]	.22382	[illegible]	[illegible]	[illegible]	.25769	[illegible]	4
57	[illegible]	[illegible]	[illegible]	[illegible]	.22410	.97457	.24108	[illegible]	[illegible]	[illegible]	3
58	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.24136	[illegible]	.25826	[illegible]	2
59	[illegible]	[illegible]	.20763	[illegible]	.22467	.97444	[illegible]	[illegible]	[illegible]	[illegible]	1
60	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.25882	[illegible]	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92118	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	69°		68°		67°		66°		65°		

′	25°		26°		27°		28°		29°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73845	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	49°		48°		47°		46°		45°		

′	0°		1°		2°		3°		4°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infin.	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	.07022	14.2411	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	.07051	14.1821	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	.07080	14.1235	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	.07110	14.0655	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	.07139	14.0079	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	.07168	13.9507	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	.07197	13.8940	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	.07227	13.8378	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	.07256	13.7821	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	.07285	13.7267	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	.07314	13.6719	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	.07344	13.6174	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	.07373	13.5634	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	.07402	13.5098	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	.07431	13.4566	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	.07461	13.4039	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	.07490	13.3515	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	.07519	13.2996	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	.07548	13.2480	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	.07578	13.1969	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	.07607	13.1461	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	.07636	13.0958	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	.07665	13.0458	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	.07695	12.9962	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	.07724	12.9469	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	.07753	12.8981	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	.07782	12.8496	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	.07812	12.8014	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	.07841	12.7536	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	.07870	12.7062	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	.07899	12.6591	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	.07929	12.6124	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	.07958	12.5660	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	.07987	12.5199	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	.08017	12.4742	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	.08046	12.4288	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	.08075	12.3838	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	.08104	12.3390	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	.08134	12.2946	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	.08163	12.2505	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	.08192	12.2067	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	.08221	12.1632	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	.08251	12.1201	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	.08280	12.0772	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	.08309	12.0346	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	.08339	11.9923	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	.08368	11.9504	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	.08397	11.9087	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	.08427	11.8673	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	.08456	11.8262	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	.08485	11.7853	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	.08514	11.7448	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	.08544	11.7045	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	.08573	11.6645	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	.08602	11.6248	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	.08632	11.5853	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	.08661	11.5461	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	.08690	11.5072	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	.08720	11.4685	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	.08749	11.4301	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	60
1	.08778	11.3919	.10540	9.48781	.12308	8.12481	.14084	7.10038	.15868	6.30189	59
2	.08807	11.3540	.10569	9.46141	.12338	8.10536	.14113	7.08546	.15898	6.29007	58
3	.08837	11.3163	.10599	9.43515	.12367	8.08600	.14143	7.07059	.15928	6.27829	57
4	.08866	11.2789	.10628	9.40904	.12397	8.06674	.14173	7.05579	.15958	6.26655	56
5	.08895	11.2417	.10657	9.38307	.12426	8.04756	.14202	7.04105	.15988	6.25486	55
6	.08925	11.2048	.10687	9.35724	.12456	8.02848	.14232	7.02637	.16017	6.24321	54
7	.08954	11.1681	.10716	9.33155	.12485	8.00948	.14262	7.01174	.16047	6.23160	53
8	.08983	11.1316	.10746	9.30599	.12515	7.99058	.14291	6.99718	.16077	6.22003	52
9	.09013	11.0954	.10775	9.28058	.12544	7.97176	.14321	6.98268	.16107	6.20851	51
10	.09042	11.0594	.10805	9.25530	.12574	7.95302	.14351	6.96823	.16137	6.19703	50
11	.09071	11.0237	.10834	9.23016	.12603	7.93438	.14381	6.95385	.16167	6.18559	49
12	.09101	10.9882	.10863	9.20516	.12633	7.91582	.14410	6.93952	.16196	6.17419	48
13	.09130	10.9529	.10893	9.18028	.12662	7.89734	.14440	6.92525	.16226	6.16283	47
14	.09159	10.9178	.10922	9.15554	.12692	7.87895	.14470	6.91104	.16256	6.15151	46
15	.09189	10.8829	.10952	9.13093	.12722	7.86064	.14499	6.89688	.16286	6.14023	45
16	.09218	10.8483	.10981	9.10646	.12751	7.84242	.14529	6.88278	.16316	6.12899	44
17	.09247	10.8139	.11011	9.08211	.12781	7.82428	.14559	6.86874	.16346	6.11779	43
18	.09277	10.7797	.11040	9.05789	.12810	7.80622	.14588	6.85475	.16376	6.10664	42
19	.09306	10.7457	.11070	9.03379	.12840	7.78825	.14618	6.84082	.16405	6.09552	41
20	.09335	10.7119	.11099	9.00983	.12869	7.77035	.14648	6.82694	.16435	6.08444	40
21	.09365	10.6783	.11128	8.98598	.12899	7.75254	.14678	6.81812	.16465	6.07340	39
22	.09394	10.6450	.11158	8.96227	.12929	7.73480	.14707	6.79936	.16495	6.06240	38
23	.09423	10.6118	.11187	8.93867	.12958	7.71715	.14737	6.78564	.16525	6.05143	37
24	.09453	10.5789	.11217	8.91520	.12988	7.69957	.14767	6.77199	.16555	6.04051	36
25	.09482	10.5462	.11246	8.89185	.13017	7.68208	.14796	6.75838	.16585	6.02962	35
26	.09511	10.5136	.11276	8.86862	.13047	7.66466	.14826	6.74483	.16615	6.01878	34
27	.09541	10.4813	.11305	8.84551	.13076	7.64732	.14856	6.73135	.16645	6.00797	33
28	.09570	10.4491	.11335	8.82252	.13106	7.63005	.14886	6.71789	.16674	5.99720	32
29	.09600	10.4172	.11364	8.79964	.13136	7.61287	.14915	6.70450	.16704	5.98646	31
30	.09629	10.3854	.11394	8.77689	.13165	7.59575	.14945	6.69116	.16734	5.97576	30
31	.09658	10.3538	.11423	8.75425	.13195	7.57872	.14975	6.67787	.16764	5.96510	29
32	.09688	10.3224	.11452	8.73172	.13224	7.56176	.15005	6.66463	.16794	5.95448	28
33	.09717	10.2913	.11482	8.70931	.13254	7.54487	.15034	6.65144	.16824	5.94390	27
34	.09746	10.2602	.11511	8.68701	.13284	7.52806	.15064	6.63831	.16854	5.93335	26
35	.09776	10.2294	.11541	8.66482	.13313	7.51132	.15094	6.62523	.16884	5.92283	25
36	.09805	10.1988	.11570	8.64275	.13343	7.49465	.15124	6.61219	.16914	5.91236	24
37	.09834	10.1683	.11600	8.62078	.13372	7.47806	.15153	6.59921	.16944	5.90191	23
38	.09864	10.1381	.11629	8.59893	.13402	7.46154	.15183	6.58627	.16974	5.89151	22
39	.09893	10.1080	.11659	8.57718	.13432	7.44509	.15213	6.57339	.17004	5.88114	21
40	.09923	10.0780	.11688	8.55555	.13461	7.42871	.15243	6.56055	.17033	5.87080	20
41	.09952	10.0483	.11718	8.53402	.13491	7.41240	.15272	6.54777	.17063	5.86051	19
42	.09981	10.0187	.11747	8.51259	.13521	7.39616	.15302	6.53503	.17093	5.85024	18
43	.10011	9.98931	.11777	8.49128	.13550	7.37999	.15332	6.52234	.17123	5.84001	17
44	.10040	9.96007	.11806	8.47007	.13580	7.36389	.15362	6.50970	.17153	5.82982	16
45	.10069	9.93101	.11836	8.44896	.13609	7.34786	.15391	6.49710	.17183	5.81966	15
46	.10099	9.90211	.11865	8.42795	.13639	7.33190	.15421	6.48456	.17213	5.80953	14
47	.10128	9.87338	.11895	8.40705	.13669	7.31600	.15451	6.47206	.17243	5.79944	13
48	.10158	9.84482	.11924	8.38625	.13698	7.30018	.15481	6.45961	.17273	5.78938	12
49	.10187	9.81641	.11954	8.36555	.13728	7.28442	.15511	6.44720	.17303	5.77936	11
50	.10216	9.78817	.11983	8.34496	.13758	7.26873	.15540	6.43484	.17333	5.76937	10
51	.10246	9.76009	.12013	8.32446	.13787	7.25310	.15570	6.42253	.17363	5.75941	9
52	.10275	9.73217	.12042	8.30406	.13817	7.23754	.15600	6.41026	.17393	5.74949	8
53	.10305	9.70441	.12072	8.28376	.13846	7.22204	.15630	6.39804	.17423	5.73960	7
54	.10334	9.67680	.12101	8.26355	.13876	7.20661	.15660	6.38587	.17453	5.72974	6
55	.10363	9.64935	.12131	8.24345	.13906	7.19125	.15689	6.37374	.17483	5.71992	5
56	.10393	9.62205	.12160	8.22344	.13935	7.17594	.15719	6.36165	.17513	5.71013	4
57	.10422	9.59490	.12190	8.20352	.13965	7.16071	.15749	6.34961	.17543	5.70037	3
58	.10452	9.56791	.12219	8.18370	.13995	7.14553	.15779	6.33761	.17573	5.69064	2
59	.10481	9.54106	.12249	8.16398	.14024	7.13042	.15809	6.32566	.17603	5.68094	1
60	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	.17633	5.67128	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.17633	5.67128	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	60
1	.17663	5.66165	.19468	5.13658	.21286	4.69791	.23117	4.32573	.24964	4.00582	59
2	.17693	5.65205	.19498	5.12862	.21316	4.69121	.23148	4.32001	.24995	4.00086	58
3	.17723	5.64248	.19529	5.12069	.21347	4.68452	.23179	4.31430	.25026	3.99592	57
4	.17753	5.63295	.19559	5.11279	.21377	4.67786	.23209	4.30860	.25056	3.99099	56
5	.17783	5.62344	.19589	5.10490	.21408	4.67121	.23240	4.30291	.25087	3.98607	55
6	.17813	5.61397	.19619	5.09704	.21438	4.66458	.23271	4.29724	.25118	3.98117	54
7	.17843	5.60452	.19649	5.08921	.21469	4.65797	.23301	4.29159	.25149	3.97627	53
8	.17873	5.59511	.19680	5.08139	.21499	4.65138	.23332	4.28595	.25180	3.97139	52
9	.17903	5.58573	.19710	5.07360	.21529	4.64480	.23363	4.28032	.25211	3.96651	51
10	.17933	5.57638	.19740	5.06584	.21560	4.63825	.23393	4.27471	.25242	3.96165	50
11	.17963	5.56706	.19770	5.05809	.21590	4.63171	.23424	4.26911	.25273	3.95680	49
12	.17993	5.55777	.19801	5.05037	.21621	4.62518	.23455	4.26352	.25304	3.95196	48
13	.18023	5.54851	.19831	5.04267	.21651	4.61868	.23485	4.25795	.25335	3.94713	47
14	.18053	5.53927	.19861	5.03499	.21682	4.61219	.23516	4.25239	.25366	3.94232	46
15	.18083	5.53007	.19891	5.02734	.21712	4.60572	.23547	4.24685	.25397	3.93751	45
16	.18113	5.52090	.19921	5.01971	.21743	4.59927	.23578	4.24132	.25428	3.93271	44
17	.18143	5.51176	.19952	5.01210	.21773	4.59283	.23608	4.23580	.25459	3.92793	43
18	.18173	5.50264	.19982	5.00451	.21804	4.58641	.23639	4.23030	.25490	3.92316	42
19	.18203	5.49356	.20012	4.99695	.21834	4.58001	.23670	4.22481	.25521	3.91839	41
20	.18233	5.48451	.20042	4.98940	.21864	4.57363	.23700	4.21933	.25552	3.91364	40
21	.18263	5.47548	.20073	4.98188	.21895	4.56726	.23731	4.21387	.25583	3.90890	39
22	.18293	5.46648	.20103	4.97438	.21925	4.56091	.23762	4.20842	.25614	3.90417	38
23	.18323	5.45751	.20133	4.96690	.21956	4.55458	.23793	4.20298	.25645	3.89945	37
24	.18353	5.44857	.20164	4.95945	.21986	4.54826	.23823	4.19756	.25676	3.89474	36
25	.18384	5.43966	.20194	4.95201	.22017	4.54196	.23854	4.19215	.25707	3.89004	35
26	.18414	5.43077	.20224	4.94460	.22047	4.53568	.23885	4.18675	.25738	3.88536	34
27	.18444	5.42192	.20254	4.93721	.22078	4.52941	.23916	4.18137	.25769	3.88068	33
28	.18474	5.41309	.20285	4.92984	.22108	4.52316	.23946	4.17600	.25800	3.87601	32
29	.18504	5.40429	.20315	4.92249	.22139	4.51693	.23977	4.17064	.25831	3.87136	31
30	.18534	5.39552	.20345	4.91516	.22169	4.51071	.24008	4.16530	.25862	3.86671	30
31	.18564	5.38677	.20376	4.90785	.22200	4.50451	.24039	4.15997	.25893	3.86208	29
32	.18594	5.37805	.20406	4.90056	.22231	4.49832	.24069	4.15465	.25924	3.85745	28
33	.18624	5.36936	.20436	4.89330	.22261	4.49215	.24100	4.14934	.25955	3.85284	27
34	.18654	5.36070	.20466	4.88605	.22292	4.48600	.24131	4.14405	.25986	3.84824	26
35	.18684	5.35206	.20497	4.87882	.22322	4.47986	.24162	4.13877	.26017	3.84364	25
36	.18714	5.34345	.20527	4.87162	.22353	4.47374	.24193	4.13350	.26048	3.83906	24
37	.18745	5.33487	.20557	4.86444	.22383	4.46764	.24223	4.12825	.26079	3.83449	23
38	.18775	5.32631	.20588	4.85727	.22414	4.46155	.24254	4.12301	.26110	3.82992	22
39	.18805	5.31778	.20618	4.85013	.22444	4.45548	.24285	4.11778	.26141	3.82537	21
40	.18835	5.30928	.20648	4.84300	.22475	4.44942	.24316	4.11256	.26172	3.82083	20
41	.18865	5.30080	.20679	4.83590	.22505	4.44338	.24347	4.10736	.26203	3.81630	19
42	.18895	5.29235	.20709	4.82882	.22536	4.43735	.24377	4.10216	.26235	3.81177	18
43	.18925	5.28393	.20739	4.82175	.22567	4.43134	.24408	4.09699	.26266	3.80726	17
44	.18955	5.27553	.20770	4.81471	.22597	4.42534	.24439	4.09182	.26297	3.80276	16
45	.18986	5.26715	.20800	4.80769	.22628	4.41936	.24470	4.08666	.26328	3.79827	15
46	.19016	5.25880	.20830	4.80068	.22658	4.41340	.24501	4.08152	.26359	3.79378	14
47	.19046	5.25048	.20861	4.79370	.22689	4.40745	.24532	4.07639	.26390	3.78931	13
48	.19076	5.24218	.20891	4.78673	.22719	4.40152	.24562	4.07127	.26421	3.78485	12
49	.19106	5.23391	.20921	4.77978	.22750	4.39560	.24593	4.06616	.26452	3.78040	11
50	.19136	5.22566	.20952	4.77286	.22781	4.38969	.24624	4.06107	.26483	3.77595	10
51	.19166	5.21744	.20982	4.76595	.22811	4.38381	.24655	4.05599	.26515	3.77152	9
52	.19197	5.20925	.21013	4.75906	.22842	4.37793	.24686	4.05092	.26546	3.76709	8
53	.19227	5.20107	.21043	4.75219	.22872	4.37207	.24717	4.04586	.26577	3.76268	7
54	.19257	5.19293	.21073	4.74534	.22903	4.36623	.24747	4.04081	.26608	3.75828	6
55	.19287	5.18480	.21104	4.73851	.22934	4.36040	.24778	4.03578	.26639	3.75388	5
56	.19317	5.17671	.21134	4.73170	.22964	4.35459	.24809	4.03076	.26670	3.74950	4
57	.19347	5.16863	.21164	4.72490	.22995	4.34879	.24840	4.02574	.26701	3.74512	3
58	.19378	5.16058	.21195	4.71813	.23026	4.34300	.24871	4.02074	.26733	3.74075	2
59	.19408	5.15256	.21225	4.71137	.23056	4.33723	.24902	4.01576	.26764	3.73640	1
60	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.26795	3.73205	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.26826	3.72771	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.26857	3.72338	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.26888	3.71907	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.26920	3.71476	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.26951	3.71046	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.26982	3.70616	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.27013	3.70188	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.27044	3.69761	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.27076	3.69335	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.27107	3.68909	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.27138	3.68485	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.27169	3.68061	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.27201	3.67638	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.27232	3.67217	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.27263	3.66796	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.27294	3.66376	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.27326	3.65957	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.27357	3.65538	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.27388	3.65121	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.27419	3.64705	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.27451	3.64289	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.27482	3.63874	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.27513	3.63461	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.27545	3.63048	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.27576	3.62636	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.27607	3.62224	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.27638	3.61814	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.27670	3.61405	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35347	2.82914	32
29	.27701	3.60996	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.27732	3.60588	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.27764	3.60181	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.27795	3.59775	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.27826	3.59370	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.27858	3.58966	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.27889	3.58562	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.27921	3.58160	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.27952	3.57758	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.27983	3.57357	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.28015	3.56957	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.28046	3.56557	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.28077	3.56159	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.28109	3.55761	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.28140	3.55364	.30033	3.32965	.31946	3.13027	.33881	2.95154	.35838	2.79033	17
44	.28172	3.54968	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.28203	3.54573	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.28234	3.54179	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.28266	3.53785	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.28297	3.53393	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.28329	3.53001	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.28360	3.52609	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.28391	3.52219	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.28423	3.51829	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.28454	3.51441	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.28486	3.51053	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.28517	3.50666	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.28549	3.50279	.30446	3.28452	.32363	3.08991	.34303	2.91522	.36265	2.75746	4
57	.28580	3.49894	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.28612	3.49509	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75247	2
59	.28643	3.49125	.30541	3.27426	.32460	3.08074	.34400	2.90696	.36364	2.74998	1
60	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	.44558	2.24428	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	.44593	2.24252	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	.44627	2.24077	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	.44662	2.23902	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	.44697	2.23727	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	.44732	2.23553	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	.44767	2.23378	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	.44802	2.23204	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	.44837	2.23030	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	.44872	2.22857	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	.44907	2.22683	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	.44942	2.22510	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	.44977	2.22337	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	.45012	2.22164	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	.45047	2.21992	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	.45082	2.21819	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	.45117	2.21647	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	.45152	2.21475	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	.45187	2.21304	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	.45222	2.21132	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	.45257	2.20961	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	.45292	2.20790	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43230	2.31271	.45327	2.20619	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	.45362	2.20449	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	.45397	2.20278	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	.45432	2.20108	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	.45467	2.19938	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	.45502	2.19769	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	.45538	2.19599	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	.45573	2.19430	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	.45608	2.19261	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	.45643	2.19092	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	.45678	2.18923	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	.45713	2.18755	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	.45748	2.18587	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	.45784	2.18419	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	.45819	2.18251	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	.45854	2.18084	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	.45889	2.17916	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	.45924	2.17749	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	.45960	2.17582	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	.45995	2.17416	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	.46030	2.17249	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	.46065	2.17083	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	.46101	2.16917	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	.46136	2.16751	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	.46171	2.16585	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	.46206	2.16420	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	.46242	2.16255	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	.46277	2.16090	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	.46312	2.15925	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	.46348	2.15760	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	.46383	2.15596	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	.46418	2.15432	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	.46454	2.15268	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	.46489	2.15104	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	.46525	2.14940	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	.46560	2.14777	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	.46595	2.14614	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	.46631	2.14451	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	69°		68°		67°		66°		65°		

′	25°		26°		27°		28°		29°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	60
1	.46666	2.14288	.48809	2.04879	.50989	1.96120	.53208	1.87941	.55469	1.80281	59
2	.46702	2.14125	.48845	2.04728	.51026	1.95979	.53246	1.87809	.55507	1.80158	58
3	.46737	2.13963	.48881	2.04577	.51063	1.95838	.53283	1.87677	.55545	1.80034	57
4	.46772	2.13801	.48917	2.04426	.51099	1.95698	.53320	1.87546	.55583	1.79911	56
5	.46808	2.13639	.48953	2.04276	.51136	1.95557	.53358	1.87415	.55621	1.79788	55
6	.46843	2.13477	.48989	2.04125	.51173	1.95417	.53395	1.87283	.55659	1.79665	54
7	.46879	2.13316	.49026	2.03975	.51209	1.95277	.53432	1.87152	.55697	1.79542	53
8	.46914	2.13154	.49062	2.03825	.51246	1.95137	.53470	1.87021	.55736	1.79419	52
9	.46950	2.12993	.49098	2.03675	.51283	1.94997	.53507	1.86891	.55774	1.79296	51
10	.46985	2.12832	.49134	2.03526	.51319	1.94858	.53545	1.86760	.55812	1.79174	50
11	.47021	2.12671	.49170	2.03376	.51356	1.94718	.53582	1.86630	.55850	1.79051	49
12	.47056	2.12511	.49206	2.03227	.51393	1.94579	.53620	1.86499	.55888	1.78929	48
13	.47092	2.12350	.49242	2.03078	.51430	1.94440	.53657	1.86369	.55926	1.78807	47
14	.47128	2.12190	.49278	2.02929	.51467	1.94301	.53694	1.86239	.55964	1.78685	46
15	.47163	2.12030	.49315	2.02780	.51503	1.94162	.53732	1.86109	.56003	1.78563	45
16	.47199	2.11871	.49351	2.02631	.51540	1.94023	.53769	1.85979	.56041	1.78441	44
17	.47234	2.11711	.49387	2.02483	.51577	1.93885	.53807	1.85850	.56079	1.78319	43
18	.47270	2.11552	.49423	2.02335	.51614	1.93746	.53844	1.85720	.56117	1.78198	42
19	.47305	2.11392	.49459	2.02187	.51651	1.93608	.53882	1.85591	.56156	1.78077	41
20	.47341	2.11233	.49495	2.02039	.51688	1.93470	.53920	1.85462	.56194	1.77955	40
21	.47377	2.11075	.49532	2.01891	.51724	1.93332	.53957	1.85333	.56232	1.77834	39
22	.47412	2.10916	.49568	2.01743	.51761	1.93195	.53995	1.85204	.56270	1.77713	38
23	.47448	2.10758	.49604	2.01596	.51798	1.93057	.54032	1.85075	.56309	1.77592	37
24	.47483	2.10600	.49640	2.01449	.51835	1.92920	.54070	1.84946	.56347	1.77471	36
25	.47519	2.10442	.49677	2.01302	.51872	1.92782	.54107	1.84818	.56385	1.77351	35
26	.47555	2.10284	.49713	2.01155	.51909	1.92645	.54145	1.84689	.56424	1.77230	34
27	.47590	2.10126	.49749	2.01008	.51946	1.92508	.54183	1.84561	.56462	1.77110	33
28	.47626	2.09969	.49786	2.00862	.51983	1.92371	.54220	1.84433	.56501	1.76990	32
29	.47662	2.09811	.49822	2.00715	.52020	1.92235	.54258	1.84305	.56539	1.76869	31
30	.47698	2.09654	.49858	2.00569	.52057	1.92098	.54296	1.84177	.56577	1.76749	30
31	.47733	2.09498	.49894	2.00423	.52094	1.91962	.54333	1.84049	.56616	1.76629	29
32	.47769	2.09341	.49931	2.00277	.52131	1.91826	.54371	1.83922	.56654	1.76510	28
33	.47805	2.09184	.49967	2.00131	.52168	1.91690	.54409	1.83794	.56693	1.76390	27
34	.47840	2.09028	.50004	1.99986	.52205	1.91554	.54446	1.83667	.56731	1.76271	26
35	.47876	2.08872	.50040	1.99841	.52242	1.91418	.54484	1.83540	.56769	1.76151	25
36	.47912	2.08716	.50076	1.99695	.52279	1.91282	.54522	1.83413	.56808	1.76032	24
37	.47948	2.08560	.50113	1.99550	.52316	1.91147	.54560	1.83286	.56846	1.75913	23
38	.47984	2.08405	.50149	1.99406	.52353	1.91012	.54597	1.83159	.56885	1.75794	22
39	.48019	2.08250	.50185	1.99261	.52390	1.90876	.54635	1.83033	.56923	1.75675	21
40	.48055	2.08094	.50222	1.99116	.52427	1.90741	.54673	1.82906	.56962	1.75556	20
41	.48091	2.07939	.50258	1.98972	.52464	1.90607	.54711	1.82780	.57000	1.75437	19
42	.48127	2.07785	.50295	1.98828	.52501	1.90472	.54748	1.82654	.57039	1.75319	18
43	.48163	2.07630	.50331	1.98684	.52538	1.90337	.54786	1.82528	.57078	1.75200	17
44	.48198	2.07476	.50368	1.98540	.52575	1.90203	.54824	1.82402	.57116	1.75082	16
45	.48234	2.07321	.50404	1.98396	.52613	1.90069	.54862	1.82276	.57155	1.74964	15
46	.48270	2.07167	.50441	1.98253	.52650	1.89935	.54900	1.82150	.57193	1.74846	14
47	.48306	2.07014	.50477	1.98110	.52687	1.89801	.54938	1.82025	.57232	1.74728	13
48	.48342	2.06860	.50514	1.97967	.52724	1.89667	.54975	1.81899	.57271	1.74610	12
49	.48378	2.06706	.50550	1.97823	.52761	1.89533	.55013	1.81774	.57309	1.74492	11
50	.48414	2.06553	.50587	1.97681	.52798	1.89400	.55051	1.81649	.57348	1.74375	10
51	.48450	2.06400	.50623	1.97538	.52836	1.89266	.55089	1.81524	.57386	1.74257	9
52	.48486	2.06247	.50660	1.97395	.52873	1.89133	.55127	1.81399	.57425	1.74140	8
53	.48521	2.06094	.50696	1.97253	.52910	1.89000	.55165	1.81274	.57464	1.74022	7
54	.48557	2.05942	.50733	1.97111	.52947	1.88867	.55203	1.81150	.57503	1.73905	6
55	.48593	2.05790	.50769	1.96969	.52985	1.88734	.55241	1.81025	.57541	1.73787	5
56	.48629	2.05637	.50806	1.96827	.53022	1.88602	.55279	1.80901	.57580	1.73671	4
57	.48665	2.05485	.50843	1.96685	.53059	1.88469	.55317	1.80777	.57619	1.73555	3
58	.48701	2.05333	.50879	1.96544	.53096	1.88337	.55355	1.80653	.57657	1.73438	2
59	.48737	2.05182	.50916	1.96402	.53134	1.88205	.55393	1.80529	.57696	1.73321	1
60	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	.57735	1.73205	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.57735	1.73205	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	60
1	.57774	1.73089	.60126	1.66318	.62527	1.59930	.64982	1.53888	.67493	1.48163	59
2	.57813	1.72973	.60165	1.66209	.62568	1.59826	.65024	1.53791	.67536	1.48070	58
3	.57851	1.72857	.60205	1.66099	.62608	1.59723	.65065	1.53693	.67578	1.47977	57
4	.57890	1.72741	.60245	1.65990	.62649	1.59620	.65106	1.53595	.67620	1.47885	56
5	.57929	1.72625	.60284	1.65881	.62689	1.59517	.65148	1.53497	.67663	1.47792	55
6	.57968	1.72509	.60324	1.65772	.62730	1.59414	.65189	1.53400	.67705	1.47699	54
7	.58007	1.72393	.60364	1.65663	.62770	1.59311	.65231	1.53302	.67748	1.47607	53
8	.58046	1.72278	.60403	1.65554	.62811	1.59208	.65272	1.53205	.67790	1.47514	52
9	.58085	1.72163	.60443	1.65445	.62852	1.59105	.65314	1.53107	.67832	1.47422	51
10	.58124	1.72047	.60483	1.65337	.62892	1.59002	.65355	1.53010	.67875	1.47330	50
11	.58162	1.71932	.60522	1.65228	.62933	1.58900	.65397	1.52913	.67917	1.47238	49
12	.58201	1.71817	.60562	1.65120	.62973	1.58797	.65438	1.52816	.67960	1.47146	48
13	.58240	1.71702	.60602	1.65011	.63014	1.58695	.65480	1.52719	.68002	1.47053	47
14	.58279	1.71588	.60642	1.64903	.63055	1.58593	.65521	1.52622	.68045	1.46962	46
15	.58318	1.71473	.60681	1.64795	.63095	1.58490	.65563	1.52525	.68088	1.46870	45
16	.58357	1.71358	.60721	1.64687	.63136	1.58388	.65604	1.52429	.68130	1.46778	44
17	.58396	1.71244	.60761	1.64579	.63177	1.58286	.65646	1.52332	.68173	1.46686	43
18	.58435	1.71129	.60801	1.64471	.63217	1.58184	.65688	1.52235	.68215	1.46595	42
19	.58474	1.71015	.60841	1.64363	.63258	1.58083	.65729	1.52139	.68258	1.46503	41
20	.58513	1.70901	.60881	1.64256	.63299	1.57981	.65771	1.52043	.68301	1.46411	40
21	.58552	1.70787	.60921	1.64148	.63340	1.57879	.65813	1.51946	.68343	1.46320	39
22	.58591	1.70673	.60960	1.64041	.63380	1.57778	.65854	1.51850	.68386	1.46229	38
23	.58631	1.70560	.61000	1.63934	.63421	1.57676	.65896	1.51754	.68429	1.46137	37
24	.58670	1.70446	.61040	1.63826	.63462	1.57575	.65938	1.51658	.68471	1.46046	36
25	.58709	1.70332	.61080	1.63719	.63503	1.57474	.65980	1.51562	.68514	1.45955	35
26	.58748	1.70219	.61120	1.63612	.63544	1.57372	.66021	1.51466	.68557	1.45864	34
27	.58787	1.70106	.61160	1.63505	.63584	1.57271	.66063	1.51370	.68600	1.45773	33
28	.58826	1.69992	.61200	1.63398	.63625	1.57170	.66105	1.51275	.68642	1.45682	32
29	.58865	1.69879	.61240	1.63292	.63666	1.57069	.66147	1.51179	.68685	1.45592	31
30	.58905	1.69766	.61280	1.63185	.63707	1.56969	.66189	1.51084	.68728	1.45501	30
31	.58944	1.69653	.61320	1.63079	.63748	1.56868	.66230	1.50988	.68771	1.45410	29
32	.58983	1.69541	.61360	1.62972	.63789	1.56767	.66272	1.50893	.68814	1.45320	28
33	.59022	1.69428	.61400	1.62866	.63830	1.56667	.66314	1.50797	.68857	1.45229	27
34	.59061	1.69316	.61440	1.62760	.63871	1.56566	.66356	1.50702	.68900	1.45139	26
35	.59101	1.69203	.61480	1.62654	.63912	1.56466	.66398	1.50607	.68942	1.45049	25
36	.59140	1.69091	.61520	1.62548	.63953	1.56366	.66440	1.50512	.68985	1.44958	24
37	.59179	1.68979	.61561	1.62442	.63994	1.56265	.66482	1.50417	.69028	1.44868	23
38	.59218	1.68866	.61601	1.62336	.64035	1.56165	.66524	1.50322	.69071	1.44778	22
39	.59258	1.68754	.61641	1.62230	.64076	1.56065	.66566	1.50228	.69114	1.44688	21
40	.59297	1.68643	.61681	1.62125	.64117	1.55966	.66608	1.50133	.69157	1.44598	20
41	.59336	1.68531	.61721	1.62019	.64158	1.55866	.66650	1.50038	.69200	1.44508	19
42	.59376	1.68419	.61761	1.61914	.64199	1.55766	.66692	1.49944	.69243	1.44418	18
43	.59415	1.68308	.61801	1.61808	.64240	1.55666	.66734	1.49849	.69286	1.44329	17
44	.59454	1.68196	.61842	1.61703	.64281	1.55567	.66776	1.49755	.69329	1.44239	16
45	.59494	1.68085	.61882	1.61598	.64322	1.55467	.66818	1.49661	.69372	1.44149	15
46	.59533	1.67974	.61922	1.61493	.64363	1.55368	.66860	1.49566	.69416	1.44060	14
47	.59573	1.67863	.61962	1.61388	.64404	1.55269	.66902	1.49472	.69459	1.43970	13
48	.59612	1.67752	.62003	1.61283	.64446	1.55170	.66944	1.49378	.69502	1.43881	12
49	.59651	1.67641	.62043	1.61179	.64487	1.55071	.66986	1.49284	.69545	1.43792	11
50	.59691	1.67530	.62083	1.61074	.64528	1.54972	.67028	1.49190	.69588	1.43703	10
51	.59730	1.67419	.62124	1.60970	.64569	1.54873	.67071	1.49097	.69631	1.43614	9
52	.59770	1.67309	.62164	1.60865	.64610	1.54774	.67113	1.49003	.69675	1.43525	8
53	.59809	1.67198	.62204	1.60761	.64652	1.54675	.67155	1.48909	.69718	1.43436	7
54	.59849	1.67088	.62245	1.60657	.64693	1.54576	.67197	1.48816	.69761	1.43347	6
55	.59888	1.66978	.62285	1.60553	.64734	1.54478	.67239	1.48722	.69804	1.43258	5
56	.59928	1.66867	.62325	1.60449	.64775	1.54379	.67282	1.48629	.69847	1.43169	4
57	.59967	1.66757	.62366	1.60345	.64817	1.54281	.67324	1.48536	.69891	1.43080	3
58	.60007	1.66647	.62406	1.60241	.64858	1.54183	.67366	1.48442	.69934	1.42992	2
59	.60046	1.66538	.62446	1.60137	.64899	1.54085	.67409	1.48349	.69977	1.42903	1
60	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.70021	1.42815	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.70064	1.42726	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.70107	1.42638	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.70151	1.42550	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.70194	1.42462	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.70238	1.42374	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.70281	1.42286	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.70325	1.42198	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.70368	1.42110	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.70412	1.42022	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.70455	1.41934	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.70499	1.41847	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.70542	1.41759	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.70586	1.41672	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.70629	1.41584	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.70673	1.41497	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.70717	1.41409	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.70760	1.41322	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.70804	1.41235	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.70848	1.41148	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.70891	1.41061	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.70935	1.40974	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.70979	1.40887	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.71023	1.40800	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.71066	1.40714	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.71110	1.40627	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.71154	1.40540	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.71198	1.40454	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.71242	1.40367	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.71285	1.40281	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.71329	1.40195	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.71373	1.40109	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.71417	1.40022	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.71461	1.39936	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.71505	1.39850	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.71549	1.39764	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.71593	1.39679	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.71637	1.39593	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.71681	1.39507	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.71725	1.39421	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.71769	1.39336	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.71813	1.39250	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.71857	1.39165	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.71901	1.39079	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.71946	1.38994	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.71990	1.38909	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.72034	1.38824	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.72078	1.38738	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.72122	1.38653	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.72167	1.38568	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.72211	1.38484	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.72255	1.38399	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.72299	1.38314	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.72344	1.38229	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.72388	1.38145	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.72432	1.38060	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.72477	1.37976	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.72521	1.37891	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.72565	1.37807	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.72610	1.37722	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	.96625	1.03493	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	.96681	1.03433	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	.96738	1.03372	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	.96794	1.03312	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	.96850	1.03252	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	.96907	1.03192	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	.96963	1.03132	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	.97020	1.03072	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	.97076	1.03012	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	.97133	1.02952	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	.97189	1.02892	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	.97246	1.02832	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	.97302	1.02772	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	.97359	1.02713	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	.97416	1.02653	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	.97472	1.02593	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	.97529	1.02533	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	.97586	1.02474	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	.97643	1.02414	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	.97700	1.02355	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	.97756	1.02295	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	.97813	1.02236	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	.97870	1.02176	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	.97927	1.02117	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	.97984	1.02057	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	.98041	1.01998	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	.98098	1.01939	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	.98155	1.01879	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	.98213	1.01820	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	.98270	1.01761	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	.98327	1.01702	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	.98384	1.01642	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	.98441	1.01583	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	.98499	1.01524	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	.98556	1.01465	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	.98613	1.01406	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	.98671	1.01347	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	.98728	1.01288	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	.98786	1.01229	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	.98843	1.01170	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	.98901	1.01112	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	.98958	1.01053	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	.99016	1.00994	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	.99073	1.00935	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	.99131	1.00876	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	.99189	1.00818	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	.99247	1.00759	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	.99304	1.00701	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	.99362	1.00642	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	.99420	1.00583	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	.99478	1.00525	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	.99536	1.00467	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	.99594	1.00408	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	.99652	1.00350	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	.99710	1.00291	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	.99768	1.00233	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	.99826	1.00175	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	.99884	1.00116	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	.99942	1.00058	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	1.00000	1.00000	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		45°		

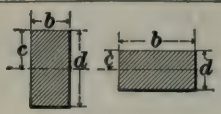
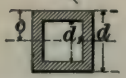
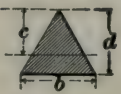
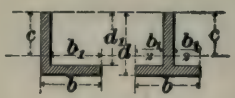
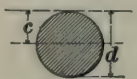
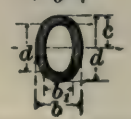
THE PROPERTIES OF SATURATED STEAM.

Pressure above Vacuum in Pounds per Square Inch.	Temperature, Fahrenheit Degrees.	Quantities of Heat in British Thermal Units.			Weight of a Cubic Foot of Steam in Pounds.	Volume.	
		Required to Raise Temperature of the Water from 32° to t.	Total Latent Heat at Pressure P.	Total Heat above 32°.		Of a Pound of Steam in Cubic Feet.	Ratio of Vol. of Steam to Vol. of Eq. Weight of Dist. Water at Temp. of Maximum Density.
1	2	3	4	5	6	7	8
p	t	q	L	H	W	V	R
1	102.018	70.040	1043.015	1113.055	.003027	330.4	20623
2	126.302	94.368	1026.034	1120.462	.005818	171.9	10730
3	141.654	109.764	1015.380	1125.144	.008522	117.3	7325
4	153.122	121.271	1007.370	1128.641	.011172	89.51	5588
5	162.370	130.563	1000.899	1131.462	.013781	72.56	4530
6	170.173	138.401	995.441	1133.842	.016357	61.14	3816
7	176.945	145.213	990.695	1135.908	.018908	52.89	3302
8	182.952	151.255	986.485	1137.740	.021436	46.65	2912
9	188.357	156.699	982.690	1139.389	.023944	41.77	2607
10	193.284	161.660	979.232	1140.892	.026437	37.83	2361
11	197.814	166.225	976.050	1142.275	.028911	34.59	2159
12	202.012	170.457	973.098	1143.555	.031376	31.87	1990
13	205.929	174.402	970.346	1144.748	.033828	29.56	1845
14	209.604	178.112	967.757	1145.869	.036265	27.58	1721
14.69	212.000	180.531	966.069	1146.600	.037928	26.37	1646
15	213.067	181.608	965.318	1146.926	.038688	25.85	1614
16	216.347	184.919	963.007	1147.926	.041109	24.33	1519
17	219.452	188.056	960.818	1148.874	.043519	22.98	1434
18	222.424	191.058	958.721	1149.779	.045920	21.78	1359
19	225.255	193.918	956.725	1150.643	.048312	20.70	1292

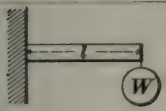
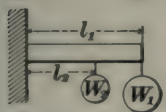
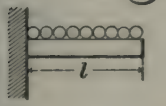
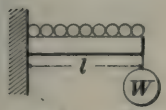
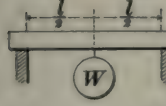
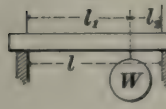
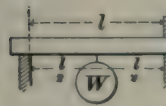
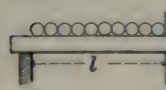
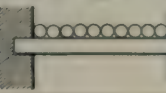
1	2	3	4	5	6	7	8
p	t	q	L	H	W	V	R
20	227.964	196.655	954.814	1151.469	.050696	19.73	1231.0
22	233.069	201.817	951.209	1153.026	.055446	18.04	1126.0
24	237.803	206.610	947.861	1154.471	.060171	16.62	1038.0
26	242.225	211.089	944.730	1155.819	.064870	15.42	962.3
28	246.376	215.293	941.791	1157.084	.069545	14.38	897.6
30	250.293	219.261	939.019	1158.280	.074201	13.48	841.3
32	254.002	223.021	936.389	1159.410	.078839	12.68	791.8
34	257.523	226.594	933.891	1160.485	.083461	11.98	748.0
36	260.883	230.001	931.508	1161.509	.088067	11.36	708.8
38	264.093	233.261	929.227	1162.488	.092657	10.79	673.7
40	267.168	236.386	927.040	1163.426	.097231	10.28	642.0
42	270.122	239.389	924.940	1164.329	.101794	9.826	613.3
44	272.965	242.275	922.919	1165.194	.106345	9.403	587.0
46	275.704	245.061	920.968	1166.029	.110884	9.018	563.0
48	278.348	247.752	919.084	1166.836	.115411	8.665	540.9
50	280.904	250.355	917.260	1167.615	.119927	8.338	520.5
52	283.381	252.875	915.494	1168.369	.124433	8.037	501.7
54	285.781	255.321	913.781	1169.102	.128928	7.756	484.2
56	288.111	257.695	912.118	1169.813	.133414	7.496	467.9
58	290.374	260.002	910.501	1170.503	.137892	7.252	452.7
60	292.575	262.248	908.928	1171.176	.142362	7.024	438.5
62	294.717	264.433	907.396	1171.829	.146824	6.811	425.2
64	296.805	266.566	905.900	1172.466	.151277	6.610	412.6
66	298.842	268.644	904.443	1173.087	.155721	6.422	400.8
68	300.831	270.674	903.020	1173.694	.160157	6.244	389.8
70	302.774	272.657	901.629	1174.286	.164584	6.076	379.3
72	304.669	274.597	900.269	1174.866	.169003	5.917	369.4
74	306.526	276.493	898.938	1175.431	.173417	5.767	360.0
76	308.344	278.350	897.635	1175.985	.177825	5.624	351.1
78	310.123	280.170	896.359	1176.529	.182229	5.488	342.6
80	311.866	281.952	895.108	1177.060	.186627	5.358	334.5
82	313.576	283.701	893.879	1177.580	.191017	5.235	326.8
84	315.250	285.414	892.677	1178.091	.195401	5.118	319.5
86	316.893	287.096	891.496	1178.592	.199781	5.006	312.5
88	318.510	288.750	890.335	1179.085	.204155	4.898	305.8

1	2	3	4	5	6	7	8
p	t	q	L	H	W	V	R
90	320.094	290.373	889.196	1179.569	.208525	4.796	299.4
92	321.653	291.970	888.075	1180.045	.212892	4.697	293.2
94	323.183	293.539	886.972	1180.511	.217253	4.603	287.3
96	324.688	295.083	885.887	1180.970	.221604	4.513	281.7
98	326.169	296.601	884.821	1181.422	.225950	4.426	276.3
100	327.625	298.093	883.773	1181.866	.230293	4.342	271.1
105	331.169	301.731	881.214	1182.945	.241139	4.147	258.9
110	334.582	305.242	878.744	1183.986	.251947	3.969	247.8
115	337.874	308.621	876.371	1184.992	.262732	3.806	237.6
120	341.058	311.885	874.076	1185.961	.273500	3.656	228.3
125	344.136	315.051	871.848	1186.899	.284243	3.518	219.6
130	347.121	318.121	869.688	1187.809	.294961	3.390	211.6
135	350.015	321.105	867.590	1188.695	.305659	3.272	204.2
140	352.827	324.003	865.552	1189.555	.316338	3.161	197.3
145	355.562	326.823	863.567	1190.390	.326998	3.058	190.9
150	358.223	329.566	861.634	1191.200	.337643	2.962	184.9
160	363.346	334.850	857.912	1192.762	.358886	2.786	173.9
170	368.226	339.892	854.359	1194.251	.380071	2.631	164.3
180	372.886	344.708	850.963	1195.671	.401201	2.493	155.6
190	377.352	349.329	847.703	1197.032	.422280	2.368	147.8
200	381.636	353.766	844.573	1198.339	.443310	2.256	140.8
210	385.759	358.041	841.556	1199.597	.464295	2.154	134.5
220	389.736	362.168	838.642	1200.810	.485237	2.061	128.7
230	393.575	366.152	835.828	1201.980	.506139	1.976	123.3
240	397.285	370.008	833.103	1203.111	.527003	1.898	118.5
250	400.883	373.750	830.459	1204.209	.547831	1.825	114.0
260	404.370	377.377	827.896	1205.273	.568626	1.759	109.8
270	407.755	380.905	825.401	1206.306	.589390	1.697	105.9
280	411.048	384.337	822.973	1207.310	.610124	1.639	102.3
290	414.250	387.677	820.609	1208.286	.630829	1.585	99.0
300	417.371	390.933	818.305	1209.238	.651506	1.535	95.8

MOMENTS OF INERTIA.

Form of Section.	Dotted Line Shows Position of Neutral Axis.	A	I	c
1. Rectangle...		bd	$\frac{1}{12}bd^3$	$\frac{1}{2}d$
2. Square..		d^2	$\frac{1}{12}d^4$	$\frac{1}{2}d$
3. Axis through Diagonal		d^2	$\frac{1}{12}d^4$	$.707d$
4. Hollow Square		$d^2-d_1^2$	$\frac{1}{12}(d^4-d_1^4)$	$\frac{1}{2}d$
5. Hollow Rectangle, I or Channel Iron		$bd-b_1d_1$	$\frac{1}{12}(bd^3-b_1d_1^3)$	$\frac{1}{2}d$
6. Triangle		$\frac{1}{2}bd$	$\frac{1}{36}bd^3$	$\frac{2}{3}d$
7. Cross....		$td+t_1b$	$\frac{1}{12}(td^3+bt_1^3)$	$\frac{1}{2}d$
8. Angle Iron.....		$bd-b_1d_1$	$\frac{(bd^2-b_1d_1^2)^2-4bdb_1d_1(d-d_1)^2}{12(bd-b_1d_1)}$	$\frac{d}{2}+\frac{b_1d_1}{2}\left(\frac{d-d_1}{bd-b_1d_1}\right)$
9. Circle....		$\frac{\pi}{4}d^2$	$\frac{\pi d^4}{64}$	$\frac{1}{2}d$
10. Hollow Circle...		$\frac{\pi}{4}(d^2-d_1^2)$	$\frac{\pi(d^4-d_1^4)}{64}$	$\frac{1}{2}d$
11. Ellipse...		$\frac{\pi}{4}bd$	$\frac{\pi bd^3}{64}$	$\frac{1}{2}d$
12. Hollow Ellipse..		$\frac{\pi}{4}(bd-b_1d_1)$	$\frac{\pi(bd^3-b_1d_1^3)}{64}$	$\frac{1}{2}d$

BENDING MOMENTS AND DEFLECTIONS.

	Manner of Supporting Beams.	Maximum Bending Moment, M.	Maximum Deflection, s.	Remarks.
1		Wl	$\frac{1}{3}\frac{Wl^3}{EI}$	Cantilever, load at free end.
2.		$W_1l_1+W_2l_2$		Cantilever, more than one load.
3		$\frac{wl^2}{2}$	$\frac{1}{8}\frac{W'l^3}{EI}$	Cantilever, uniform load w lb. per unit of length. $W'=wl$.
4.		$\frac{wl^2}{2}+Wl$	$\frac{1}{3}\frac{Wl^3}{EI}+\frac{1}{8}\frac{W'l^3}{EI}$	Cantilever, load partly uniform, partly concentrated.
5.		$\frac{Wl}{4}$	$\frac{1}{48}\frac{Wl^3}{EI}$	Simple beam, load at middle.
6.		$W\frac{l_1l_2}{l}$		Simple beam, load at some other point than the middle.
7.		$\frac{wl^2}{8}$	$\frac{5}{384}\frac{W'l^3}{EI}$	Simple beam, uniformly loaded.
8.		$\frac{3}{16}Wl$	$.0182\frac{Wl^3}{EI}$	One end fixed, other end supported, load in the middle.
9.		$\frac{wl^2}{8}$	$.0054\frac{W'l^3}{EI}$	One end fixed, other end supported, uniformly loaded.
10.		$\frac{Wl}{8}$	$\frac{1}{192}\frac{Wl^3}{EI}$	Both ends fixed, load in the middle.
11.		$\frac{wl^2}{12}$	$\frac{1}{384}\frac{W'l^3}{EI}$	Both ends fixed, uniformly loaded.

SPECIFIC GRAVITIES AND WEIGHTS PER CUBIC FOOT.

METALS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Osmium	23.00	1,437.5
Platinum	21.50	1,343.8
Gold	19.50	1,218.8
Mercury	13.60	850.0
Lead (cast)	11.35	709.4
Silver	10.50	656.3
Copper (cast)	8.79	549.4
Brass	8.38	523.8
Wrought Iron	7.68	480.0
Cast Iron	7.21	450.0
Steel	7.84	490.0
Tin (cast)	7.29	455.6
Zinc (cast)	6.86	428.8
Antimony	6.71	419.4
Aluminum	2.50	156.3

WOODS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Ash	.845	52.80
Beech	.852	53.25
Cedar	.561	35.06
Cork	.240	15.00
Ebony (American)	1.331	83.19
Lignum-vitæ	1.333	83.30
Maple	.750	46.88
Oak (old)	1.170	73.10
Spruce	.500	31.25
Pine (yellow)	.660	41.20
Pine (white)	.554	34.60
Walnut	.671	41.90

LIQUIDS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Acetic Acid	1.062	66.4
Nitric Acid	1.217	76.1
Sulphuric Acid	1.841	115.1
Muriatic Acid	1.200	75.0
Alcohol	.800	50.0
Turpentine	.870	54.4
Sea Water (ordinary)	1.026	64.1
Milk	1.032	64.5

GASES.

At 32° F., and under a Pressure of One Atmosphere.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Atmospheric Air	1.0000	.08073
Carbonic Acid	1.5290	.12344
Carbonic Oxide	.9674	.07810
Chlorine	2.4400	.19700
Oxygen	1.1056	.08925
Nitrogen	.9736	.07860
Smoke (bituminous coal)	.1020	.00815
Smoke (wood)	.0900	.00727
*Steam at 212° F.	.4700	.03790
Hydrogen	.0692	.00559

* The specific gravity of steam at any temperature and pressure compared with air at the same temperature and pressure is 0.622.

MISCELLANEOUS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Emery	4.00	250
Glass (average)	2.80	175
Chalk	2.78	174
Granite	2.65	166
Marble	2.70	169
Stone (common)	2.52	158
Salt (common)	2.13	133
Soil (common)	1.98	124
Clay	1.93	121
Brick	1.90	118
Plaster Paris (average)	2.00	125
Sand	1.80	113

MISCELLANEOUS TABLES.

SPECIFIC HEATS UNDER CONSTANT PRESSURE.

SOLIDS.

Copper	0.0951	Cast Iron	0.1298
Gold	0.0324	Lead	0.0314
Wrought Iron	0.1138	Platinum	0.0324
Steel (soft)	0.1165	Silver	0.0570
Steel (hard)	0.1175	Tin	0.0562
Zinc	0.0956	Ice	0.5040
Brass	0.0939	Sulphur	0.2026
Glass	0.1937	Charcoal	0.2410

LIQUIDS.

Water	1.0000	Lead (melted)	0.0402
Alcohol	0.7000	Sulphur (melted)	0.2340
Mercury	0.0333	Tin (melted)	0.0637
Benzine	0.4500	Sulphuric Acid	0.3350
Glycerine	0.5550	Oil of Turpentine	0.4260

GASES.

	Constant Pressure.	Constant Volume.
Air	0.23751	0.16847
Oxygen	0.21751	0.15507
Nitrogen	0.24380	0.17273
Hydrogen	3.40900	2.41226
Superheated Steam	0.48050	0.34600
Carbonic Oxide	0.24790	0.17580
Carbonic Acid	0.40400	0.15350

VOLUMES AND WEIGHTS OF GASES.

Gas.	Volume of one Pound at 32°F. and a Tension of 1 Atmosphere (14.7 Lb. per Sq. In.). Cu. Ft.	Weight of one Cu. Ft. at 32° F. and a Tension of 1 Atmosphere (14.7 Lb. per Sq. In.). Lb.	R.
Air	12.388	.08073	.37052
Oxygen	11.2056	.08925	.33552
Nitrogen	12.7226	.0786	.38143
Hydrogen	178.891	.00559	5.34946

TEMPERATURES OF FUSION AND VAPORIZATION.

Substance.	Temperature of Fusion.	Temperature of Vaporization.	Latent Heat of Fusion.	Latent Heat of Vaporization.
Water	32°	212°	142.65	966.6
Mercury	— 37.8°	662°	5.09	157
Sulphur	228.3°	824°	13.26	
Tin	446°		25.65	
Lead	626°		9.67	
Zinc	680°	1900°	50.63	493
Alcohol	Unknown	173°		372
Oil of Turpentine	14°	313°		124
Linseed Oil		600°		
Aluminum	1400°			
Copper	2100°			
Cast Iron	2192°	3300°		
Wrought Iron	2912°	5000°		
Steel	2520°			
Platinum	3632°			
Iridium	4892°			

COEFFICIENTS OF EXPANSION.

Name of Substance.	Linear Expansion.	Surface Expansion.	Cubic Expansion.
Cast Iron	.00000617	.00001234	.00001850
Copper	.00000955	.00001910	.00002864
Brass	.00001037	.00002074	.00003112
Silver	.00000690	.00001390	.00002070
Bar Iron	.00000686	.00001372	.00002058
Steel (untempered)	.00000599	.00001198	.00001798
Steel (tempered)	.00000702	.00001404	.00002106
Zinc	.00001634	.00003268	.00004903
Tin	.00001410	.00002820	.00004229
Mercury	.00003334	.00006668	.00010010
Alcohol	.00019259	.00038518	.00057778
Gases			.00203252

UNITED STATES STANDARD SCREW THREADS.

Diameter of Screw in Inches. d	Number of Threads per Inch. n	Diameter at Bottom of Threads in Inches. d_1	Area at Bottom of Threads in Square Inches. a
$\frac{1}{4}$	20	.185	.0269
$\frac{5}{16}$	18	.240	.0452
$\frac{3}{8}$	16	.294	.0679
$\frac{7}{16}$	14	.345	.0935
$\frac{1}{2}$	13	.400	.1257
$\frac{9}{16}$	12	.454	.1619
$\frac{5}{8}$	11	.507	.2019
$\frac{3}{4}$	10	.620	.3019
$\frac{7}{8}$	9	.731	.4197
1	8	.838	.5515
$1\frac{1}{8}$	7	.939	.6925
$1\frac{1}{4}$	7	1.064	.8892
$1\frac{3}{8}$	6	1.158	1.0532
$1\frac{1}{2}$	6	1.283	1.2928
$1\frac{5}{8}$	$5\frac{1}{2}$	1.389	1.5153
$1\frac{3}{4}$	5	1.490	1.7437
$1\frac{7}{8}$	5	1.615	2.0485
2	$4\frac{1}{2}$	1.711	2.2993
$2\frac{1}{4}$	$4\frac{1}{2}$	1.961	3.0203
$2\frac{1}{2}$	4	2.175	3.7154
$2\frac{3}{4}$	4	2.425	4.6186
3	$3\frac{1}{2}$	2.629	5.4284
$3\frac{1}{4}$	$3\frac{1}{2}$	2.879	6.5099
$3\frac{1}{2}$	$3\frac{1}{4}$	3.100	7.5477
$3\frac{3}{4}$	3	3.317	8.6414
4	3	3.567	9.9930
$4\frac{1}{4}$	$2\frac{7}{8}$	3.798	11.3292
$4\frac{1}{2}$	$2\frac{3}{4}$	4.027	12.7366
$4\frac{3}{4}$	$2\frac{5}{8}$	4.255	14.2197
5	$2\frac{1}{2}$	4.480	15.7633
$5\frac{1}{4}$	$2\frac{1}{2}$	4.730	17.5717
$5\frac{1}{2}$	$2\frac{3}{8}$	4.953	19.2676
$5\frac{3}{4}$	$2\frac{3}{8}$	5.203	21.2617
6	$2\frac{1}{4}$	5.423	23.0978

STANDARD DIMENSIONS OF WROUGHT-IRON STEAM, GAS, AND WATER PIPES.

Nominal Diameter in Inches.	Thickness in Inches.	Actual Internal Diameter in Inches.	Actual External Diameter in Inches.	Threads per Inch. n	Pitch of Threads.
$\frac{1}{8}$	.068	.270	.405	27	.037
$\frac{1}{4}$	.088	.364	.540	18	.056
$\frac{3}{8}$	.091	.494	.675	18	.056
$\frac{1}{2}$	.109	.623	.840	14	.071
$\frac{3}{4}$	.113	.824	1.050	14	.071
1	.134	1.048	1.315	$11\frac{1}{2}$	.087
$1\frac{1}{4}$	.140	1.380	1.660	$11\frac{1}{2}$	.087
$1\frac{1}{2}$	.145	1.611	1.900	$11\frac{1}{2}$	.087
2	.154	2.067	2.375	$11\frac{1}{2}$	.087
$2\frac{1}{2}$	.204	2.468	2.875	8	.125
3	.217	3.061	3.500	8	.125
$3\frac{1}{2}$	.226	3.548	4.000	8	.125
4	.237	4.026	4.500	8	.125
$4\frac{1}{2}$	.247	4.508	5.000	8	.125
5	.259	5.045	5.563	8	.125
6	.280	6.065	6.625	8	.125
7	.301	7.023	7.625	8	.125
8	.322	7.982	8.625	8	.125
9	.344	9.001	9.688	8	.125
10	.366	10.019	10.750	8	.125

STANDARD PIPE FLANGES.

Inside Diam. of Pipe.	Thickness of Pipe.	Diam. of Bolts.	Length of Bolts.	No. of Bolts.	Thickness of Flange.	Diam. of Bolt Circle.	Diam. of Flange.
2.0	.409	$\frac{5}{8}$	2.0	4	$\frac{5}{8}$	4.75	6.0
2.5	.429	$\frac{5}{8}$	2.25	4	$\frac{11}{16}$	5.25	7.0
3.0	.448	$\frac{5}{8}$	2.5	4	$\frac{3}{4}$	6.0	7.5
3.5	.466	$\frac{5}{8}$	2.5	4	$\frac{13}{16}$	6.5	8.5
4.0	.486	$\frac{3}{4}$	2.75	4	$\frac{15}{16}$	7.25	9.0
4.5	.498	$\frac{3}{4}$	3.0	8	$\frac{15}{16}$	7.75	9.25
5	.525	$\frac{3}{4}$	3.0	8	$\frac{15}{16}$	8.5	10.0
6	.563	$\frac{3}{4}$	3.0	8	1	9.625	11.0
7	.600	$\frac{3}{4}$	3.25	8	$1\frac{1}{16}$	10.75	12.5
8	.639	$\frac{3}{4}$	3.5	8	$1\frac{1}{8}$	11.75	13.5
9	.678	$\frac{3}{4}$	3.5	12	$1\frac{1}{8}$	13.0	15.0
10	.713	$\frac{7}{8}$	3.625	12	$1\frac{3}{16}$	14.25	16.0
12	.790	$\frac{7}{8}$	3.75	12	$1\frac{1}{4}$	16.5	19.0
14	.864	1	4.25	12	$1\frac{3}{8}$	18.75	21.0
15	.904	1	4.25	16	$1\frac{3}{8}$	20.0	22.25
16	.946	1	4.25	16	$1\frac{7}{16}$	21.25	23.5
18	1.020	$1\frac{1}{8}$	4.75	16	$1\frac{9}{16}$	22.75	25.0
20	1.090	$1\frac{1}{8}$	5.0	20	$1\frac{11}{16}$	25.0	27.5
22	1.180	$1\frac{1}{4}$	5.5	20	$1\frac{13}{16}$	27.25	29.5
24	1.250	$1\frac{1}{4}$	5.5	20	$1\frac{7}{8}$	29.5	32.0
26	1.300	$1\frac{1}{4}$	5.75	24	2	31.75	34.25
28	1.380	$1\frac{1}{4}$	6.0	28	$2\frac{1}{16}$	34.0	36.5
30	1.480	$1\frac{3}{8}$	6.25	28	$2\frac{1}{8}$	36.0	38.75
36	1.710	$1\frac{3}{8}$	6.5	32	$2\frac{3}{8}$	42.75	45.75
42	1.870	$1\frac{1}{2}$	7.25	36	$2\frac{5}{8}$	49.5	52.75
48	2.170	$1\frac{1}{2}$	7.75	44	$2\frac{3}{4}$	56.0	59.5

LAP-WELDED AMERICAN CHARCOAL IRON BOILER TUBES.

STANDARD DIMENSIONS.

External Diameter.	Standard Thickness.	Wire Gauge.	Internal Diameter.	Internal Circumference.	External Circumference.	Length of Tube per sq. ft. Inside Surface.	Length of Tube per sq. ft. Out. Surface.	Internal Area.	External Area.	Weight per foot.
In.	In.		In.	In.	In.	Ft.	Ft.	Sq. In.	Sq. In.	Lb.
1	.072	15	.856	2.689	3.142	4.463	3.819	.575	.785	.708
1½	.083	14	1.334	4.191	4.712	2.863	2.547	1.396	1.767	1.250
2	.098	13	1.804	5.667	6.283	2.118	1.909	2.556	3.142	1.981
2½	.109	12	2.283	7.172	7.854	1.673	1.528	4.094	4.909	2.755
3	.109	12	2.783	8.743	9.425	1.373	1.273	6.083	7.069	3.333
3½	.119	11	3.262	10.248	10.996	1.171	1.091	8.357	9.621	4.272
4	.130	10	3.741	11.753	12.566	1.021	.955	10.992	12.566	5.320
4½	.130	10	4.241	13.323	14.137	.901	.849	14.126	15.904	6.010
5	.140	9.5	4.720	14.828	15.708	.809	.764	17.497	19.635	7.276
6	.151	9	5.699	17.904	18.850	.670	.637	25.509	28.274	9.346

PROPORTIONS OF RIVETED JOINTS.

Thickness of Plate. t	Diameter of Rivet.	Diameter of Hole. d	Pitch.		Efficiency of Joint.	
			Single.	Double.	Single.	Double.
$\frac{1}{4}''$	$\frac{5}{8}''$	$\frac{11}{16}''$	$2''$	$3''$	.66	.77
$\frac{5}{16}''$	$\frac{11}{16}''$	$\frac{3}{4}''$	$2\frac{1}{16}''$	$3\frac{1}{8}''$	.64	.76
$\frac{3}{8}''$	$\frac{3}{4}''$	$\frac{13}{16}''$	$2\frac{1}{8}''$	$3\frac{1}{4}''$	.62	.75
$\frac{7}{16}''$	$\frac{13}{16}''$	$\frac{7}{8}''$	$2\frac{3}{16}''$	$3\frac{3}{8}''$	.60	.74
$\frac{1}{2}''$	$\frac{7}{8}''$	$\frac{15}{16}''$	$2\frac{1}{4}''$	$3\frac{1}{2}''$	.58	.73

PROPORTIONS OF GEAR TEETH.

C = Circular Pitch.

	Rough Cast Gears.	Cut Gears.
Addendum	0.30 C	0.30 C
Root	0.40 C	0.35 C
Whole Depth	0.70 C	0.65 C
Thickness of Tooth	0.48 C	0.495 C
Width of Space	0.52 C	0.505 C

RELATION BETWEEN DIAMETRAL AND CIRCULAR PITCH.

Diametral Pitch.	Circular Pitch.	Circular Pitch.	Diametral Pitch.
2	1.571 inches.	2 inches.	1.571
$2\frac{1}{4}$	1.396 "	$1\frac{7}{8}$ "	1.676
$2\frac{1}{2}$	1.257 "	$1\frac{3}{4}$ "	1.795
$2\frac{3}{4}$	1.142 "	$1\frac{5}{8}$ "	1.933
3	1.047 "	$1\frac{1}{2}$ "	2.094
$3\frac{1}{2}$	.898 "	$1\frac{7}{16}$ "	2.185
4	.785 "	$1\frac{3}{8}$ "	2.285
5	.628 "	$1\frac{5}{16}$ "	2.394
6	.524 "	$1\frac{1}{4}$ "	2.513
7	.449 "	$1\frac{3}{16}$ "	2.646
8	.393 "	$1\frac{1}{8}$ "	2.793
9	.349 "	$1\frac{1}{16}$ "	2.957
10	.314 "	1 "	3.142
11	.286 "	$\frac{15}{16}$ "	3.351
12	.262 "	$\frac{7}{8}$ "	3.590
14	.224 "	$\frac{13}{16}$ "	3.867
16	.196 "	$\frac{3}{4}$ "	4.189
18	.175 "	$\frac{11}{16}$ "	4.570
20	.157 "	$\frac{5}{8}$ "	5.027

BREAKING STRENGTH OF AND POWER TRANSMITTED BY WIRE ROPES (42 WIRES).

Diameter of Ropes, Inches.	Diameter of Pulleys, Feet.	Revolutions per Minute.	Breaking Stress of Rope per Pound.	Horse-power Transmitted.	Velocity of Rope in Feet per Second.
$\frac{7}{16}$	5	100	4,260	8.6	26
$\frac{15}{32}$	6	100	5,660	13.4	31
$\frac{1}{2}$	7	100	8,200	21.1	36
$\frac{5}{8}$	8	100	11,600	27.5	42
$\frac{5}{8}$	8	120	11,600	33.0	50
$\frac{5}{8}$	9	100	11,600	51.9	47
$\frac{5}{8}$	9	120	11,600	62.2	56
$\frac{11}{16}$	10	100	15,200	73.0	52
$\frac{11}{16}$	10	120	15,200	87.6	62
$\frac{11}{16}$	10	140	15,200	102.2	73
$\frac{11}{16}$	12	100	15,200	116.7	63
$\frac{3}{4}$	12	120	17,600	148.9	75
$\frac{3}{4}$	12	140	17,600	173.7	87
$\frac{3}{4}$	14	100	17,600	185.0	73
$\frac{3}{4}$	14	120	17,600	222.0	87
$\frac{3}{4}$	15	120	17,600	300.0	94

SAFE PRESSURE ON BEARINGS AND SLIDES.

Kind of Journal Bearing.	Pressure per Sq. In. of Projected Area, p.
Bearings on which the load is intermittent and the speed slow, such as crank-pins of shearing machines	3,000 lb.
Cross-head neck journals	1,200 lb.
Crank-pins of large, slow-speed engines	800 to 900 lb.
Crank-pins of marine engines, usually	400 to 500 lb.
Main crank-shaft bearings—marine engines (slow)	600 lb.

Main crank-shaft bearings—marine engines (fast)	400 lb.
Locomotive driving-axle journal	180 to 350 lb.
Railway journals	200 lb.
Fly-wheel shaft journals	150 to 250 lb.
Small engine crank-pins	150 to 200 lb.
Slipper slide blocks, marine engines	100 lb.
Stationary engine slide blocks	25 to 125 lb.
Stationary engine slide blocks, usually	30 to 60 lb.
Propeller thrust bearings	50 to 70 lb.
Shaft in cast-iron steps or seats	15 lb.

EQUIVALENTS OF WIRES.

0000	2–0	4–3	8–6	16–9	32–12	64–15
000	2–1	4–4	8–7	16–10	32–13	64–16
00	2–2	4–5	8–8	16–11	32–14	64–17
0	2–3	4–6	8–9	16–12	32–15	
1	2–4	4–7	8–10	16–13	32–16	
2	2–5	4–8	8–11	16–14	32–17	
3	2–6	4–9	8–12	16–15	32–18	
4	2–7	4–10	8–13	16–16		
5	2–8	4–11	8–14	16–17		
6	2–9	4–12	8–15	16–18		
7	2–10	4–13	8–16			
8	2–11	4–14	8–17			
9	2–12	4–15	8–18			
10	2–13	4–16				
11	2–14	4–17				
12	2–15	4–18				
13	2–16	4–19				
14	2–17					
15	2–18					
16	2–19					

AMERICAN, OR B. & S., GAUGE.

Gauge No.	Diameter in Mils or 1-1000 in.	Area in Circular Mils.	Weight in Pounds per 1,000 Feet.	Feet per Pound.	Resistance of Pure Copper at 75° F.		
					Ohms per 1,000 Feet.	Feet per Ohm.	Ohms per Pound.
0000	460.000	211,600.00	639.33	1.56	.051	19,605.69	.0000708
000	409.640	167,805.00	507.01	1.97	.064	15,547.87	.0001270
00	364.800	133,079.40	402.09	2.49	.081	12,330.36	.0002020
0	324.865	105,534.50	319.04	3.13	.102	9,783.63	.0003200
1	289.300	83,694.20	252.88	3.95	.129	7,754.66	.0005100
2	257.630	66,373.00	200.54	4.99	.163	6,149.78	.0008110
3	229.420	52,634.00	159.03	6.29	.205	4,876.73	.0012890
4	204.310	41,742.00	126.12	7.93	.259	3,867.62	.0020500
5	181.940	33,102.00	100.01	10.00	.326	3,067.06	.0032600
6	162.020	26,250.50	79.32	12.61	.411	2,432.22	.0051800
7	144.280	20,816.00	62.90	15.90	.519	1,928.75	.0082400
8	128.490	16,509.00	49.88	20.05	.652	1,529.69	.0131100
9	114.430	13,094.00	39.56	25.28	.824	1,213.22	.0208300
10	101.890	10,381.00	31.37	31.88	1.040	961.91	.0331400
11	90.742	8,234.00	24.88	40.20	1.311	762.93	.0526900
12	80.808	6,529.90	19.73	50.69	1.653	605.03	.0837700
13	71.961	5,178.40	15.65	63.91	2.084	479.80	.1332100
14	64.084	4,106.80	12.41	80.59	2.628	380.51	.2118000
15	57.068	3,256.70	9.84	101.63	3.314	301.75	.3368000
16	50.820	2,582.90	7.81	128.14	4.179	239.32	.5355000
17	45.257	2,048.20	6.19	161.59	5.269	189.78	.8515000
18	40.303	1,624.30	4.91	203.76	6.645	150.50	1.3539000
19	35.390	1,288.10	3.78	264.26	8.617	116.05	2.2772000
20	31.961	1,021.50	3.09	324.00	10.566	94.65	3.4230000
21	28.462	810.10	2.45	408.56	13.323	75.06	5.4430000
22	25.347	642.70	1.94	515.15	16.799	59.53	8.6540000
23	22.571	509.45	1.54	649.66	21.185	47.20	13.7630000
24	20.100	404.01	1.22	819.21	26.713	37.43	21.8850000
25	17.900	320.40	.97	1,032.96	33.684	29.69	34.7950000
26	15.940	254.01	.77	1,302.61	42.477	23.54	55.3310000
27	14.195	201.50	.61	1,642.55	53.563	18.68	87.9790000
28	12.641	159.79	.48	2,071.22	67.542	14.81	139.8930000
29	11.257	126.72	.38	2,611.82	85.170	11.74	222.4490000
30	10.025	100.50	.30	3,293.97	107.391	9.31	353.7420000
31	8.928	79.71	.24	4,152.22	135.402	7.39	562.2210000
32	7.950	63.20	.19	5,236.66	170.765	5.86	894.2420000
33	7.080	50.13	.15	6,602.71	215.312	4.64	1,421.6460000
34	6.304	39.74	.12	8,328.30	271.583	3.68	2,261.8200000
35	5.614	31.52	.10	10,501.35	342.443	2.92	3,596.1040000
36	5.000	25.00	.08	13,238.83	431.712	2.32	5,715.3600000
37	4.453	19.83	.06	16,691.06	544.287	1.84	9,084.7100000
38	3.965	15.72	.05	20,854.65	686.511	1.46	14,320.2600000
39	3.531	12.47	.04	26,302.23	865.046	1.16	22,752.6000000
40	3.144	9.89	.03	33,175.94	1,091.865	.92	36,223.5900000

COPPER WIRE TABLE.

Number, American, or B. & S. Gauge.	Copper Conductor.			Single Cotton-Covered Wire.			Double Cotton-Covered Wire.		
	Diameter in Mils.	Area in Circular Mils.	Resistance, Ohms per 1,000 Feet at 70° F.	Maximum Diameter in Mils.	Turns per Linear Inch.	Turns per Square Inch.	Maximum Diameter in Mils.	Turns per Linear Inch.	Turns per Square Inch.
0000	460.000	211,600.000	0.04904	472.000	1.80	3.60	478.00	1.70	3.21
000	409.600	167,800.000	0.06182	423.600	2.08	4.81	429.00	2.00	4.44
00	364.800	133,100.000	0.07798	376.800	2.38	6.29	384.00	2.32	5.98
0	324.900	105,500.000	0.09833	336.900	2.72	8.22	342.90	2.65	7.80
1	289.300	83,690.000	0.12400	301.300	3.07	10.37	307.30	2.99	9.93
2	257.600	66,370.000	0.15640	269.600	3.48	13.45	275.60	3.36	12.54
3	229.400	52,630.000	0.19720	241.400	4.00	17.33	247.40	3.80	16.04
4	204.300	41,740.000	0.24860	216.300	4.52	22.70	226.40	4.28	20.35
5	181.900	33,100.000	0.31330	193.900	5.05	27.22	207.90	4.83	25.92
6	162.000	26,250.000	0.39530	172.000	5.60	34.84	189.00	5.44	32 45
7	144.300	20,820.000	0.49840	154.300	6.23	43.12	173.30	6.08	41.07
8	128.500	16,510.000	0.62850	137.500	6.94	53.51	157.50	6.80	51 38
9	114.400	13,090.000	0.79200	122.400	7.68	65.53	142.50	7.64	64.96
10	101.900	10,380.000	0.99950	117.900	8.55	81.22	127.90	8.51	80.47
11	90.740	8,234.000	1.25900	96.740	9.60	102.40	112.70	9.58	101.97
12	80.810	6,530.000	1.59000	86.810	10.80	129.60	94.80	10.62	125.30
13	71.960	5,178.000	2.00400	77.960	12.06	161.60	80.96	11.88	156.80
14	64.080	4,107.000	2.52700	70.080	13.45	201.00	73.08	13.10	190.70
15	57.070	3,257.000	3.18600	63.070	14.90	246.60	66.07	14.68	239.40
16	50.820	2,583.000	4.01800	56.820	16.60	306.10	59.82	16.35	300.00
17	45.260	2,048.000	5.06700	51.260	18.20	368.10	54.26	18.08	363.20
18	40.300	1,624.000	6.38900	46.300	20.20	448.00	49.30	19.90	440.00
19	35.890	1,288.000	8.29000	41.890	22.60	567.10	44.89	21.83	528.50
20	31.960	1,022.000	10.16000	37.960	25.30	763.00	40.96	23.91	634.80
21	28.460	810.100	12.81000	34.460	28.60	908.80	37.40	26.20	762.70
22	25.350	642.400	16.17000	31.350	31.00	1,065.00	29.12	28.58	907.00
23	22.570	509.500	20.37000	28.570	34.30	1,307.00	30.60	31.12	1,075.00
24	20.100	404.000	25.69000	26.100	37.70	1,579.00	28.10	33.60	1,254.00
25	17.900	320.400	32.39000	23.900	41.50	1,914.00	25.90	36.20	1,456.00
26	15.940	254.100	40.85000	21.940	45.30	2,280.00	23.94	39.90	1,770.00
27	14.200	201.500	51.50000	20.200	49.40	2,711.00	22.20	42.60	2,016.00
28	12.640	159.800	64.94000	18.640	54.00	3,240.00	20.64	45.50	2,300.00
29	11.260	126.700	81.89000	17.260	58.80	3,841.00	19.36	48.00	2,560.00
30	10.030	100.500	103.20000	16.030	64.40	4,608.00	18.03	51.10	2,901.00
31	8.928	79.700	130.20000	14.930	69.00	5,290.00	16.93	56.80	3,585.00
32	7.950	63.210	164.20000	13.950	75.00	6,250.00	15.95	60.20	4,027.00
33	7.080	50.130	207.10000	13.080	81.00	7,290.00	15.08	64.30	4,594.00
34	6.305	39.750	261.10000	12.310	87.60	8,527.00	14.31	68.60	5,230.00
35	5.615	31.520	329.20000	11.620	94.20	9,860.00	13.61	73.00	5,921.00
36	5.000	25.000	415.20000	11.000	101.00	11,330.00	13.00	78.50	6,847.00
37	4.453	19.830	523.40000	10.450	108.00	12,960.00	12.45	84.00	7,392.00
38	3.965	15.720	660.00000	9.965	115.00	13,580.00	11.96	89.10	8,821.00
39	3.531	12.470	832.40000	9.531	122.50	16,670.00	11.53	95.00	8,805.00
40	3.145	9.888	105.00000	9.145	130.00	18,780.00	11.15	102.50	11,650.00

DIFFERENT STANDARDS FOR WIRE GAUGE IN USE IN THE UNITED STATES.

Dimensions of Wires in Decimal Parts of an Inch.

Number of Wire Gauge.	American, or Brown & Sharpe.	Birmingham, or Stubs.	Washburn & Moen M'f'g Co., Worcester, Mass.	Trenton Iron Co., Trenton, N. J.	G. W. Prentiss, Holyoke, Mass.	Old English from Brass Mfrs'. List.	Number of Wire Gauge.
000000	.....	....	.4600	....	....	....	000000
00000	.....	....	.4300	.4500	....	....	00000
0000	.460000	.454	.3930	.4000	....	....	0000
000	.409640	.425	.3620	.3600	.3586	....	000
00	.364800	.380	.3310	.3300	.3282	....	00
0	.324860	.340	.3070	.3050	.2994	....	0
1	.289300	.300	.2830	.2850	.2777	....	1
2	.257630	.284	.2630	.2650	.2591	....	2
3	.229420	.259	.2440	.2450	.2401	....	3
4	.204310	.238	.2250	.2250	.2230	....	4
5	.181940	.220	.2070	.2050	.2047	....	5
6	.162020	.203	.1920	.1900	.1885	....	6
7	.144280	.180	.1770	.1750	.1758	....	7
8	.128490	.165	.1620	.1600	.1605	....	8
9	.114430	.148	.1480	.1450	.1471	....	9
10	.101890	.134	.1350	.1300	.1351	....	10
11	.090742	.120	.1200	.1175	.1205	....	11
12	.080808	.109	.1050	.1050	.1065	....	12
13	.071961	.095	.0920	.0925	.0928	....	13
14	.064084	.083	.0800	.0800	.0816	.08300	14
15	.057068	.072	.0720	.0700	.0726	.07200	15
16	.050820	.065	.0630	.0610	.0627	.06500	16
17	.045257	.058	.0540	.0525	.0546	.05800	17
18	.040303	.049	.0470	.0450	.0478	.04900	18
19	.035890	.042	.0410	.0400	.0411	.04000	19
20	.031961	.035	.0350	.0350	.0351	.03500	20
21	.028462	.032	.0320	.0310	.0321	.03150	21
22	.025347	.028	.0280	.0280	.0290	.02950	22
23	.022571	.025	.0250	.0250	.0261	.02700	23
24	.020100	.022	.0230	.0225	.0231	.02500	24
25	.017900	.020	.0200	.0200	.0212	.02300	25
26	.015940	.018	.0180	.0180	.0194	.02050	26
27	.014195	.016	.0170	.0170	.0182	.01875	27
28	.012641	.014	.0160	.0160	.0170	.01650	28
29	.011257	.013	.0150	.0150	.0163	.01550	29
30	.010025	.012	.0140	.0140	.0156	.01375	30
31	.008928	.010	.0135	.0130	.0146	.01225	31
32	.007950	.009	.0130	.0120	.0136	.01125	32
33	.007080	.008	.0110	.0110	.0130	.01025	33
34	.006304	.007	.0100	.0100	.0118	.00950	34
35	.005614	.005	.0095	.0095	.0109	.00900	35
36	.005000	.004	.0090	.0090	.0100	.00750	36
37	.004453	....	.0085	.0085	.0095	.00650	37
38	.003965	....	.0080	.0080	.0090	.00575	38
39	.003531	....	.0075	.0075	.0083	.00500	39
40	.003144	....	.0070	.0070	.0078	.00450	40

ENERGY EQUIVALENTS.

Heat Energy.		Mechanical Energy.		Electrical Energy.
1.000000	B.T.U. =	778.0000 foot-pounds	=	1,055.2000 joules.
.001285	B.T.U. =	1.0000 foot-pound	=	1.3563 joules.
.0009477	B.T.U. =	.7373 foot-pound	=	1.0000 joule.

TEMPERATURE CONSTANTS.

The following is a table giving the constants by which the resistance of a copper conductor (at the observed temperature) must be multiplied to correct its resistance to 75° F.:

Temperature in Degrees F.	Factor.	Temperature in Degrees F.	Factor.	Temperature in Degrees F.	Factor.	Temperature in Degrees F.	Factor.
100	.9484	82	.9853	64	1.0236	46	1.0634
99	.9504	81	.9874	63	1.0258	45	1.0657
98	.9524	80	.9895	62	1.0280	44	1.0679
97	.9544	79	.9916	61	1.0301	43	1.0702
96	.9564	78	.9937	60	1.0323	42	1.0725
95	.9585	77	.9958	59	1.0345	41	1.0748
94	.9605	76	.9979	58	1.0367	40	1.0771
93	.9626	75	1.0000	57	1.0389	39	1.0793
92	.9646	74	1.0021	56	1.0411	38	1.0816
91	.9666	73	1.0042	55	1.0433	37	1.0839
90	.9687	72	1.0064	54	1.0455	36	1.0862
89	.9708	71	1.0085	53	1.0478	35	1.0885
88	.9728	70	1.0106	52	1.0500	34	1.0908
87	.9749	69	1.0128	51	1.0522	33	1.0932
86	.9769	68	1.0149	50	1.0544	32	1.0954
85	.9790	67	1.0160	49	1.0567		
84	.9811	66	1.0193	48	1.0589		
83	.9832	65	1.0214	47	1.0612		

The following is a table for the temperature correction of copper wire. It is based on Matthiessen's standard.

Resistance per Mil-Foot in Legal Ohms.	Temperature in Degrees F.	Resistance per Mil-Foot in Legal Ohms.	Temperature in Degrees F.	Resistance per Mil-Foot in Legal Ohms.	Temperature in Degrees F.
10.00	50.47	10.60	78.01	11.10	100.04
10.10	55.15	10.70	82.47	11.20	104.36
10.20	59.79	10.80	86.90	11.30	108.64
10.30	64.40	10.90	91.31	11.40	112.90
10.40	68.97	11.00	95.69	11.50	117.14
10.50	73.51				

RESISTANCE OF COPPER WIRE.

Resistance in B. A. units of copper wire 1 foot long, weighing 1 grain.

Temperature in Degrees F.	Resistance.	Temperature in Degrees F.	Resistance.	Temperature in Degrees F.	Resistance.
32.0	.2064	66.2	.2220	84.2	.2305
41.0	.2102	68.0	.2228	86.0	.2313
50.0	.2144	69.8	.2237	87.8	.2322
51.8	.2153	71.6	.2242	89.6	.2328
53.6	.2161	73.4	.2253	91.4	.2340
55.4	.2170	75.2	.2262	93.2	.2348
57.2	.2178	77.0	.2271	95.0	.2357
59.0	.2186	78.8	.2279	96.8	.2365
60.8	.2194	80.6	.2287	98.6	.2376
62.6	.2203	82.4	.2296	100.4	.2383
64.4	.2211				

POWER EXPENDED BY HYSTERESIS.

B	Watts Expended per Cubic Inch, 1 Cycle per Sec.
25,800	.002320
32,250	.002715
38,700	.004340
45,150	.005320
51,600	.006370
64,500	.009040
77,400	.011920
90,300	.015180
103,200	.018780
109,650	.022850
116,100	.028150

TESTS OF WIRE.

Sample Mark and B. W. Gauge.	Mechanical.					Electrical.	
	Weight per Mile in Pounds.	Percentage of Elongation.	Number of Twists that 6 In. Will Stand.	Actual Breaking Stress in Pounds.	Relative Breaking Stress.	Percentage Conductivity.	Resistance per Mile in Ohms at 60° F.
E. B. B. 12	190.83	11.50	15.00	417.50	11,552.20	14.40	30.50
E. B. B. 8	381.66	17.70	26.50	937.50	12,930.50	17.30	12.67
E. B. B. 11	222.64	17.20	21.50	577.50	13,639.40	15.60	24.20
$\frac{151}{151}$ 9½	282.80	10.00	26.50	770.00	14,375.90	21.90	16.10
E. B. B. 10	254.44	17.70	28.50	697.50	14,478.10	17.80	18.42
$\frac{140}{140}$ 9½	287.50	16.00	29.00	832.50	15,288.86	21.90	16.10
E. B. B. 6	508.88	11.40	21.50	1,587.50	16,462.40	17.70	9.21
E. B. B. 9	318.05	19.30	17.50	1,007.50	16,725.10	16.90	15.54
Nashua 8	381.66	15.10	26.50	1,535.00	21,183.00	14.70	15.00
M. S. plain 6	528.00	10.40	19.50	2,137.50	21,375.00	13.50	11.78
$\frac{442}{442}$ 8	378.10	10.00	31.00	1,635.00	22,301.40	16.50	16.10
A. H. 9½	293.50	16.00	27.50	1,257.50	22,635.00	15.10	22.70

GALVANIZED IRON WIRE.

The following table gives the sizes and principal properties of three grades of galvanized iron wire. The sizes are according to the Birmingham Wire Gauge, which is most commonly used in connection with iron wire:

Number B. W. G.	Diameter in Mils = d.	Area in Circular Mils = d^2.	Weight in Pounds.		Breaking Strengths in Pounds.		Resistance per Mile.		
			1,000 Feet.	One Mile.	Iron.	Steel.	E. B. B.	B. B.	Steel.
0	340	115,600	304.0	1,607	4,821	9,079	2.93	3.42	4.05
1	300	90,000	237.0	1,251	3,753	7,068	3.76	4.40	5.20
2	284	80,656	212.0	1,121	3,363	6,335	4.19	4.91	5.80
3	259	67,081	177.0	932	2,796	5,268	5.04	5.90	6.97
4	238	56,644	149.0	787	2,361	4,449	5.97	6.99	8.26
5	220	48,400	127.0	673	2,019	3,801	6.99	8.18	9.66
6	203	41,209	109.0	573	1,719	3,237	8.21	9.60	11.35
7	180	32,400	85.0	450	1,350	2,545	10.44	12.21	14.43
8	165	27,225	72.0	378	1,134	2,138	12.42	14.53	17.18
9	148	21,904	58.0	305	915	1,720	15.44	18.06	21.35
10	134	17,956	47.0	250	750	1,410	18.83	22.04	26.04
11	120	14,400	38.0	200	600	1,131	23.48	27.48	32.47
12	109	11,881	31.0	165	495	933	28.46	33.30	39.36
13	95	9,025	24.0	125	375	709	37.47	43.85	51.82
14	83	6,889	18.0	96	288	541	49.08	57.44	67.88
15	72	5,184	13.7	72	216	407	65.23	76.33	90.21
16	65	4,225	11.1	59	177	332	80.03	93.66	110.70
17	58	3,364	8.9	47	141	264	100.50	120.40	139.00
18	49	2,401	6.3	33	99	189	140.80	164.80	194.80

SAFE CARRYING CAPACITY OF CONDUCTORS.

The amount of current which a wire may carry in practice is limited by the temperature which it will acquire by the passage of such current. The insurance corporations of this country have decided that the temperature of a wire carrying current should never exceed 10° C., or 18° F., above the surrounding air.

Though this limit is often exceeded in practice, it is the standard for good work, and the following table therefore gives the amount of current in amperes that a wire under certain conditions may carry without having its temperature raised beyond the limit:

Amperes.	Number in B. & S.	Circular Mils.	Amperes.	Number in B. & S.	Circular Mils.
1,000.00	....	2,160,900	61.00	3	52,634
900.00	....	1,876,900	52.00	4	41,742
800.00	....	1,612,900	43.00	5	33,102
700.00	....	1,345,600	36.00	6	26,244
600.00	....	1,100,401	30.00	7	20,822
550.00	....	976,144	25.00	8	16,512
500.00	....	861,184	22.00	9	13,110
475.00	....	804,609	18.00	10	10,381
450.00	....	748,225	15.00	11	8,226
425.00	....	692,224	12.00	12	6,528
400.00	....	640,000	10.50	13	5,184
375.00	....	586,756	9.00	14	4,110
350.00	....	535,824	7.25	15	3,260
325.00	....	485,809	6.00	16	2,581
300.00	....	435,600	5.50	17	2,044
275.00	....	388,129	4.00	18	1,624
250.00	....	342,225	3.25	19	1,253
225.00	....	297,025	2.75	20	1,024
200.00	....	254,016	2.25	21	820
174.00	0000	211,600	2.00	22	626
147.00	000	167,805	1.75	23	510
124.00	00	133,079	1.50	24	404
103.00	0	105,592	1.25	25	320
87.00	1	83,694	1.00	26	254
73.00	2	66,373			

CURRENT CAPACITY OF OVERHEAD WIRES CORRESPONDING TO A TEMPERATURE RISE OF 10°, 20°, 30°, AND 40° C., RESPECTIVELY.

Diameter in Inches.	Area in Circular Mils.	Amperes.				Gauge Number.	Diameter in Mils, or $\frac{1}{1000}$ In.	Area in Circular Mils.	Amperes.			
		10°	20°	30°	40°				10°	20°	30°	40°
1.00	1,000,000	410	570	700	800	0000	460.000	211,600	145	200	240	275
.95	902,500	380	530	650	750	000	409.640	167,805	125	170	210	235
.90	810,000	350	490	600	700	00	364.800	133,079	110	145	180	205
.85	722,500	320	455	560	645	0	324.950	105,592	95	125	150	180
.80	640,000	290	420	520	595	1	289.300	83,694	80	110	130	150
.75	562,500	265	385	470	545	2	257.630	66,373	70	95	115	130
.70	490,000	240	350	430	500	3	229.420	52,634	60	80	100	115
.65	422,500	220	310	390	450	4	204.310	41,742	55	70	85	100
.60	360,000	200	280	350	405	5	181.940	33,102	50	63	75	87
.55	302,500	180	250	310	360	6	162.020	26,250	40	52	64	75
.50	250,000	160	220	270	310	7	144.280	20,816	35	45	55	65

VALUES OF B, H, AND μ FOR IRON AND STEEL.

Gray Cast Iron.

B	H	μ
10,000	64	156.3
20,000	105	190.5
30,000	164	182.9
40,000	262	152.9
50,000	430	116.3
60,000	718	83.6
65,000	1,030	63.1

Cast Steel—Unannealed.

B	H	μ
10,000	18	555.5
20,000	28	714.3
30,000	35	857.1
40,000	43	930.2
50,000	54	925.9
60,000	72	833.3
70,000	99	707.1
80,000	146	547.3
90,000	225	400.0
100,000	375	266.6
110,000	730	150.7
115,000	1,015	113.3

Sheet Iron—Annealed.

B	H	μ
10,000	16	625.0
20,000	23	869.6
30,000	28	1,071.4
40,000	33	1,212.1
50,000	42	1,190.4
60,000	53	1,132.0
70,000	68	1,029.4
80,000	94	851.0
90,000	138	652.2
100,000	214	467.3
110,000	374	294.1
120,000	725	165.5
125,000	1,075	116.3

Wrought-Iron Forgings.

B	H	μ
10,000	12.0	833.3
20,000	15.0	1,333.3
30,000	18.8	1,595.7
40,000	23.0	1,739.1
50,000	30.0	1,666.6
60,000	44.0	1,363.6
70,000	65.0	1,076.9
80,000	104.0	769.2
90,000	200.0	450.0
100,000	430.0	232.6
105,000	630.0	166.6
110,000	1,035.0	106.3

DEPOLARIZING EFFECT OF VARIOUS SUBSTANCES.

Substance. Solids.	E. M. F. Volts.	Substance. Liquid.	E. M. F. Volts.
MnO_2(ordinary).....	.33	HNO_3 (concentrated).....	.35
Pb_2O_3.....	.81	H_2CrO_4.....	.47
		Cl gas dissolved in water.....	.64

THE ELECTRIC SERIES.

1. Fur.	6. Cotton.	11. Sealing-wax.
2. Flannel.	7. Silk.	12. Resin.
3. Ivory.	8. The body.	13. Sulphur.
4. Crystals.	9. Wood.	14. Gutta-percha.
5. Glass.	10. Metals.	15. Guncotton.

CONDUCTORS AND INSULATORS IN ORDER OF THEIR VALUE.

Conductors.	Insulators (Non-Conductors).	
Silver.	Dry Air.	Glass.
Copper.	Shellac.	Mica.
Other Metals.	Paraffin.	Ebonite.
Charcoal.	Amber.	India-rubber.
Plumbago.	Resin.	Silk.
Moist Earth.	Sulphur.	Paper.
Water.	Wax.	Oils.

A general idea of these values may be obtained from the fact that *water* has 6,754 million times greater resistance than copper.

INSULATORS IN ORDER OF THEIR INDUCTIVE CAPACITY VALUES.

Glass.	India-rubber.	Air.
Shellac.	Petroleum.	Hydrogen.
Sulphur.	Paraffin (solid).	Vacuum.
Ebonite.	Carbonic Acid.	

THE ELECTROMOTIVE SERIES.

1. + Sodium.
2. Magnesium.
3. Zinc.
4. Lead.
5. Tin.
6. Iron.
7. Copper.
8. Silver.
9. Gold.
10. Platinum.
11. — Graphite (carbon).

THE PRINCIPAL ELEMENTS.

1. Name of Element.	2. Symbol.	3. Atomic Weight.	4. Valency.	5. Chemical Equivalent.	6. Electro-chemical Equivalent. Grams per Coulomb.
Aluminum	*Al*	27.00	III	9.00	.00009324
Antimony	*Sb*	120.00	V	24.00	.00024860
Arsenic	*As*	75.00	V	15.00	.00015540
Barium	*Ba*	137.00	II	68.50	.00070960
Bismuth	*Bi*	208.90	V	41.78	.00043280
Boron	*B*	11.00	III	3.66	.00003792
Bromine	*Br*	79.95	I	79.95	.00082100
			VII	11.42	.00011840
Cadmium	*Cd*	112.00	II	56.00	.00058020
Calcium	*Ca*	40.00	II	20.00	.00020720
Carbon	*C*	12.00	IV	3.00	.00003098
Chlorine	*Cl*	35.45	I	35.45	.00036730
			VII	5.07	.00005252
Chromium	*Cr*	52.10	II	26.05	.00026990
			VI	7.44	.00007708
Cobalt	*Co*	59.00	II	29.50	.00030560
			VIII	7.38	.00007646
Copper	*Cu*	63.40	I	63.40	.00065680
			II	31.70	.00032840
Fluorine	*Fl*	19.00	I	19.00	.00019680
			VII	2.57	.00002663
Gold	*Au*	197.30	I	197.30	.00204400
			III	65.77	.00068140
Hydrogen	*H*	1.00	I	1.00	.00001036

THE PRINCIPAL ELEMENTS—*Continued.*

1. Name of Element.	2. Symbol.	3. Atomic Weight.	4. Valency.	5. Chemical Equivalent.	6. Electro-chemical Equivalent. Grams per Coulomb.
Iodine	*I*	125.85	I	125.85	.00130300
			VII	17.98	.00018630
Iron	*Fe*	56.00	II	28.00	.00029010
			IV	14.00	.00014500
Lead	*Pb*	206.95	II	103.48	.00107200
			IV	51.74	.00053600
Magnesium	*Mg*	24.30	II	12.15	.00012590
Manganese	*Mn*	55.00	II	27.50	.00028490
			VII	7.86	.00008143
Mercury	*Hg*	200.00	I	200.00	.00207200
			II	100.00	.00103600
Nickel	*Ni*	58.00	II	29.00	.00030040
			VIII	7.25	.00007510
Nitrogen	*N*	14.03	V	2.81	.00002911
Oxygen	*O*	16.00	II	8.00	.00008288
			VI	2.67	.00002766
Phosphorus	*P*	31.00	V	6.20	.00006423
Platinum	*Pt*	195.00	II	97.50	.00101000
			IV	48.75	.00050500
Potassium	*K*	39.11	I	39.11	.00040520
Selenium	*Se*	79.00	II	39.50	.00040920
			VI	13.17	.00013640
Silicon	*Si*	28.40	IV	7.10	.00007355
Silver	*Ag*	107.90	I	107.90	.00111800
Sodium	*Na*	23.05	I	23.05	.00023880
Strontium	*Sr*	87.60	II	43.80	.00045370
Sulphur	*S*	32.06	II	16.03	.00016610
			VI	5.34	.00005532
Tellurium	*Te*	125.00	II	62.50	.00064750
Tin	*Sn*	119.00	II	59.50	.00061640
			IV	29.75	.00030820
Zinc	*Zn*	65.30	II	32.65	.00033820

The names of the non-metallic elements are printed in *Italics*.

VARIOUS VALUES OF THE OHM.

Name.	Height of Mercury Column.	Cross-Section of Mercury Column.	Use.
Siemens' Unit..	100 cm.	1 sq. mm.	Out of use, because incorrect.
British Association Unit, written B. A. U...	104.8 cm.	1 sq. mm.	Out of use, because incorrect.
Legal Ohm (commonly called Ohm).....	106.0 cm.	1 sq. mm.	In all technical measurements and calculations, as well as in this Course.
International Ohm.....	106.3 cm.	1 sq. mm.	Latest and most exact determination. Correct within $\frac{1}{5000}$ part. Not yet in general use.

The relative values of these units are given by the following list:

1 legal ohm = 1.0112 B. A. U.
1 legal ohm = 1.0600 Siemens' Units
1 B. A. U. = .9889 legal ohm.
1 B. A. U. = 1.0483 Siemens' Units.
1 Siemens' Unit = .9540 B. A. U.
1 Siemens' Unit = .9434 legal ohm.

HEAT OF COMBINATION WITH OXYGEN.

1 Gram of	Compound Formed.	Calories or Gram Degrees of Heat Produced.
Hydrogen	H_2O	34,000
Carbon	CO_2	8,000
Sulphur	SO_2	2,300
Phosphorus	P_2O_5	5,747
Zinc	ZnO	1,301
Iron	Fe_3O_4	1,576
Tin	SnO_2	1,233
Copper	CuO	602

RESISTANCES OF METALS.

Name of Metal.	Resistance in Microhms of 1 Cu. In. at 32° F.	Relative Resistance.
Silver, annealed	.5921	1.000
Copper, annealed	.6292	1.063
Silver, hard-drawn	.6433	1.086
Copper, hard-drawn	.6433	1.086
Gold, annealed	.8102	1.369
Gold, hard-drawn	.8247	1.393
Aluminum, annealed	1.1470	1.935
Zinc, pressed	2.2150	3.741
Platinum, annealed	3.5650	6.022
Iron, annealed	3.8250	6.460
Nickel, annealed	4.9070	8.285
Tin, pressed	5.2020	8.784
Lead, pressed	7.7280	13.050
German Silver	8.2400	13.920
Antimony, pressed	13.9800	23.600
Mercury	37.1500	62.730
Bismuth, pressed	51.6500	87.230

Resistances of the same metals in microhms per *cubic centimeter* at a temperature of 0° C.

Metal.	Resistance.	Metal.	Resistance.
Silver, annealed	1.504	Iron, annealed.....	9.716
Copper, annealed ...	1.598	Nickel, annealed....	12.470
Silver, hard-drawn..	1.634	Tin, pressed.....	13.210
Copper, hard-drawn.	1.634	Lead, pressed.....	19.630
Gold, annealed.....	2.058	German Silver.....	20.930
Gold, hard-drawn...	2.094	Antimony, pressed..	35.500
Aluminum, annealed	2.912	Mercury	94.320
Zinc, pressed.....	5.626	Bismuth, pressed ...	131.200
Platinum, annealed..	9.057		

INCANDESCENT LAMPS.

The following table is taken from tests made on standard incandescent lamps of various makes at present in use:

Volts.	C. P.	Amperes.	Watts per Lamp.	Watts per C. P.
52	16	1.04	54.1	3.38
52	32	2.08	108.2	3.38
52	100	6.50	338.0	3.38
100	16	.64	64.0	4.00
100	32	1.28	128.0	4.00
100	100	4.00	400.0	4.00
110	16	.58	64.0	4.00
110	32	1.16	128.0	4.00
110	100	3.64	400.0	4.00

ILLUMINATION BY INCANDESCENT LAMPS.

The following table gives the general effect produced by different numbers of 16 candle-power lamps placed at a given height above the floor:

Number of 16 C. P. Lamps per 100 Sq. Ft.	Approximate Effect.
1.0	Dull
1.5	Medium
2.0	Good
3.0	Bright
4.0	Brilliant

The following table gives the number of square feet illuminated by a 16 candle-power incandescent lamp at stated heights and under certain conditions; the table fulfils all practical requirements:

Nature of Room.	Height.	Number of Square Feet per 16 C. P. Lamp.
Offices with desks	10 feet	50, also localized near desks.
Reading-rooms with tables	15 feet	50, also localized near tables.
Stores	15 feet	80.
Dance halls	20 feet	80.
Halls and lobbies	18 feet	100.
Machine-shops	15 feet	150 and localized at machines.
Large town halls	25 feet	120.
Churches	40 feet	200.

CURRENT REQUIRED BY AVERAGE LAMPS.

110-Volt.		55-Volt.
.5 ampere	16 c. p. lamp	1 ampere
1 ampere	32 c. p. lamp	2 amperes
1.5 amperes	50 c. p. lamp	3 amperes
3 amperes	100 c. p. lamp	6 amperes

The number of lamps on a circuit may therefore be expressed in terms of 16 c. p. lamps. Thus

one 32 c. p. lamp = 2 of 16 c. p.
one 50 c. p. lamp = 3 of 16 c. p.
one 100 c. p. lamp = 6 of 16 c. p.

HORIZONTAL COMPONENT OF THE EARTH'S MAGNETISM.

Locality.	Value of Component. Lines of Force per Square Centimeter.
London, England	.180
Paris	.188
Berlin	.178
Rome	.240
Montreal	.147
Niagara	.167
Halifax	.159
Boston	.170
New York	.184
Philadelphia	.194
Washington	.200
Chicago	.184
Cleveland	.184
San Francisco	.255

SPECIFIC RESISTANCES OF INSULATORS.

Substance.	Specific Resistance.
Mica	84 tregohms
Gutta-percha	449 tregohms
Hard rubber	28 quegohms
Paraffin	34 quegohms
Porcelain	540 quegohms
Flint glass	16,700 quegohms
Olive oil	1 tregohm
Lard oil	350 begohms

SPECIFIC RESISTANCES OF ELECTROLYTES.

At 50° Fahrenheit.

Liquid.	Specific Gravity.	Specific Resistance. Ohms.
Copper sulphate Saturated solution	1.205	29.30
Zinc sulphate Saturated solution	1.440	33.60
Zinc sulphate	Solution giving least resistance.	28.22
Common salt		4.70
Sal ammoniac		2.50
Sulphate of soda		11.30
Sulphuric acid		1.38
Nitric acid		1.29
Hydrochloric acid		1.32

SIZES OF STANDARD EYE-BOLTS.

A	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2
B	$1\frac{1}{4}$	$1\frac{1}{4}$	$1\frac{5}{16}$	$1\frac{1}{2}$	$1\frac{9}{16}$	$1\frac{5}{8}$	$1\frac{11}{16}$	$2\frac{1}{8}$	$2\frac{3}{4}$	$3\frac{1}{8}$
C	$1\frac{1}{8}$	$1\frac{1}{2}$	$1\frac{11}{16}$	$1\frac{7}{8}$	$2\frac{1}{4}$	$2\frac{5}{8}$	2	$2\frac{1}{2}$	3	4

STANDARD HEXAGONAL HEAD TAP-BOLTS.

Diameter of Screw.....	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$
Short Diameter of Head	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{13}{16}$
Long Diameter of Head	.51	.58	.65	.72	.87	.94
Length Under Head....	$\frac{3}{4}$ to 3	$\frac{3}{4}$ to $3\frac{1}{2}$	$\frac{3}{4}$ to $3\frac{1}{2}$	$\frac{3}{4}$ to $3\frac{3}{4}$	$\frac{3}{4}$ to 4	$\frac{3}{4}$ to $4\frac{1}{2}$

Diameter of Screw.....	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$
Short Diameter of Head	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$
Long Diameter of Head	1.01	1.15	1.30	1.45	1.59	1.73
Length Under Head....	1 to $4\frac{1}{2}$	$1\frac{1}{2}$ to $4\frac{1}{2}$	$1\frac{1}{2}$ to 5	$1\frac{3}{4}$ to 5	2 to 5	2 to 5

SIZES OF KEYS FOR SHAFTS.

Diameter of Shaft.	Width.	Thickness.	Diameter of Shaft.	Width.	Thickness.
$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{16}$	$3\frac{1}{8}$	$\frac{3}{4}$	$\frac{7}{16}$
$\frac{3}{8}$	$\frac{3}{32}$	$\frac{1}{16}$	$3\frac{1}{4}$	$\frac{3}{4}$	$\frac{7}{16}$
$\frac{1}{2}$	$\frac{1}{8}$	$\frac{3}{32}$	$3\frac{3}{8}$	$\frac{7}{8}$	$\frac{1}{2}$
$\frac{5}{8}$	$\frac{5}{32}$	$\frac{3}{32}$	$3\frac{1}{2}$	$\frac{7}{8}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$3\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{2}$
$\frac{7}{8}$	$\frac{3}{16}$	$\frac{1}{8}$	$3\frac{3}{4}$	$\frac{7}{8}$	$\frac{1}{2}$
1	$\frac{1}{4}$	$\frac{5}{32}$	$3\frac{7}{8}$	1	$\frac{5}{8}$
$1\frac{1}{8}$	$\frac{1}{4}$	$\frac{5}{32}$	4	1	$\frac{5}{8}$
$1\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{16}$	$4\frac{1}{4}$	1	$\frac{5}{8}$
$1\frac{3}{8}$	$\frac{5}{16}$	$\frac{3}{16}$	$4\frac{1}{2}$	$1\frac{1}{8}$	$\frac{5}{8}$
$1\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$4\frac{3}{4}$	$1\frac{1}{8}$	$\frac{5}{8}$
$1\frac{5}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	5	$1\frac{1}{4}$	$\frac{3}{4}$
$1\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$5\frac{1}{4}$	$1\frac{1}{4}$	$\frac{3}{4}$
$1\frac{7}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$5\frac{1}{2}$	$1\frac{1}{4}$	$\frac{3}{4}$
2	$\frac{1}{2}$	$\frac{5}{16}$	6	$1\frac{1}{2}$	$1\frac{5}{16}$
$2\frac{1}{8}$	$\frac{1}{2}$	$\frac{5}{16}$	$6\frac{1}{2}$	$1\frac{5}{8}$	1
$2\frac{1}{4}$	$\frac{1}{2}$	$\frac{5}{16}$	7	$1\frac{3}{4}$	$1\frac{1}{16}$
$2\frac{3}{8}$	$\frac{5}{8}$	$\frac{3}{8}$	$7\frac{1}{2}$	$1\frac{7}{8}$	$1\frac{1}{8}$
$2\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{8}$	8	2	$1\frac{1}{4}$
$2\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{8}$	$8\frac{1}{2}$	$2\frac{1}{8}$	$1\frac{1}{4}$
$2\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{8}$	9	$2\frac{1}{4}$	$1\frac{3}{8}$
$2\frac{7}{8}$	$\frac{3}{4}$	$\frac{7}{16}$	$9\frac{1}{2}$	$2\frac{3}{8}$	$1\frac{3}{8}$
3	$\frac{3}{4}$	$\frac{7}{16}$	10	$2\frac{1}{2}$	$1\frac{1}{2}$

ANALOGIES BETWEEN THE FLOW OF WATER AND ELECTRICITY.

	Water in Pipes.	Electricity in Conductors.
I.	Difference of level tends to make water fall from the upper level to the lower level.	Difference of potential tends to make electric current fall from point of high potential to point of low potential.
II.	Difference of level hence acts as a pressure tending to cause a flow.	Difference of potential or E. M. F., hence acts as a pressure tending to cause a flow.
III.	If not entirely obstructed, this pressure actually produces a flow of water.	If not entirely obstructed, this pressure or E. M. F. actually produces a flow of current.
IV.	Some of this pressure is lost by friction of the water against inside walls of pipe.	Some of this pressure is lost by the electrical resistance of the conductor. The loss is called *drop of potential.*
V.	This loss by friction is directly proportional to the length of the pipe, and inversely proportional to the diameter of the pipe.	This loss or drop due to resistance is directly proportional to the length of the conductor, and inversely proportional to its area of cross-section.
VI.	No quantity of water can flow through a pipe without suffering some loss in this manner; in other words, there is no such thing as an absolutely frictionless pipe.	No quantity of electricity can flow through a conductor without suffering some loss in this manner; in other words, there is no such thing as an absolutely resistanceless conductor.

TEMPERATURE COEFFICIENTS FOR VARIOUS METALS.

Name of Metal.	For Centigrade. (*a*)	For Fahrenheit. (*b*)
Silver	.00377	.002094
Copper	.00388	.002156
Gold	.00365	.002028
Aluminum	.00390	.002167
Platinum	.00247	.001372
Iron	.00453	.002517
Tin	.00365	.002028
Lead	.00387	.002150
Antimony	.00389	.002161
Bismuth	.00354	.001967
Mercury	.00088	.000489
German silver	.00044	.000244

REDUCTION OF METALS.

The E. M. F.'s and the Compositions of the Electrolytes for Various Metals.

COPPER. The electrolyte is a saturated solution of sulphate of copper, mixed with ¼ its volume of water containing about 10 per cent. of sulphuric acid for conducting the current. The E. M. F. is from 0.5 to 1 volt.

SILVER. The solution consists of cyanide of silver dissolved in cyanide of potassium. The E. M. F. is from 1 to 2 volts.

LEAD. The solution consists of plumbic sulphate dissolved in sodic acetate. The E. M. F. is from 0.5 to 1 volt.

ALUMINUM. The electrolyte is chloride of aluminum, ammonium, and sodium, but, instead of being in solution, it is used in a melted or fused state. The E. M. F. is from 15 to 20 volts.

GOLD. { The solution is cyanide of gold in cyanide of potassium. E. M. F. is from .5 to 3 volts.

NICKEL. { The solution is the double sulphate or chloride of nickel and ammonia. E. M. F. is from .5 to 1 volt.

TABLE OF CENTIGRADE AND FAHRENHEIT DEGREES.

Deg. C.	Deg. F.	Deg. C.	Deg. F.	Deg. C.	Deg. F.	Deg. C.	Deg. F.
0	32.0	26	78.8	51	123.8	76	168.8
1	33.8	27	80.6	52	125.6	77	170.6
2	35.6	28	82.4	53	127.4	78	172.4
3	37.4	29	84.2	54	129.2	79	174.2
4	39.2	30	86.0	55	131.0	80	176.0
5	41.0	31	87.8	56	132.8	81	177.8
6	42.8	32	89.6	57	134.6	82	179.6
7	44.6	33	91.4	58	136.4	83	181.4
8	46.4	34	93.2	59	138.2	84	183.2
9	48.2	35	95.0	60	140.0	85	185.0
10	50.0	36	96.8	61	141.8	86	186.8
11	51.8	37	98.6	62	143.6	87	188.6
12	53.6	38	100.4	63	145.4	88	190.4
13	55.4	39	102.2	64	147.2	89	192.2
14	57.2	40	104.0	65	149.0	90	194.0
15	59.0	41	105.8	66	150.8	91	195.8
16	60.8	42	107.6	67	152.6	92	197.6
17	62.6	43	109.4	68	154.4	93	199.4
18	64.4	44	111.2	69	156.2	94	201.2
19	66.2	45	113.0	70	158.0	95	203.0
20	68.0	46	114.8	71	159.8	96	204.8
21	69.8	47	116.6	72	161.6	97	206.6
22	71.6	48	118.4	73	163.4	98	208.4
23	73.4	49	120.2	74	165.2	99	210.2
24	75.2	50	122.0	75	167.0	100	212.0
25	77.0						

COMMON ACIDS.

1. Name of Acid.	2. Chemical Formula.	Specific Gravity.	
		3. Pure Acid.	4. Commercial Acid. (Average.)
Hydrochloric.....	HCl	1.227	1.14 to 1.16
Hydrobromic.....	HBr	1.515	.....
Nitric.....	HNO_3	1.530	1.33 to 1.41
Sulphuric.....	H_2SO_4	1.846	1.70 to 1.83
Chromic	H_2CrO_4	.....	.....

NOTE.—*HCl* and *HBr* are in reality gases, which dissolve readily in water, forming the liquid known by the above names. The specific gravity given is that of the solution (maximum).

ELECTROLYTES USED IN ELECTROPLATING.

COPPER PLATING.

Acid Solution.

Sulphate of copper..... 1 lb.
Sulphuric acid..... 1 lb.
Water..... 1 gallon.

The sulphate of copper is dissolved in hot water, after which cold water is added until the quantity named is made up. The sulphuric acid is then added, and after the solution has become cool, it is poured into the depositing vessel. This may be of glass or of wood, and should be thoroughly cleaned before using.

Cyanide Solution With Copper Sulphate.—For this bath 8 ounces of sulphate of copper is dissolved in 1 quart of hot water and set aside to cool. When cold, liquor ammonia, specific gravity .880, is added gradually, the solution being continually stirred with a glass rod or a piece

of wood until the precipitate which at first appears is again dissolved. One quart of cold water is then added. A solution of cyanide of potassium, made by dissolving 1¼ lb. in 2 quarts of water, is *gradually* mixed in, with constant stirring, until the blue color disappears. Then the remaining cyanide solution is added, and the mixture is allowed to stand until the clear liquor rises to the top, when it may be siphoned off and is ready for use in the bath. This solution is generally used cold, although a quicker action may be obtained if it is warmed to a temperature approximating 120° F. In using cyanide of potassium, great care must be exercised in handling, as this chemical is a strong poison and very deadly. If the hands are cut or scratched, blood-poisoning will probably result from contact with cyanide, so that the greatest precautions should always be adopted in preparing solutions of which it forms an ingredient.

Cyanide Solution With Acetate of Copper.—In making up this bath, the following proportions of ingredients are necessary:

Acetate of copper, crystallized...	20 parts.
Carbonate of soda, crystals.....	20 "
Bisulphite of soda.....	20 "
Cyanide of potassium, pure.....	20 "
Water.....	1,000 "

The acetate of soda is reduced to powder and made into paste with a little water; to this is added the carbonate of soda previously dissolved in 200 parts of water, and the mixture is well stirred. The bisulphite of soda is then dissolved in 200 parts of water and added to the mixture, when the green precipitate first formed becomes of a yellow color. The cyanide of potassium is dissolved in 600 parts of water and added, the mixture being well stirred. The precipitate will be dissolved by the cyanide and the solution will become clear. Should this not take place, more cyanide should be added. This bath may be used hot or cold.

NICKEL PLATING.

The plating bath is made as follows:

Double sulphate nickel and ammonia (pure)	12 to 14 oz.
Water	1 gallon.

The nickel salts are put into a wooden tank and hot water is poured upon them until they dissolve, this being hastened by stirring with a clean wooden stick. The liquid may be poured off into the plating tank after a short time, without waiting for all the salts to dissolve; then more hot water is poured upon the salts, this being repeated until the salts are completely dissolved. When this is accomplished, cold water is added to bring the solution to the required measure.

This plating bath has a rather high resistance, and good results may usually be obtained by adding to the solution 10 per cent. of common salt. This has the effect of whitening the nickel deposit and making it more tough, so that the film of metal will not readily strip, and the resistance of the bath is decreased.

The nickel bar or plate used as an anode must be pure, in order to secure good results.

Cast anodes are more readily dissolved by the action of the current than are rolled anodes, and are therefore to be preferred. For small work, however, rolled anodes are more generally used, as they are more easily obtained.

All the water used in plating baths should be distilled, or rain water may be employed if desired.

GOLD PLATING.

The electrolyte used in gold plating is made of the following ingredients:

Fine gold	10 parts.
Pure cyanide of potassium	20 "
Or, commercial cyanide	30 to 40 "
Distilled water	1,000 "

The gold must be formed into chloride in the following manner: The gold is placed in a flask, and a mixture of

2 parts of hydrochloric and 1 part of nitric acid is poured upon it. The proportion of this acid to gold will be about 4 to 1, that is, 4 ounces of mixed acid to 1 ounce of gold, depending, however, upon the quality of the acid, of which too much should not be used. This mixture is then to be evaporated on a hot sand bath until nearly all the acid is driven off and the solution begins to move sluggishly on rocking the dish. It is then allowed to cool. If too much heat has been applied, the gold will be converted into a metallic state. It should in that case be washed with distilled water and redissolved with acid. This chloride of gold is now to be dissolved in about 200 parts of water. The cyanide is dissolved in the remainder of the water. After filtering, the solutions are mixed and then boiled for a short time.

SILVER PLATING.

For silver plating the following ingredients are required, in the proportions given:

Pure silver	25 parts.
Cyanide of potassium, pure..	50 "
Water	1,000 "

The amount of silver used in the bath varies from ½ ounce to 5 or 6 ounces to 1 gallon of water, but 2 ounces or less will usually give excellent results. It is always advisable to use distilled or rain water for the plating bath, as there are likely to be impurities in river water which may injuriously affect the quality of the deposit.

CURRENT DENSITY USED IN ELECTROPLATING.

The usual densities for different metals are:

Copper, acid bath.....	5 to 10	amperes per sq. foot.
" cyanide bath....	3 to 5	" " " "
Nickel, double sulphate..	6 to 8	" " " "
Gold, chloride in cyanide.	1 to 2	" " " "
Silver, double cyanide...	2 to 5	" " " "

In general, a low current density produces a hard, close-grained surface, and a high current density a softer, more open deposit, which may even become of a powdery appearance.

E. M. F. OF THE FORMATION OF VARIOUS SULPHATES.

Metal.	Formula of Sulphate.	E. M. F. Volts.
Potassium	K_2SO_4	2.35
Zinc	$ZnSO_4$	1.35
Cadmium	$CdSO_4$	1.05
Lead	$PbSO_4$	.90
Tin	$SnSO_4$	.88
Iron	$FeSO_4$	.83
Aluminum	$Al_2(SO_4)_3$	.70
Copper	$CuSO_4$	.47
Silver	Ag_2SO_4	.30
Platinum	None	.00

E. M. F. OF ZINC (PURE) WITH VARIOUS ELECTROLYTES.

Electrolyte. Acids.	E. M. F. Volts.	Electrolyte. Saline Solutions.	E. M. F. Volts.
H_2SO_4	1.35	$NaCl$	1.08
HCl	1.40	$ZnSO_4$	1.32
HNO_3	1.43	NH_4Cl	1.15
		$NaOH$	1.35
		KOH	1.38
		Ordinary H_2O	0.90

POWER USED IN ELECTRIC WELDING.

Iron and Steel.					Brass.					Copper.				
Area in Sq. In.	Watts in Primary of Welders.	Time in Seconds.	H. P. Applied to Dynamos.	Foot-Pounds.	Area in Sq. In.	Watts in Primary of Welders.	Time in Seconds.	H. P. Applied to Dynamos.	Foot-Pounds.	Area in Sq. In.	Watts in Primary of Welders.	Time in Seconds.	H. P. Applied to Dynamos.	Foot-Pounds.
.5	8,550	33	14.4	260,000	.25	7,500	17	12.6	117,000	.125	6,000	8	10.0	44,000
1.0	16,700	45	28.0	692,000	.50	13,500	22	22.6	281,000	.250	14,000	11	23.4	142,000
1.5	23,500	55	39.4	1,191,000	.75	19,000	29	31.8	508,000	.375	19,000	13	31.8	227,000
2.0	29,000	65	48.6	1,738,000	1.00	25,000	33	42.0	760,000	.500	25,000	16	42.0	369,000
2.5	34,000	70	57.0	2,194,000	1.25	31,000	38	52.0	1,087,000	.625	31,000	18	51.9	513,000
3.0	39,000	78	65.4	2,804,000	1.50	36,000	42	60.3	1,390,000	.750	36,500	21	61.2	700,000
3.5	44,000	85	73.7	3,447,000	1.75	40,000	45	67.0	1,659,000	.875	43,000	22	72.9	872,000
4.0	50,000	90	83.8	4,148,000	2.00	44,000	48	73.7	1,947,000	1.000	49,000	23	82.1	1,039,000

IMPEDANCE AND REACTANCE OF WIRE.

Gauge No. B. & S. Wire.	Resistance in Ohms per Mile of Wire.	Reactance and Impedance in Ohms per Mile of Wire at a Frequency of 60.						Reactance and Impedance in Ohms per Mile of Wire at a Frequency of 125.					
		12 Inches Between Centers.		18 Inches Between Centers.		24 Inches Between Centers.		12 Inches Between Centers.		18 Inches Between Centers.		24 Inches Between Centers.	
		React.	Imp.	React.	Imp.	React.	Imp.	React.	Imp.	React.	Imp.	React.	Imp.
0000	.259	.508	.570	.557	.615	.591	.646	1.06	1.092	1.17	1.190	1.23	1.260
000	.324	.523	.616	.573	.658	.607	.686	1.09	1.138	1.20	1.237	1.26	1.305
00	.412	.534	.682	.588	.725	.618	.749	1.12	1.194	1.23	1.297	1.29	1.357
0	.519	.550	.756	.603	.796	.633	.818	1.15	1.258	1.26	1.360	1.32	1.415
1	.655	.565	.865	.614	.896	.648	.920	1.18	1.349	1.28	1.436	1.35	1.500
2	.826	.580	1.008	.629	1.038	.663	1.060	1.21	1.466	1.31	1.550	1.38	1.610
3	1.041	.591	1.196	.644	1.223	.674	1.240	1.24	1.610	1.34	1.700	1.41	1.750
4	1.313	.606	1.448	.656	1.467	.690	1.480	1.26	1.820	1.37	1.890	1.44	1.940
5	1.656	.620	1.760	.670	1.780	.704	1.800	1.30	2.100	1.40	2.170	1.47	2.220
6	2.088	.633	2.180	.685	2.200	.720	2.210	1.32	2.460	1.43	2.510	1.49	2.560
7	2.633	.647	2.710	.700	2.720	.730	2.730	1.35	2.930	1.46	3.000	1.52	3.040
8	3.320	.662	3.380	.712	3.390	.742	3.400	1.38	3.590	1.48	3.630	1.55	3.660
9	4.186	.677	4.210	.727	4.220	.761	4.230	1.41	4.390	1.51	4.430	1.58	4.450
10	5.280	.688	5.320	.742	5.330	.776	5.340	1.44	5.470	1.54	5.500	1.62	5.530

NUMBER OF POLES, OUTPUT, AND SPEED OF SMALL ALTERNATORS.

No. Poles.	60 Cycle.		No. Poles.	125 Cycle.	
	Output K. W.	Speed R. P. M.		Output K. W.	Speed R. P. M.
8	75	900	10	30	1,500
12	150	600	14	120	1,070
16	250	450	16	200	940

NUMBER OF POLES, OUTPUT, AND SPEED OF INDUCTION MOTORS.

Poles.	H. P.	Speed.
4	1	1,800
6	5	1,200
6	10	1,200
8	10	900
8	20	900
10	50	720
12	75	600

RULES AND FORMULAS.

FORMULAS USED IN ALGEBRA.

Let a and b be any two quantities, then,

$(a+b)^2 = a^2 + 2ab + b^2$. **(1.)** Art. **432.**

$(a-b)^2 = a^2 - 2ab + b^2$. **(2.)** Art. **432.**

$(a+b)(a-b) = a^2 - b^2$. **(3.)** Art. **432.**

$a^2+2ab+b^2 = (a+b)(a+b) = (a+b)^2$. **(4.)** Art. **455.**

$a^2-2ab+b^2 = (a-b)(a-b) = (a-b)^2$. **(5.)** Art. **455.**

$a^2 - b^2 = (a+b)(a-b)$. **(6.)** Art. **462.**

Let $ax^2 + bx = c$ be any quadratic equation (it must be borne in mind that b and c may be positive or negative);

then,

$$x = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 + \frac{c}{a}} = \frac{-b \pm \sqrt{b^2+4ac}}{2a}.$$ Art. **597.**

THE TRIGONOMETRIC FUNCTIONS.

Art. **754.**

Rule 1.—$Sine = \frac{side\ opposite}{hypotenuse}$.

Rule 2.—*Side opposite* = *hypotenuse* × *sine*.

Rule 3.—$Cosine = \frac{side\ adjacent}{hypotenuse}$.

Rule 4.—*Side adjacent* = *hypotenuse* × *cosine*.

Rule 5.—$Tangent = \frac{side\ opposite}{side\ adjacent}$.

Rule 6.—*Side opposite = side adjacent × tangent.*

Rule 7.—$Cotangent = \frac{side\ adjacent}{side\ opposite}$.

Rule 8.—*Side adjacent = side opposite × cotangent.*

Rule 9.—$Hypotenuse = \frac{side\ opposite}{sine}$.

Rule 10.—$Hypotenuse = \frac{side\ adjacent}{cosine}$.

RULES FOR USING TABLE OF LOGARITHMS OF NUMBERS.

Arts. **625–636.**

I. **To find the Characteristic.**—*For a number greater than 1 the characteristic is one less than the number of integral places in the number. For a number wholly decimal the characteristic is negative, and is numerically one greater than the number of ciphers between the decimal point and the first digit of the decimal.*

II. **To find the Logarithm of a Number not having more than four figures.**—*Find the first three significant figures of the number whose logarithm is desired in the left-hand column; find the fourth figure in the column at the top (or bottom) of the page, and in the column under (or above) this figure, and opposite the first three figures previously found, will be the mantissa, or decimal part, of the logarithm. The characteristic being found as described above, write it at the left of the mantissa, and the resulting expression will be the logarithm of the required number.*

III. **To find the Logarithm of a Number consisting of five or more figures.**

(*a*) *If the number consists of more than five figures, and the sixth figure is 5 or greater, increase the fifth figure by 1, and write ciphers in place of the sixth and remaining figures.*

(*b*) *Find the mantissa corresponding to the logarithm of the first four figures, and subtract this mantissa from the next greater mantissa in the table; the remainder is the difference.*

(*c*) *Find in the secondary table headed P. P. a column headed by the same number as that just found for the difference, and in this column opposite the number corresponding to the fifth figure (or fifth figure increased by 1) of the given number (this figure is always situated at the left of the dividing line of the column) will be found the P. P. (proportional part) for that number. The P. P. thus found is to be added to the mantissa found in* (*b*), *and the result is the mantissa of the logarithm of the given number, as nearly as may be found with five-place tables.*

IV. **To find a Number whose Logarithm is given.**—

(*a*) *Consider the mantissa first. Glance along the different columns of the table which are headed 0 until the first two figures of the mantissa are found. Then glance down the same column until the third figure is found (or 1 less than the third figure). Having found the first three figures, glance to the right along the row in which they are situated until the last three figures of the mantissa are found. Then the number which heads the column in which the last three figures of the mantissa are found is the fourth figure of the required number, and the first three figures lie in the column headed N, and in the same row in which lie the last three figures of the mantissa.*

(*b*) *If the mantissa cannot be found in the table, find the mantissa which is nearest to, but less than, the given mantissa, and which call the* **next less mantissa.** *Subtract the next less mantissa from the next greater mantissa in the table to obtain the difference. Also subtract the next less mantissa from the mantissa of the given logarithm, and call the remainder the P. P. Looking in the secondary table headed P. P. for the column headed by the difference just found, find the number opposite the P. P. just found (or the P. P. corresponding most*

nearly to that just found); this number is the fifth figure of the required number; the fourth figure will be found at the top of the column containing the next less mantissa, and the first three figures in the column headed N, and in the same row which contains the next less mantissa.

(*c*) *Having found the figures of the number as above directed, locate the decimal point by the rules for the characteristic, annexing ciphers to bring the number up to the required number of figures if the characteristic is greater than 4.*

RULES FOR USING TRIGONOMETRIC TABLES.

Given, an angle, to find its sine, cosine, tangent, and cotangent.

Rule 11.—*Find in the table the sine, cosine, tangent, or cotangent corresponding to the degrees and minutes of the angle.*

For the seconds, find the difference of the values of the sine, cosine, tangent, or cotangent taken from the table, between which the seconds of the angle fall; multiply this difference by a fraction whose numerator is the number of seconds in the given angle, and whose denominator is 60.

If sine or tangent, add this correction to the value first found; if cosine or cotangent, subtract the correction. Art. **756.**

Given, the sine, cosine, tangent, or cotangent to find the angle corresponding.

To find the angle corresponding to a given sine, cosine, tangent, or cotangent whose exact value is not contained in the table:

Rule 12.—*Find the difference of the two numbers in the table between which the given sine, cosine, tangent, or cotangent falls, and use the number of parts in this difference as the denominator of a fraction.*

Find the difference between the number belonging to the **smaller angle,** *and the given sine, cosine, tangent, or cotangent, and use the number of parts in the difference just found as the numerator of the fraction mentioned above. Multiply this fraction by 60, and the result will be the number of seconds to be added to the* **smaller angle.** Art. **758.**

RULES FOR MENSURATION.

THE TRIANGLE.

Rule.—*The area of any triangle equals one-half the product of the base and the altitude.* Art. **766.**

THE QUADRILATERAL.

To find the area of a parallelogram:

Rule.—*The area of any parallelogram equals the product of the base and the altitude.* Art. **777.**

To find the area of a trapezoid:

Rule.—*The area of a trapezoid equals one-half the sum of the parallel sides multiplied by the altitude.* Art. **778.**

To find the area of an irregular figure bounded by straight lines:

Rule.—*Divide the figure into triangles, and find the area of each triangle separately. The sum of the areas of all the triangles will be the area of the figure.* Art. **779.**

THE CIRCLE.

To find the circumference or diameter of a circle:

Rule.—*The circumference of a circle equals the diameter multiplied by 3.1416.* Art. **780.**

Rule.—*The diameter of a circle equals the circumference divided by 3.1416.* Art. **780.**

To find the length of an arc of a circle:

Rule.—*The length of an arc of a circle equals the circumference of the circle of which the arc is a part multiplied by the number of degrees in the arc, and divided by 360.* Art. **781.**

To find the area of a circle:

Rule.—*Square the diameter, and multiply by .7854.* Art. **782.**

Given, the area of a circle to find its diameter:

Rule.—*Divide the area by .7854, and extract the square root of the quotient.* Art. **783.**

To find the area of a sector:

Rule.—*Divide the number of degrees in the arc of a sector by 360. Multiply the result by the area of the circle of which the sector is a part.* Art. **784.**

To find the area of a segment of a circle:

Rule.—*Draw radii from the center of the circle to the extremities of the arc of the segment; find the area of the sector thus formed, subtract from this the area of the triangle formed by the radii and the chord of the arc of the segment, and the result is the area of the segment.* Art. **785.**

THE ELLIPSE.

To find the perimeter of an ellipse:

Rule.—*Multiply the major axis by 1.82, and the minor axis by 1.315. The sum of the results will be the perimeter.* Art. **788.**

To find the exact area of an ellipse:

Rule.—*The area of an ellipse is equal to the product of its two diameters multiplied by .7854.* Art. **789.**

To find the area of any plane figure:

Rule.—*The area of any plane figure may be found by dividing it into triangles, quadrilaterals, circles or parts of circles, and ellipses, finding the area of each part separately, and adding them together.* Art. **790.**

THE PRISM AND CYLINDER.

To find the area of the convex surface of any right prism or right cylinder:

Rule.—*Multiply the perimeter of the base by the altitude.* Art. **803.**

To find the volume of a right prism or cylinder:

Rule.—*The volume of any right prism or cylinder equals the area of the base multiplied by the altitude.* Art. **804.**

THE PYRAMID AND CONE.

To find the area of a right pyramid or right cone:

Rule.—*The convex area of a right pyramid or cone equals the perimeter of the base multiplied by one-half the slant height.* Art. **809.**

To find the volume of a right pyramid or cone:

Rule.—*The volume of a right pyramid or cone equals the area of the base multiplied by one-third of the altitude.* Art. **810.**

THE FRUSTUM OF A PYRAMID OR CONE.

To find the convex area of a frustum of a right pyramid or right cone:

Rule.—*The convex area of a frustum of a right pyramid or right cone equals one-half the sum of the perimeters of its bases multiplied by the slant height of the frustum.* Art. **814.**

To find the volume of the frustum of a pyramid or cone:

Rule.—*Add the areas of the upper base, the lower base, and the square root of the product of the areas of the two bases; multiply this sum by one-third of the altitude.* Art. **815.**

THE SPHERE.

To find the area of the surface of a sphere:

Rule.—*The area of the surface of a sphere equals the square of the diameter multiplied by 3.1416.* Art. **817.**

To find the volume of a sphere:

Rule.—*The volume of a sphere equals the cube of the diameter multiplied by .5236.* Art. **818.**

FORMULAS USED IN ELEMENTARY MECHANICS.

UNIFORM MOTION.

Let S = the length of space passed over uniformly;
t = the time occupied in passing over the space S;
V = the velocity.

$$V = \frac{S}{t}. \qquad (7.) \text{ Art. } \mathbf{859.}$$

$$S = Vt. \qquad (8.) \text{ Art. } \mathbf{859.}$$

$$t = \frac{S}{V}. \qquad (9.) \text{ Art. } \mathbf{859.}$$

MASS, WEIGHT, AND GRAVITY.

If the mass of the body be represented by m, its weight by W, and the force of gravity at the place where the body was weighed by g, we have

$$\text{mass} = \frac{\text{weight of body}}{\text{force of gravity}}, \text{ or } m = \frac{W}{g}. \qquad (10.) \text{ Art. } \mathbf{888.}$$

FORMULAS FOR GRAVITY PROBLEMS.

Let W = weight of body at the surface;

w = weight of a body at a given distance above or below the surface;

d = distance between the center of the earth and the center of the body;

R = radius of the earth = 4,000 miles.

Formula for weight when the body is below the surface:

$$w R = d W. \qquad (11.) \text{ Art. } \mathbf{891.}$$

Formula for weight when the body is above the surface:

$$w d^2 = W R^2. \qquad (12.) \text{ Art. } \mathbf{891.}$$

FALLING BODIES.

Let g = force of gravity = constant accelerating force due to the attraction of the earth;

t = number of seconds the body falls;

v = velocity at the end of the time t;

h = distance that a body falls during the time t.

$$v = g t. \qquad (13.) \text{ Art. } \mathbf{896.}$$

That is, the velocity acquired by a freely falling body at the end of t seconds equals 32.16 multiplied by the time in seconds.

$$t = \frac{v}{g}. \qquad (14.) \text{ Art. } \mathbf{896.}$$

That is, the number of seconds during which a body must have fallen to acquire a given velocity equals the given velocity in feet per second divided by 32.16.

$$h = \frac{v^2}{2g}. \qquad \textbf{(15.)} \quad \text{Art. } \textbf{896.}$$

That is, the height from which a body must fall to acquire a given velocity equals the square of the given velocity divided by 2 × 32.16.

$$v = \sqrt{2gh}. \qquad \textbf{(16.)} \quad \text{Art. } \textbf{896.}$$

That is, the velocity that a body will acquire in falling through a given height equals the square root of the product of twice 32.16 and the given height.

$$h = \tfrac{1}{2} g t^2. \qquad \textbf{(17.)} \quad \text{Art. } \textbf{896.}$$

That is, the distance a body will fall in a given time equals 32.16 ÷ 2 multiplied by the square of the number of seconds.

$$t = \sqrt{\frac{2h}{g}} \qquad \textbf{(18.)} \quad \text{Art. } \textbf{896.}$$

That is, the time it will take a body to fall through a given height equals the square root of twice the height divided by 32.16.

CENTRIFUGAL FORCE.

The value of the centrifugal force of any revolving body, expressed in pounds, is

$$F = .00034\ W R N^2; \qquad \textbf{(19.)} \quad \text{Art. } \textbf{903.}$$

in which F = centrifugal force;
W = total weight of body in pounds;
R = radius, usually taken as the distance between the center of motion and the center of gravity of the revolving body, in feet;
N = number of revolutions per *minute*.

THE CENTER OF GRAVITY OF TWO BODIES.

Let l = the distance between the centers of the bodies;
l_1 = the short arm;
w = weight of small body;
W = weight of large body.

$$l_1 = \frac{w\,l}{W + w}. \qquad \textbf{(20.)} \quad \text{Art. } \textbf{911.}$$

THE EFFICIENCY OF A MACHINE.

Let F = the force applied to the machine;
V = the velocity ratio of the machine;
W = the weight actually lifted or equivalent resistance overcome;
E = the efficiency of the machine;

Then, $$\mathsf{E} = \frac{W}{F\,V}. \qquad \textbf{(22.)} \quad \text{Art. } \textbf{950.}$$

WORK.

If the force necessary to overcome the resistance be represented by F, the space through which the resistance acts by S, and the work done by U, then $U = F\,S$.

If W = the weight of a body, and h = the height through which it is raised, $U = W\,h$. Hence the work done

$$U = F\,S = W\,h. \qquad \textbf{(23.)} \quad \text{Art. } \textbf{953.}$$

POWER.

The power of a machine may always be determined by *dividing the work done in foot-pounds by the time in minutes required to do the work; i. e.*,

$$\text{Power} = \frac{F\,S}{T}. \qquad \textbf{(24.)} \quad \text{Art. } \textbf{954.}$$

KINETIC ENERGY.

Let W = the weight of the body in pounds;
v = its velocity in feet per second;
h = the height in feet through which the body must fall to produce the velocity v;
m = the mass of the body $= \frac{W}{g}$. (See formula **10.**)

The work necessary to raise a body through a height h is Wh. The velocity produced in falling a height h is $v = \sqrt{2gh}$, and $h = \frac{v^2}{2g}$. (See formulas **15** and **16.**)

Therefore, work $= Wh = W\frac{v^2}{2g} = \frac{1}{2} \times \frac{W}{g} \times v^2 = \frac{1}{2} m v^2$, or

$$Wh = \tfrac{1}{2} m v^2. \qquad \textbf{(25.)} \quad \text{Art. } \textbf{957.}$$

DENSITY.

The **density of a body** is its mass divided by its volume in cubic feet.

Let D be the density; then the density of a body is

$$D = \frac{m}{V}. \quad \text{Since } m = \frac{W}{g}, \; D = \frac{W}{gV}. \qquad \textbf{(26.)} \quad \text{Art. } \textbf{962.}$$

RULES AND FORMULAS USED IN HYDRO-MECHANICS.

PASCAL'S LAW.

Rule I.—*The pressure per unit of area exerted anywhere upon a mass of liquid is transmitted undiminished in all directions, and acts with the same force upon all surfaces in a direction at* **right angles** *to those surfaces.*

THE GENERAL LAW FOR THE DOWNWARD PRESSURE UPON THE BOTTOM OF ANY VESSEL.

Rule II.—*The pressure upon the bottom of a vessel containing a fluid is independent of the shape of the vessel, and is equal to the weight of a prism of the fluid whose base has the same area as the bottom of the vessel, and whose altitude is the distance between the bottom and the upper surface of the fluid plus the pressure per unit of area upon the upper surface of the fluid multiplied by the area of the bottom of the vessel.*

GENERAL LAW FOR UPWARD PRESSURE.

Rule III.—*The upward pressure on any submerged horizontal surface equals the weight of a prism of the liquid*

whose base has an area equal to the area of the submerged surface, and whose altitude is the distance between the submerged surface and the upper surface of the liquid plus the pressure per unit of area on the upper surface of the fluid multiplied by the area of the submerged surface.

GENERAL LAW FOR LATERAL PRESSURE.

Rule IV.—*The pressure upon any vertical surface due to the weight of a liquid is equal to the weight of a prism of the liquid whose base has the same area as the vertical surface, and whose altitude is the depth of the center of gravity of the vertical surface below the level of the liquid.*

Any additional pressure is to be added as in the previous cases.

GENERAL LAW FOR PRESSURE.

Rule V.—*The pressure exerted by a fluid in any direction upon* **any** *surface is equal to the weight of a prism of the fluid whose base is the projection of the surface at right angles to the direction considered, and whose height is the depth of the center of gravity of the surface below the level of the liquid.*

SPECIFIC GRAVITY.

Let W be the weight of the solid in air and W' the weight in water; then, $W - W'$ = the weight of a volume of water equal to the volume of the solid, and

$$\text{Sp. Gr.} = \frac{W}{W - W'}. \qquad \textbf{(27.)} \quad \text{Art. } \textbf{991.}$$

If the body be lighter than water, a piece of iron or other heavy substance must be attached to it sufficiently heavy to sink both. *Then weigh both bodies in air and both in water.*

Let W = weight of both bodies in air;
W' = weight of both bodies in water;
w = weight of light body in air;
W_1 = weight of heavy body in air;
W_2 = weight of heavy body in water.

Then, the specific gravity of the light body is given by

$$\text{Sp. Gr.} = \frac{w}{(W - W') - (W_1 - W_2)}. \qquad \textbf{(28.)} \quad \text{Art. } \textbf{992.}$$

To find the specific gravity of a liquid:

Weigh an empty flask; fill it with water, then weigh it, and find the difference between the two results; this will equal the weight of the water. Then weigh the flask filled with the liquid and subtract the weight of the flask; the result is the weight of a volume of the liquid equal to the volume of the water. The weight of the liquid divided by the weight of the water is the specific gravity of the liquid.

Let W = the weight of the flask and liquid;
W' = the weight of the flask and water;
w = the weight of the flask.

Then, $$\text{Sp. Gr.} = \frac{W - w}{W' - w}. \qquad \textbf{(29.)} \quad \text{Art. } \textbf{993.}$$

FORMULAS FOR FLOW OF WATER.

MEAN VELOCITY.

Let Q = the quantity in cubic feet which passes any section in 1 second;
A = the area of the section in square feet;
v_m = the mean velocity in feet per second.

Then, $$Q = A\,v_m, \qquad \textbf{(31.)} \quad \text{Art. } \textbf{1003.}$$

and $$v_m = \frac{Q}{A}. \qquad \textbf{(32.)} \quad \text{Art. } \textbf{1003.}$$

VELOCITY OF EFFLUX FROM AN ORIFICE.

Let v = the velocity of efflux in feet per second;
h = the head in feet on the orifice considered;
h_1 = the head equivalent to a pressure p;
W = the weight of the water in pounds flowing through the aperture per second.

The kinetic energy of the issuing water $= \frac{W v^2}{2g}$.

The work the issuing water can do $= W h$.

$W h = \frac{W v^2}{2g}$, or $v = \sqrt{2gh}$.

$h = \frac{p}{.434}$, where h_1 is in feet, and p in pounds per square inch.

$h_1 = \frac{p}{62.5}$, where h_1 is in feet, and p in pounds per square foot.

$h + h_1 =$ the *total head.*

$v = \sqrt{2g(h_1 + h)}$. **(33.)** Art. **1007.**

If a is the area of a large orifice in the bottom of a small vessel whose area is A, the velocity is

$$v\sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}}. \qquad \textbf{(35.)} \text{ Art. } \textbf{1010.}$$

Actual velocity of efflux from a small square-edged orifice:

$v = .98\sqrt{2gh}$. **(36.)** Art. **1013.**

Actual quantity discharged from a small square-edged orifice:

$Q = .615 A v$. **(37.)** Art. **1015.**

THEORETICAL AND ACTUAL DISCHARGE.

Let Q = theoretical number of cubic feet discharged per second;

v_m = theoretical mean velocity through orifice in feet per second $= Q \div A$;

A = area of orifice in square feet;

h = theoretical head necessary to give a mean velocity v_m.

Q_a = actual quantity discharged in cubic feet per second.

Then, for an orifice in a thin plate, or a square-edged orifice (the hole itself may be of any shape, triangular,

square, circular, etc., but the edges must not be rounded), the actual quantity discharged is:

$$Q_a = .615\,Q = .615\,A\,v_m = .615\,A\sqrt{2gh}. \qquad \textbf{(38.)}$$

Art. **1017.**

For a discharge through a short tube,

$$Q_a = .815\,Q = .815\,A\,v_m = .815\,A\sqrt{2gh}. \quad \textbf{(39.)}$$ Art. **1017.**

For a discharge through a mouthpiece,

$$Q_a = .97\,Q = .97\,A\,v_m = .97\,A\sqrt{2gh}. \qquad \textbf{(40.)}$$ Art. **1017.**

For a discharge through the compound mouthpiece, the area of the orifice being taken as the area of the smallest section,

$$Q_a = 1.5526\,Q = 1.5526\,A\,v_m = 1.5526\,A\sqrt{2gh}. \qquad \textbf{(41.)}$$

Art. **1017.**

In these four formulas it is taken for granted that there is a constant head.

FLOW OF WATER THROUGH WEIRS.

If d = the depth of the opening in feet, and b its breadth in feet, the area of the opening is $A = d \times b$, and the theoretical discharge is $Q = d \times b \times v_m = d\,b \times \frac{2}{3}\sqrt{2gh} = \frac{2}{3}\,b\,d\sqrt{2gd}$, the head for this case being taken as d.

The actual discharge is

$$Q_a = .615\,Q = .615 \times \tfrac{2}{3}\,b\,d\sqrt{2gd} = .41\,b\sqrt{2gd^3}. \qquad \textbf{(42.)}$$

Art. **1020.**

That is, *the actual discharge through a weir in cubic feet per second, whose top is on a level with the upper surface of the water, is equal to .41 multiplied by the breadth of the weir, multiplied by the square root of 2 g times the cube of the depth of the weir. All dimensions are to be taken in feet.*

To obtain the mean velocity v_m, divide the actual discharge by the area of the weir, or

$$v_m = \frac{Q_a}{A} = \frac{Q_a}{b\,d}. \qquad \textbf{(43.)}$$ Art. **1020.**

For a weir whose upper edge is below the level of the upper surface of the water, let h_1 be the depth in feet of the top of the weir below the surface of the water, and h the depth in feet of the bottom of the weir below the surface of the water. The actual discharge Q_a in cubic feet per second is

$$Q_a = .41\, b\sqrt{2g}\,(\sqrt{h^3} - \sqrt{h_1^3}). \qquad \textbf{(44.)} \quad \text{Art. } \textbf{1022.}$$

FLOW OF WATER IN PIPES.

THE MEAN VELOCITY OF DISCHARGE.

For straight cylindrical pipes of uniform diameter:

Let v_m = mean velocity of discharge in feet per second;
h = total head in feet = the vertical distance between the level of the water in the reservoir and the point of discharge;
l = length of pipe in feet;
d = diameter of pipe in inches;
f = coefficient of friction.

Then, $$v_m = 2.315\sqrt{\frac{h\,d}{f\,l + \frac{1}{8}\,d}}. \qquad \textbf{(45.)} \quad \text{Art. } \textbf{1024.}$$

When the pipe is very long, compared with the diameter, the following formula may be used:

$$v_m = 2.315\sqrt{\frac{h\,d}{f\,l}}. \qquad \textbf{(46.)} \quad \text{Art. } \textbf{1027.}$$

THE ACTUAL HEAD.

The actual head necessary to produce a certain velocity v_m may be calculated by the formula

$$h = \frac{f\,l\,v^2{}_m}{5.36\,d} + .0233\,v^2{}_m. \qquad \textbf{(47.)} \quad \text{Art. } \textbf{1029.}$$

THE QUANTITY DISCHARGED FROM PIPES.

Let d = the diameter of the pipe in inches, then the discharge Q in gallons per second is

$$Q = .0408\,d^2\,v_m. \qquad \textbf{(48.)} \quad \text{Art. } \textbf{1030.}$$

If the diameter of the pipe and the discharge are known, the mean velocity v_m is

$$v_m = \frac{24.51\,Q}{d^2}. \qquad \textbf{(49.)} \quad \text{Art. } \textbf{1031.}$$

If the head, the length of the pipe, and the diameter of the pipe are given, to find the discharge use the formula

$$Q = .09445\,d^2\sqrt{\frac{h\,d}{f\,l + \frac{1}{2}\,d}}. \qquad \textbf{(50.)} \quad \text{Art. } \textbf{1032.}$$

To find the value of f, calculate v_m by formula **46,** assuming that $f = .025$, and get the final value of f from the following table. Art. **1033.**

$v_m =$	0.1	0.2	0.3	0.4	0.5	0.6
$f =$	.0686	.0527	.0457	.0415	.0387	.0365
$v_m =$	0.7	0.8	0.9	1	1¼	1½
$f =$	.0349	.0336	.0325	.0315	.0297	.0284
$v_m =$	2	3	4	6	8	12
$f =$	.0265	.0243	.0230	.0214	.0205	.0193

NOTE.—The values given in the above table are calculated by the formula $f = .014392 + \frac{.017156}{\sqrt{v_m}}$.

TO CALCULATE THE DIAMETER OF A PIPE.

Neglecting the fraction $\frac{1}{2}\,d$ in formula **50,** and solving for d,

$$d = 2.57\sqrt[5]{\frac{f\,l\,Q^2}{h}}.$$

Assuming that $f = .025$, for a trial value, the above equation then becomes

$$d = 1.229\sqrt[5]{\frac{l\,Q^2}{h}}. \qquad \textbf{(51.)} \quad \text{Art. } \textbf{1035.}$$

Formula **50** may also be written

$$d = 2.57 \sqrt[5]{\frac{(f l + \frac{1}{8} d)\, Q^2}{h}}. \qquad \textbf{(52.)} \text{ Art. } \textbf{1035.}$$

By aid of formulas **51** and **52**, the diameter of a pipe may be approximated as follows:

Rule.—*Find the value of d by formula* ***51****; substitute this value in formula* ***49****, and find the value of* v_m. *Then find from the table the value of f corresponding to this value of* v_m. *Substitute the values of d and f just found in the right-hand member of formula* ***52****, and solve for d; the result will be the diameter of the pipe, nearly enough for all practical purposes.*

FORMULAS USED IN PNEUMATICS.

PRESSURE, VOLUME, DENSITY, AND WEIGHT OF AIR WHEN THE TEMPERATURE IS CONSTANT:

Mariotte's Law.—*The temperature remaining the same, the volume of a given quantity of gas varies inversely as the pressure.*

Let p = pressure for one position of the piston;
p_1 = pressure for any other position of the piston;
v = volume corresponding to the pressure p;
v_1 = volume corresponding to the pressure p_1.

Then, $p\, v = p_1\, v_1$. **(53.)** Art. **1049.**

Let D be the density corresponding to the pressure p and volume v, and D_1 be the density corresponding to the pressure p_1 and volume v_1; then,

$$p : D = p_1 : D_1, \text{ or } p\, D_1 = p_1\, D, \qquad \textbf{(54.)} \text{ Art. } \textbf{1052.}$$

and $v : D_1 = v_1 : D$, or $v\, D = v_1\, D_1$. **(55.)** Art. **1052.**

Thus, let W be the weight of a cubic foot of air or other gas, whose volume is v, and pressure is p; let W_1 be the weight of a cubic foot when the volume is v_1, and pressure is p_1; then,

$$p\, W_1 = p_1\, W. \qquad \textbf{(56.)} \text{ Art. } \textbf{1052.}$$

$$v\, W = v_1\, W_1. \qquad \textbf{(57.)} \text{ Art. } \textbf{1052.}$$

PRESSURE AND VOLUME OF A GAS WITH VARIABLE TEMPERATURE:

Gay-Lussac's Law.—*If the pressure remains constant, every increase of temperature of 1° F. produces in a given quantity of gas an expansion of $\frac{1}{492}$ of its volume at 32° F.*

If the pressure remains constant it will also be found that every decrease of temperature of 1° F. will cause a decrease of $\frac{1}{492}$ of the volume at 32° F.

Let v = original volume of gas;
v_1 = final volume of gas;
t = temperature corresponding to volume v;
t_1 = temperature corresponding to volume v_1.

Then, $$v_1 = v\left(\frac{460 + t_1}{460 + t}\right). \qquad (58.)$$ Art. **1054.**

That is, *the volume of gas after heating (or cooling) equals the original volume multiplied by 460 plus the final temperature divided by 460 plus the original temperature.*

Let p = the original tension;
t = the corresponding temperature;
p_1 = final tension;
t_1 = final temperature.

Then, $$p_1 = p\left(\frac{460 + t_1}{460 + t}\right). \qquad (59.)$$ Art. **1055.**

Let p = pressure in pounds per square inch;
V = volume of air in cubic feet;
T = absolute temperature;
W = weight in pounds.

Then, $$p\,V = .37052\,T. \qquad (60.)$$ Art. **1056.**

If the weight of the air be greater or less than 1 pound, the following formula must be used:

$$p\,V = .37052\,W\,T. \qquad (61.)$$ Art. **1057.**

Let p_1, V_1, and T_1 represent the pressure, volume, and temperature of the same weight of air in another state; then,

$$\frac{p\,V}{T} = \frac{p_1\,V_1}{T_1}. \qquad (62.)$$ Art. **1058.**

MIXTURE OF TWO GASES HAVING UNEQUAL VOLUMES AND PRESSURES.

Let v and p be the volume and pressure, respectively, of one of the gases.

Let v_1 and p_1 be the volume and pressure, respectively, of the other gas.

Let V and P be the volume and pressure, respectively, of the mixture. Then, if the temperature remains the same,

$$V P = v p + v_1 p_1. \quad \textbf{(63.)} \quad \text{Art. } \textbf{1062.}$$

MIXTURE OF TWO VOLUMES OF AIR HAVING UNEQUAL PRESSURES, VOLUMES, AND TEMPERATURES.

If a body of air having a temperature t_1, a pressure p_1, and a volume v_1, be mixed with another volume of air having a temperature t_2, a pressure p_2, and a volume v_2, to form a volume V having a pressure P, and a temperature t, then, either the new temperature t, the new volume V, or the new pressure P may be found, if the other two quantities are known, by the following formula, in which T_1, T_2, and T are the absolute temperatures corresponding to t_1, t_2, and t:

$$PV = \left[\frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}\right] T. \quad \textbf{(64.)} \quad \text{Art. } \textbf{1063.}$$

FORMULAS USED IN HEAT.

TO CHANGE FAHRENHEIT TEMPERATURES TO CENTIGRADE, AND CENTIGRADE TO FAHRENHEIT.

Let t_c = temperature Centigrade, and t_f = temperature Fahrenheit. Then,

$$t_f = \tfrac{9}{5}\, t_c + 32^\circ, \quad \textbf{(65.)} \quad \text{Art. } \textbf{1104.}$$

$$\text{and } t_c = (t_f - 32^\circ)\, \tfrac{5}{9}. \quad \textbf{(66.)} \quad \text{Art. } \textbf{1104.}$$

EXPANSION OR CONTRACTION OF BODIES FROM CHANGES OF TEMPERATURE.

Let L = length of any body;

l = amount of expansion or contraction due to heating or cooling the body;

A = area of any section of the body;

a = increase or decrease of area of the same section after heating or cooling the body;
V = volume of the body;
v = increase or decrease in volume due to heating or cooling the body;
C_1 = coefficient of expansion taken from column 1, Table 1;
C_2 = coefficient taken from column 2, Table 1;
C_3 = coefficient taken from column 3, Table 1;
t = difference in degrees of temperature between the original temperature and the temperature of the body after it has been heated or cooled.

Then, $l = L\,C_1\,t$; **(67.)** Art. **1113.**

$a = A\,C_2\,t$; **(68.)** Art. **1113.**

$v = V\,C_3\,t$. **(69.)** Art. **1113.**

RESULTING TEMPERATURES AND SPECIFIC HEATS OF THE MIXTURE OF SEVERAL BODIES HAVING UNEQUAL TEMPERATURES.

$$t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3 + \text{etc.}}{W_1 s_1 + W_2 s_2 + W_3 s_3 + \text{etc.}}, \quad \left\{ \begin{array}{l} \textbf{(73.)} \\ \text{Art. } \textbf{1136.} \end{array} \right.$$

$$\text{or } s_3 = \frac{W_1 s_1 (t_1 - t) + W_2 s_2 (t_2 - t) + \text{etc.}}{W_3 (t - t_3)}, \quad \left\{ \begin{array}{l} \textbf{(74.)} \\ \text{Art. } \textbf{1137.} \end{array} \right.$$

in which t is the final temperature of the mixture; W_1, s_1, and t_1, the weight, specific heat, and temperature, respectively, of one body; W_2, s_2, and t_2 the same for second body; and W_3, s_3, and t_3 the same for a third body, etc.

WORK DONE BY THE ISOTHERMAL EXPANSION OF A GAS.

L = the work in foot-pounds;
P = the total initial pressure in pounds per square foot;
P_1 = the total final pressure in pounds per square foot;
V = the initial volume in cubic feet;
V_1 = the final volume in cubic feet.

$$L = 2.3026\,PV \log \frac{V_1}{V}. \qquad \textbf{(75.)} \quad \text{Art. } \textbf{1160.}$$

Since $PV = P_1V_1$, $\frac{P}{P_1} = \frac{V_1}{V}$, and formula **75** might be written

$$L = 2.3026\,PV \log \frac{P}{P_1}. \qquad \textbf{(76.)} \quad \text{Art. } \textbf{1160.}$$

Whichever formula is used it must be kept in mind that the fraction $\frac{V_1}{V}$ or $\frac{P}{P_1}$ must *always be greater than 1; that is, the numerator must always be greater than the denominator.*

WORK REQUIRED TO COMPRESS A GAS ISOTHERMALLY.

$$L = 2.3026\,PV \log \frac{V}{V_1}, \qquad \textbf{(77.)} \text{ and}$$

$$L = 2.3026\,PV \log \frac{P_1}{P}, \qquad \textbf{(78.)} \quad \text{Art. } \textbf{1162.}$$

in which the letters have the same meaning as before, and *the larger volume or pressure is always in the numerator.*

Formulas **75, 76, 77,** and **78** will be easier to use if the pressure be taken in pounds per square inch, and $144 \times 2.3026 = 331.5744$ be substituted for 2.3026. As before, the volume must always be taken in cubic feet. Formulas **75** and **76** then become

$$L = 331.5744\,p\,V \log \frac{V_1}{V}, \qquad \textbf{(79.)}$$

$$L = 331.5744\,p\,V \log \frac{p}{p_1}, \qquad \textbf{(80.)} \quad \text{Art. } \textbf{1163.}$$

in which p = pressure in pounds per square inch.

RELATION BETWEEN PRESSURES AND VOLUMES OF AIR DURING ADIABATIC EXPANSION OR COMPRESSION.

Let p and v = the volume and pressure during one stage;

p_1 and v_1 = the volume and pressure during the next stage;

p_2 and v_2 = the volume and pressure during the third stage, etc.;

then $p\,v^{1.41} = p_1\,v_1^{1.41} = p_2\,v_2^{1.41} = \text{etc.}$ **(81.)** Art. **1164.**

WORK DONE BY THE ADIABATIC EXPANSION OF AIR.

Let L = work in foot-pounds;
P = initial pressure in pounds per square foot;
P_1 = final pressure in pounds per square foot;
V = initial volume in cubic feet;
V_1 = final volume in cubic feet.

$$L = 2.44\,PV\left[1 - \left(\frac{V}{V_1}\right)^{.41}\right], \qquad \textbf{(82.)}\ \text{Art. } \textbf{1165.}$$

$$L = 2.44\,PV\left[1 - \left(\frac{P_1}{P}\right)^{.29078}\right]. \qquad \textbf{(83.)}\ \text{Art. } \textbf{1166.}$$

Let p and p_1 be the initial and final pressures, respectively, in pounds per square inch, then,

$$L = 351.36\,pV\left[1 - \left(\frac{V}{V_1}\right)^{.41}\right], \qquad \textbf{(84.)}$$

$$L = 351.36\,pV\left[1 - \left(\frac{p_1}{p}\right)^{.29078}\right]. \qquad \textbf{(85.)}\ \text{Art. } \textbf{1167.}$$

The relation between the pressures and temperatures, and the volumes and temperatures, during the adiabatic expansion of air, are given by the following formulas:

$$\left(\frac{P}{P_1}\right)^{.29078} = \frac{T}{T_1}. \qquad \textbf{(86.)}\ \text{Art. } \textbf{1170.}$$

$$\left(\frac{V_1}{V}\right)^{.41} = \frac{T}{T_1}. \qquad \textbf{(87.)}\ \text{Art. } \textbf{1170.}$$

To obtain the area of the diagram representing the adiabatic compression of air, the following formula may be used which gives it directly when p and p_1 are the greater and lesser pressures, respectively, and V and V_1 their corresponding volumes:

$$\frac{pV - p_1V_1}{.41} = \text{area}. \qquad \textbf{(88.)}\ \text{Art. } \textbf{1172.}$$

EFFICIENCY OF A PERFECT HEAT ENGINE.

Let $c = \frac{P_1V_1}{T_1}$, and $r = \frac{V}{V_1}$, then, the efficiency of a perfect heat engine $= \frac{2.3026\,c\log r_1(T_1 - T_2)}{2.3026\,c\,T_1\log r.} = E = \frac{T_1 - T_2}{T_1}$. **(89.)** Art. **1182.**

That is, *for a perfect heat engine, operating through a reversible cycle process, the efficiency of the machine is the ratio of the difference of the absolute temperatures of the sources of heat and of cold to the absolute temperature of the source of heat.*

FORMULAS USED IN STEAM AND STEAM ENGINES.

RELATION BETWEEN THE PRESSURE AND TEMPERATURE OF SATURATED STEAM.

Let t = temperature, Fahrenheit;

T = absolute temperature = $t + 460°$;

p = absolute pressure (pressure above vacuum) in pounds per sq. in.

$$\text{Log}\, p = 6.1007 - \frac{2{,}719.78}{T} - \frac{400{,}215}{T^2}. \quad \left\{ \begin{array}{l} \textbf{(90.)} \\ \text{Art. } \textbf{1197.} \end{array} \right.$$

TOTAL HEAT OF VAPORIZATION.

Let H = total heat of vaporization;

t = temperature;

$H = 1{,}081.94 + .305\, t.$ **(91.)** Art. **1200.**

SPECIFIC VOLUME OF SATURATED STEAM.

$p\, V^{\frac{17}{16}} = 475,$ **(92.)** Art. **1203.**

in which p is the pressure in pounds per square inch, and V the volume in cubic feet of a pound of steam at the given pressure.

WORK DONE IN THE CYLINDER OF A STEAM ENGINE.

Let P = the pressure on the piston in pounds per sq. ft.;

p = pressure on piston in pounds per sq. in.;

V = volume swept through by the piston;

W = work in foot-pounds.

Then, $W = P\, V = 144\, p\, V.$ **(93.)** Art. **1212.**

RELATION BETWEEN CLEARANCE, CUT-OFF, AND NUMBER OF EXPANSIONS, OR RATIO OF EXPANSION.

Let c = the number of expansions;

i = the clearance, expressed as a per cent. of the stroke;

k = the real cut-off;
k_1 = the apparent cut-off;
r = the apparent number of expansions $= \frac{1}{k_1}$.

Then, $c = \frac{1}{k}$ and $k = \frac{1}{c}$. **(96.)** Art. **1254.**

$$k = \frac{k_1 + i}{1 + i}. \qquad \textbf{(97.)} \text{ Art. } \textbf{1254.}$$

TO FIND THE AREA OF A THEORETICAL DIAGRAM AND THE WORK REPRESENTED BY THE DIAGRAM.

Let A = total area of diagram in square inches;
p_1 = initial pressure measured in inches;
V_1 = volume at cut-off measured in inches;
L = work in foot-pounds;
a = net area of diagram;
h = scale used to lay off pressures;
h_1 = scale used to lay off volumes.

$$A = 2.3026\, p_1 V_1 \log \frac{V}{V_1}. \qquad \textbf{(94.)} \text{ Art. } \textbf{1215.}$$

$$L = 144\, a\, h\, h_1. \qquad \textbf{(95.)} \text{ Art. } \textbf{1215.}$$

INDICATED HORSEPOWER AND MEAN EFFECTIVE PRESSURE.

Let P = the M. E. P. in lb. per sq. in.;
A = the area of piston in sq. in.;
L = the length of stroke in ft.
N = the No. of strokes per min.

Then, the work done per minute is $P\,L\,A\,N$ foot-pounds.
One horsepower = 33,000 foot-pounds per min.
Therefore, the indicated horsepower of the engine is found from the formula

$$\text{I. H. P.} = \frac{P\,L\,A\,N}{33{,}000}. \qquad \textbf{(98.)} \text{ Art. } \textbf{1268.}$$

When the point of real cut-off, and the steam pressure

at the beginning of the stroke, are known, the M. E. P. may be found approximately by the following formula:

$$\text{M. E. P.} = \frac{.9\,P(1 + 2.3 \log e)}{e} - .9\,p, \qquad \textbf{(99.)}$$

Art. **1269.**

in which P = absolute steam pressure = gauge pressure + 14.7 pounds;

e = ratio of expansion;

p = absolute back pressure.

p is usually taken as about 3 pounds for condensing engines, and 17 pounds for non-condensing engines.

RELATION BETWEEN LENGTH OF STROKE, NUMBER OF REVOLUTIONS, AND PISTON SPEED.

Let L = length of stroke in inches;

R = number of revolutions per minute;

S = piston speed in feet per minute.

$$\left.\begin{aligned} S &= \frac{L\,R}{6}. \\ L &= \frac{6\,S}{R}. \\ R &= \frac{6\,S}{L}. \end{aligned}\right\} \qquad \textbf{(100.)}$$

Art. **1270.**

RATIO OF EXPANSION IN A COMPOUND OR TRIPLE-EXPANSION ENGINE.

Let e = ratio of expansion in high-pressure cylinder;

E = total ratio of expansion;

v = volume of cylinder receiving steam from the boiler;

V = volume of cylinder exhausting into atmosphere or condenser.

Then, $$E = \frac{e\,V}{v}. \qquad \textbf{(102.)}$$

Art. **1298.**

RATIO BETWEEN THE CYLINDERS OF A COMPOUND ENGINE.

Letting the letters have the same meaning as in the last formula, then either of the following formulas may be used:

$$E = 2.72\,\frac{V}{v}, \text{ or } \frac{V}{v} = \frac{E}{2.72}. \qquad \textbf{(103.)} \text{ Art. } \textbf{1306.}$$

$$\frac{V}{v} = \sqrt{E}. \qquad \textbf{(104.)} \text{ Art. } \textbf{1306.}$$

AMOUNT OF WATER AND THE COOLING SURFACE FOR CONDENSERS.

Water Required by a Condenser.

Let t_1 = the temperature of departing condensing water;
t_2 = the temperature of entering condensing water;
t_3 = the temperature of the condensed steam upon leaving the condenser;
H = total heat of one pound of steam at the pressure of the exhaust;
W = the weight of water required per pound of steam condensed.

$$W = \frac{H - t_3 + 32}{t_1 - t_2}. \qquad \textbf{(105.)} \text{ Art. } \textbf{1323.}$$

Cooling Surface.

Let S = the required surface in square feet;
W = total weight of steam used per hour.

Then, $S = .0944\ W$. **(106.)** Art. **1325.**

TO FIND THE WEIGHT OF A FLY-WHEEL.

Let V_1 = the greatest velocity of the crank-pin in ft. per sec.;
V_2 = the least velocity of the crank-pin in ft. per sec.;
V_0 = average velocity of crank-pin in ft. per sec.;
W = required weight of fly-wheel in pounds;
H = the number of foot-pounds per sq. in. of piston represented by the excess of crank effect over the resistance;
A = area of piston in square inches;
n = ratio between radius of fly-wheel and length of crank;

$$E = \frac{V_1 - V_2}{V_0} = \text{coefficient of unsteadiness.}$$

$$W = \frac{A\,H\,g}{n^2\,E\,V_0^2}. \qquad \textbf{(107.)} \text{ Art. } \textbf{1328.}$$

FORMULAS USED IN STRENGTH OF MATERIALS.

UNIT STRESS, UNIT STRAIN, AND COEFFICIENT OF ELASTICITY.

Let P = the total stress in pounds;
A = area of cross-section in square inches;
S = unit stress in pounds per square inch;
l = length of body in inches;
e = elongation in inches;
s = unit strain;
E = coefficient of elasticity.

$$S = \frac{P}{A}, \text{ or } P = A\,S. \qquad \textbf{(108.)} \quad \text{Art. } \textbf{1345.}$$

$$s = \frac{e}{l}, \text{ or } e = l s. \qquad \textbf{(109.)} \quad \text{Art. } \textbf{1346.}$$

$$E = \frac{S}{s} = \frac{P}{A} \div \frac{e}{l} = \frac{P l}{A e}. \qquad \textbf{(110.)} \quad \text{Art. } \textbf{1352.}$$

STRENGTH OF PIPES AND CYLINDERS.

Let d = inside diameter of pipe in inches;
l = length of pipe in inches;
p = pressure in pounds per square inch;
P = total pressure; then, $P = p\,l\,d$;
t = thickness of pipe;
S = working strength of the material.

For longitudinal rupture

$$p\,l\,d = 2\,t\,l\,S, \text{ or}$$

$$p\,d = 2\,t\,S. \qquad \textbf{(111.)} \quad \text{Art. } \textbf{1365.}$$

For transverse rupture

$$p\,d = 4\,t\,S. \qquad \textbf{(112.)} \quad \text{Art. } \textbf{1366.}$$

Since, for longitudinal rupture, $p\,d = 2\,t\,S$, it is seen that a cylinder is twice as strong against transverse rupture as against longitudinal rupture.

For pipes and cylinders whose thickness is greater than $\frac{1}{10}$ of the radius, use the following formula, in which r = the inner radius, and the other letters have the same meaning as before.

$$p = \frac{S t}{r + t}. \qquad \textbf{(113.)} \quad \text{Art. } \textbf{1367.}$$

The following formula gives the collapsing pressure in lb. per sq. in. for wrought-iron pipe:

$$p = 9{,}600{,}000 \frac{t^{2.18}}{l d}. \qquad \textbf{(114.)} \quad \text{Art. } \textbf{1368.}$$

MOMENT OF INERTIA, RESISTING MOMENT, AND BENDING MOMENT OF BEAMS.

Let I = moment of inertia;
A = area of cross-section;
r = radius of gyration;
c = distance from neutral axis to outermost fiber;
S_1 = ultimate strength of flexure;
f = factor of safety;
M = bending moment.

$$I = A r^2. \qquad \textbf{(115.)} \quad \text{Art. } \textbf{1396.}$$

The resisting moment is given by the expression

$$\frac{S}{c} A r^2 = \frac{S}{c} I, \text{ or } S \frac{I}{c}.$$

For the bending moment

$$M = S_1 \frac{I}{c}, \qquad \textbf{(116.)} \quad \text{Art. } \textbf{1398.}$$

and, when a factor of safety is used,

$$M = \frac{S_1 I}{f c}. \qquad \textbf{(117.)} \quad \text{Art. } \textbf{1401.}$$

DEFLECTION OF A BEAM.

Let a = a constant depending on the manner of loading the beam and the condition of the ends;
s = the deflection;
E = the coefficient of elasticity;
l = the length of the beam in inches;
W = the total weight supported in pounds;
I = moment of inertia about the neutral axis.

$$s = a \frac{W l^3}{E I}. \qquad \textbf{(118.)} \quad \text{Art. } \textbf{1404.}$$

STRENGTH OF COLUMNS.

Let W = load on a column;
S_1 = ultimate strength for compression;
A = area of section of column;
f = factor of safety;
l = length of column in inches;
g = a constant to be taken from table;
I = least moment of inertia of cross-section;
b = length of longer side of a rectangular column.
d = length of shorter side of a rectangular column, or the diameter of a circular column;
c = length of one side of a square column.

$$W = \frac{S_1 A}{f\left(1 + \frac{A l^2}{g I}\right)}. \qquad \textbf{(119.)} \text{ Art. } \textbf{1411.}$$

$$c = \sqrt{\frac{Wf}{2S_1} + \sqrt{\frac{Wf}{S_1}\left(\frac{Wf}{4S_1} + \frac{12 l^2}{g}\right)}}. \quad \left\{\begin{matrix}\textbf{(120.)} \\ \text{Art. } \textbf{1413.}\end{matrix}\right.$$

For a circular column,

$$d = 1.4142 \sqrt{\frac{.3183\, Wf}{S_1} + \sqrt{\frac{.3183\, Wf}{S_1}\left(\frac{.3183\, Wf}{S_1} + \frac{16 l^2}{g}\right)}}.$$

(121.) Art. **1413.**

For a rectangular column, assume d, then,

$$b = \frac{Wf\left(1 + \frac{12 l^2}{d^2 g}\right)}{d S_1}. \qquad \textbf{(122.)} \text{ Art. } \textbf{1413.}$$

STRENGTH OF SHAFTS.

Let d = diameter of a round shaft, or side of a square shaft, in inches;
c = a constant taken from Table 31;
c_1 = a constant taken from Table 31;
P = a force applied at the end of a lever arm in pounds;
r = length of lever arm in inches;

H = horsepower transmitted by shaft;
N = number of revolutions per minute;
k = a constant taken from Table 32;
k_1 = a constant taken from Table 32;
q = a constant taken from Table 32;
q_1 = a constant taken from Table 32.

For all solid shafts below 11 inches in diameter use the formula

$$d = c\sqrt[3]{Pr} = c_1\sqrt[4]{\frac{H}{N}}. \qquad \textbf{(123.)} \text{ Art. } \textbf{1415.}$$

If the diameter of a wrought-iron shaft is greater than 12.4″, of a cast-iron shaft greater than 10.3″, or of a steel shaft greater than 13.6″, the following formula should be used:

$$d = k\sqrt[3]{Pr} = k_1\sqrt[3]{\frac{H}{N}}. \qquad \textbf{(124.)} \text{ Art. } \textbf{1416.}$$

If the shaft is hollow (round) either of the two following formulas may be used:

$$P = q\left(\frac{d_1^4 - d_2^4}{d_1 r}\right), \qquad \textbf{(125.)} \text{ Art. } \textbf{1416.}$$

$$\text{or } H = q_1 N\left(\frac{d_1^4 - d_2^4}{d_1}\right). \qquad \textbf{(126.)} \text{ Art. } \textbf{1416.}$$

SHAFTS SUBJECTED TO COMBINED BENDING AND TWISTING.

Let B = bending moment;
T = twisting moment;
T_1 = the twisting moment which would have the same effect as B and T acting together.

Then, $$T_1 = B + \sqrt{B^2 + T^2}. \qquad \textbf{(255.)} \text{ Art. } \textbf{1991.}$$

The twisting moment T_1 is called the **ideal twisting moment,** and should be used in place of Pr in formulas **123** and **124** to determine the diameter of a shaft subjected to combined bending and twisting.

VALUES OF c AND c_1 TO BE USED IN FORMULA 123.

Material.	c		c_1	
	Round.	Square.	Round.	Square.
Wrought Iron	.31	.272	4.92	4.31
Cast Iron.....	.353	.309	5.59	4.89
Steel	.297	.26	4.7	4.11

VALUES OF k, k_1, q, AND q_1 TO BE USED IN FORMULAS 124, 125, AND 126.

Material.	k	k_1	q	q_1
Wrought Iron	.0909	3.62	1,335	.0212
Cast Iron	.1145	4.56	669	.0106
Steel	.0828	3.3	1,767	.028

STRENGTH OF ROPES AND CHAINS.

Let P = working or safe load in pounds;
C = circumference of rope in inches;
d = diameter of the link of a chain in inches.

For manila ropes, hemp ropes, or tarred hemp ropes,

$$P = 100\ C^2. \qquad \textbf{(127.)}\ \text{Art. } \textbf{1417.}$$

For iron wire rope of 7 strands, 19 wires to the strand,

$$P = 600\ C^2. \qquad \textbf{(128.)}\ \text{Art. } \textbf{1418.}$$

For the best steel wire rope, 7 strands, 19 wires to the strand,

$$P = 1{,}000\ C^2. \qquad \textbf{(129.)}\ \text{Art. } \textbf{1418.}$$

For open-link chains made from a good quality of wrought iron,

$$P = 12{,}000\ d^2, \qquad \textbf{(130.)}\ \text{Art. } \textbf{1421.}$$

and for stud-link chains,

$$P = 18{,}000\ d^2. \qquad \textbf{(131.)}\ \text{Art. } \textbf{1421.}$$

FORMULAS USED IN APPLIED MECHANICS.

RELATION BETWEEN THE DIAMETERS AND NUMBER OF REVOLUTIONS OF PULLEYS.

Let D_1 = diameter of driving pulley;
N_1 = number of revolutions of driving pulley;
F_1 = diameter of driven pulley;
n_1 = number of revolutions of driven pulley.

$$\frac{N_1}{n_1} = \frac{F_1}{D_1}. \qquad \textbf{(136.)} \quad \text{Art. } \textbf{1477.}$$

$$D_1 N_1 = F_1 n_1.$$

HORSEPOWER OF BELTS.

Let H = horsepower to be transmitted;
W = width of belt in inches;
S = belt speed in feet per minute.

FOR SINGLE BELTS.

$$H = \frac{WS}{900}. \qquad \textbf{(138.)} \quad \text{Art. } \textbf{1488.}$$

$$W = \frac{900\,H}{S}. \qquad \textbf{(139.)} \quad \text{Art. } \textbf{1488.}$$

$$S = \frac{900\,H}{W}. \qquad \textbf{(140.)} \quad \text{Art. } \textbf{1488.}$$

FOR DOUBLE BELTS.

$$H = \frac{WS}{630}. \qquad \textbf{(141.)} \quad \text{Art. } \textbf{1489.}$$

$$W = \frac{630\,H}{S}. \qquad \textbf{(142.)} \quad \text{Art. } \textbf{1489.}$$

$$S = \frac{630\,H}{W}. \qquad \textbf{(143.)} \quad \text{Art. } \textbf{1489.}$$

TEETH OF GEAR-WHEELS.

Let P = diametral pitch;
D = diameter of pitch circle of large gear;
d = diameter of pitch circle of small gear;
$O\,D$ = outside diameter.

C = circular pitch;
N = number of teeth;
A = distance between centers of wheels;
V = velocity; i. e., revolutions per minute of large gear.
v = velocity; i. e., revolutions per minute of small gear.

$$P = \frac{3.1416}{C}, \text{ and} \qquad \textbf{(149.)}$$

$$C = \frac{3.1416}{P}. \qquad \textbf{(150.)} \text{ Art. } \textbf{1556.}$$

$$N = DP. \qquad \textbf{(151.)} \text{ Art. } \textbf{1557.}$$

$$OD = \frac{N}{P} + 2 \times \frac{1}{P} = \frac{N+2}{P}. \qquad \textbf{(152.)} \text{ Art. } \textbf{1557.}$$

$$D = \frac{C \times N}{3.1416}. \qquad \textbf{(153.)} \text{ Art. } \textbf{1557.}$$

Given the distance between the centers of two gears and their velocities, the formulas for their diameters are

$$d = \frac{2AV}{V+v}; \qquad \textbf{(154.)}$$

and $$D = \frac{2Av}{v+V}. \qquad \textbf{(155.)} \text{ Art. } \textbf{1557.}$$

THE PRONY BRAKE.

Let $H.P.$ = number of horsepower absorbed;
R = length of lever arm in feet;
W = weight in scale pan;
N = number of revolutions per minute;
r = length of lever arm in inches.

$$H.P. = \frac{2 \times 3.1416 \times R \times W \times N}{33{,}000}, \text{ or}$$

$$H.P. = .0001904\, R\, W\, N. \qquad \textbf{(156.)} \text{ Art. } \textbf{1593.}$$

$$H.P. = .00001586\, r\, W\, N. \qquad \textbf{(157.)} \text{ Art. } \textbf{1595.}$$

FORMULAS USED IN STEAM BOILERS.

STRENGTH OF BOILER SHELLS AND TUBES.

Let p = the pressure of steam in pounds per square inch;
t = thickness of plate in shell or tube;
d = diameter of shell or tube;
S_1 = ultimate tensile strength of material;
f = factor of safety;
y = efficiency of joint;
l = length of a tube subjected to external pressure

For tubes without riveted joints,

$$t = \frac{f p d}{2 S_1}. \qquad \textbf{(164.)} \quad \text{Art. } \textbf{1730.}$$

$$p = \frac{2 t S_1}{f d}. \qquad \textbf{(166.)} \quad \text{Art. } \textbf{1731.}$$

For tubes or shells with riveted joints,

$$t = \frac{f p d}{2 S_1 y}, \qquad \textbf{(165.)}$$

$$p = \frac{2 t S_1 y}{f d}. \qquad \textbf{(167.)} \quad \text{Art. } \textbf{1731.}$$

For tubes subjected to external pressure,

$$p = 1{,}600{,}000 \frac{t^2}{l d} \qquad \textbf{(168.)} \quad \text{Art. } \textbf{1732.}$$

RIVETED JOINTS.

Let d = diameter of rivet hole in inches;
h = pitch of rivets in inches;
t = thickness of plate in inches.

$$h = \left(1 + \frac{7 d}{8 t}\right) d. \qquad \textbf{(169.)} \quad \text{Art. } \textbf{1742.}$$

For double-riveted lap joints or single butt joints, formula **169** becomes

$$h = \left(1 + \frac{7 d}{4 t}\right) d. \qquad \textbf{(170.)} \quad \text{Art. } \textbf{1742.}$$

For double-riveted double butt joints, formula **170** becomes

$$h = \left(1 + \frac{7\,d}{2\,t}\right) d. \qquad \textbf{(171.)} \quad \text{Art. } \textbf{1742.}$$

The d in the above formulas refers to the diameter of the rivet hole, or of the rivet after being headed.

The diameter of the cold rivet is $\frac{1}{16}''$ less.

For single and double-riveted lap joints and butt joints, with one cover plate, the diameter of the rivet is

$$d = t + \tfrac{3}{8}'' \qquad \textbf{(172.)} \quad \text{Art. } \textbf{1742.}$$

for values of t between $\frac{1}{4}''$ and $\frac{3}{4}''$. For butt joints with double cover plates, the diameter of rivet may be taken about $1\frac{1}{4}$ times the thickness of the plate.

FLAT SURFACES.

The strength of a circular plate supported at the edges, as, for example, the head of a boiler shell, is given by the following formula:

$$S = \frac{2\,r^2\,p}{3\,t^2}, \qquad \textbf{(174.)} \quad \text{Art. } \textbf{1751.}$$

$$t = r\sqrt{\frac{2\,p}{3\,S}}, \qquad \textbf{(175.)} \quad \text{Art. } \textbf{1751.}$$

where S represents the safe tensile strength of the material, r the radius of the plate in inches, t the thickness of the plate in inches, and p the pressure in lb. per sq. in.

The following formula gives the strength of a square unstayed flat plate fixed along the edge (the length of side is a):

$$\left.\begin{aligned} S &= \frac{a^2\,p}{4\,t^2}, \\ \text{or } t &= \frac{a}{2}\sqrt{\frac{p}{S}}. \end{aligned}\right\} \qquad \textbf{(176.)} \quad \text{Art. } \textbf{1752.}$$

BOILER STAYS.

Let A = area in sq. in. of plate supported by one stay;

d = smallest diameter of stay in inches;

p = pressure of steam in pounds per sq. in.;

T = safe stress per sq. in. allowed on stay;
a = pitch for stays arranged at equal distances apart, both horizontally and vertically.

$$\left.\begin{array}{l} \frac{1}{4}\pi d^2 T = A p, \\ \text{or } d = 1.13\sqrt{\frac{A p}{T}}. \end{array}\right\} \qquad \textbf{(177.)} \text{ Art. } \textbf{1756.}$$

$$d = 1.13\, a\sqrt{\frac{p}{T}}, \qquad \textbf{(178.)} \text{ Art. } \textbf{1756.}$$

$$\text{and } a = .885\, d\sqrt{\frac{T}{p}}. \qquad \textbf{(179.)} \text{ Art. } \textbf{1756.}$$

The values of T used in formulas **177, 178,** and **179** may be taken as follows:

For copper screw staybolts..... $T = 4{,}000$

Iron Stays.—For screw stays, and for other stays not exceeding $1\frac{1}{4}$ inches in diameter, and for all stays which are welded..... $T = 6{,}000$

For unwelded stays above $1\frac{1}{2}$ inches effective diameter..... $T = 7{,}500$

Steel Stays.—For screw stays, and for other stays not exceeding $1\frac{1}{2}$ inches effective diameter..... $T = 8{,}000$

For stays above $1\frac{1}{2}$ inches effective diameter..... $T = 9{,}000$

No steel stays are to be welded.

The size of a diagonal stay may be calculated from formula **178** by substituting $\frac{p}{\cos B}$ for p, B being the angle between the stay and the shell. Formula **178** thus becomes

$$d = 1.13\sqrt{\frac{A p}{T \cos B}}. \qquad \textbf{(180.)} \text{ Art. } \textbf{1758.}$$

For the strength of a girder stay:

Let l = length of girder in inches;
h = distance between adjacent girders in inches;
p = steam pressure in lb. per sq. in.;
b = breadth of girder in inches = $\frac{1}{4}\, d$;
d = depth of girder in inches.

$$d = \sqrt[3]{\frac{p\,h\,l^2}{6{,}000}}. \qquad \textbf{(181.)} \quad \text{Art. } \textbf{1761.}$$

$$p = \frac{6{,}000\,d^3}{h\,l^2}. \qquad \textbf{(182.)} \quad \text{Art. } \textbf{1761.}$$

For the safe pressure allowable on a flat surface, strengthened by screw staybolts:

Let p = pressure in lb. per sq. in.;
h = pitch of staybolts;
t = thickness of plate;
k = a constant.

$$\text{Then, } p = k\frac{t^2}{h^2}, \qquad \textbf{(183.)}$$

$$\text{and } t = h\sqrt{\frac{p}{k}}. \qquad \textbf{(184.)} \quad \text{Art. } \textbf{1763.}$$

The constant k has the following values:

For iron plates and iron bolts ..... $k = 24{,}000$
For low steel plates and iron bolts ..... $k = 25{,}000$
For low steel plates and low steel bolts ..... $k = 28{,}000$
For iron plates and iron bolts with nuts ..... $k = 40{,}000$
For copper plates and iron bolts ..... $k = 14{,}500$

SAFETY VALVES.

For a dead-weight safety valve:

Let A = area of opening in valve seat in sq. in.;
p = pressure at which the valve is to blow off (in lb. per sq. in.);
W = dead weight in pounds (including weight of valve).

$$W = A\,p. \qquad \text{Art. } \textbf{1774.}$$

For a weighted-lever safety valve:

Let $d = FB$ = distance from fulcrum to weight in inches;
$c = FG$ = distance from fulcrum to center of gravity of lever in inches;

a = FV = distance from fulcrum to center of valve in inches;
A = area of orifice beneath bottom of valve in inches;
W = weight of weight P in pounds;
W_1 = weight of valve and stem in pounds;
W_2 = weight of lever in pounds;
P = blow-off pressure in pounds per sq. in.;
$W_3 = W_1 + W_2$ = combined weight of valve and lever;
b = distance from fulcrum of center of gravity of W_3.

$$W = \frac{a(pA - W_1) - W_2 c}{d}, \qquad \textbf{(185.)} \text{ Art. } \textbf{1776.}$$

and $$d = \frac{a(pA - W_1) - W_2 c}{W}. \qquad \textbf{(186.)} \text{ Art. } \textbf{1776.}$$

Also $$d = \frac{pAa - W_3 b}{W}. \qquad \textbf{(187.)} \text{ Art. } \textbf{1778.}$$

To find the area of a safety valve:

Let a = area of the valve opening in square inches;
w = maximum weight of steam generated in pounds per hour;
p = gauge pressure in pounds per square inch.

$$a = \frac{.5\,w}{p + 10}. \qquad \textbf{(188.)} \text{ Art. } \textbf{1781.}$$

DIMENSIONS OF CHIMNEYS.

Let H = height of chimney in feet;
$H.P.$ = horsepower of boiler or boilers;
A = actual area of chimney in square feet;
E = effective area of chimney in square feet;
S = side of square chimney in inches;
d = diameter of round chimney in inches;
p = pressure of draft in inches of water;
T_a = absolute temperature of outside air;
T_e = absolute temperature of gas in chimney.

$$p = H\left(\frac{7.6}{T_a} - \frac{7.9}{T_e}\right). \qquad \textbf{(192.)} \text{ Art. } \textbf{1825.}$$

$$H = \frac{p}{\left(\frac{7.6}{T_a} - \frac{7.9}{T_e}\right)}. \qquad \textbf{(193.)} \text{ Art. } \textbf{1826.}$$

$$E = \frac{.3\ H.\ P.}{\sqrt{H}} = A - .6\sqrt{A}. \qquad \textbf{(194.)}\ \text{Art. } \textbf{1827.}$$

$$H.\ P. = 3.33\ E\sqrt{H}. \qquad \textbf{(195.)}\ \text{Art. } \textbf{1827.}$$

$$S = 12\sqrt{E} + 4. \qquad \textbf{(196.)}\ \text{Art. } \textbf{1827.}$$

$$d = 13.54\sqrt{E} + 4. \qquad \textbf{(197.)}\ \text{Art. } \textbf{1827.}$$

The *maximum* rates of combustion attainable under natural draft are given by the following formulas:

Let F = weight in pounds of coal per hour per square foot of grate area;
H = height in feet of chimney or stack.

Then, for anthracite coal burned under the most favorable conditions,

$$F = 2\sqrt{H} - 1, \qquad \textbf{(198.)}\ \text{Art. } \textbf{1829.}$$

and under ordinary conditions,

$$F = 1.5\sqrt{H} - 1. \qquad \textbf{(199.)}\ \text{Art. } \textbf{1829.}$$

For best semianthracite and bituminous coals,

$$F = 2.25\sqrt{H}, \qquad \textbf{(200.)}\ \text{Art. } \textbf{1829.}$$

and for less valuable soft coals,

$$F = 3\sqrt{H}. \qquad \textbf{(201.)}\ \text{Art. } \textbf{1829.}$$

GRATE AREA.

Let G = area of grate in sq. ft.;
F = rate of combustion in lb. per sq. ft. of grate surface per hour;
W = weight of steam generated per hour by boiler or boilers;
e = evaporation in pounds of water per pound of coal per hour.

Then, $$G = \frac{W}{Fe}. \qquad \textbf{(202.)}\ \text{Art. } \textbf{1830.}$$

In using this formula care must be taken that W and e are taken at the same pressure and temperature.

TO REDUCE THE ACTUAL EVAPORATION TO AN EQUIVALENT EVAPORATION.

Let W = the actual evaporation;
H = total heat of steam above 32° at pressure of actual evaporation;
t = observed temperature of feed-water;
W' = equivalent evaporation from and at 212°.

Then, $W(H - t + 32) = 966.1\ W'$,

or $W' = \dfrac{W(H - t + 32)}{966.1}$. **(203.)** Art. **1838.**

The quantity $\left(\dfrac{H - t + 32}{966.1}\right)$ which changes actual evaporation to equivalent evaporation from and at 212° is called the **factor of evaporation.**

TO FIND THE QUALITY OF STEAM BY MEANS OF THE BARREL CALORIMETER.

Let W = original weight of water in barrel;
w = weight of steam and water condensed in the cold water;
l = latent heat of steam at observed pressure;
t = temperature of steam at observed pressure;
t_1 = initial temperature of water;
t_2 = final temperature of water;
x = the portion of w which is dry steam;
Q = quality of steam; that is, the percentage of the mixture evaporated, which is pure dry steam.

$$Q = \frac{x}{w} = \frac{1}{l}\left[\frac{W}{w}(t_2 - t_1) - (t - t_2)\right]. \quad \textbf{(204.)} \text{ Art. } \textbf{1853.}$$

TO FIND THE QUALITY OF STEAM BY MEANS OF THE SEPARATOR CALORIMETER.

Let W = weight of dry steam discharged from the instrument;
w = weight of water drawn from separator;
R = water condensed by radiation;
Q = quality of steam.

$$Q = \frac{W + R}{W + R + w}. \quad \textbf{(205.)} \text{ Art. } \textbf{1855.}$$

FORMULAS USED IN MACHINE DESIGN.

PROPORTIONS OF BOLT HEADS, NUTS, AND WASHERS.

d = diameter of bolt.

Diameter of nut or head across flats:

$$\left.\begin{aligned} D &= 1\tfrac{1}{2}\,d + \tfrac{1}{8}'' \text{ for rough work.} \\ D &= 1\tfrac{1}{2}\,d + \tfrac{1}{16}'' \text{ for finished work.} \end{aligned}\right\} \quad \textbf{(220.)}\ \text{Art. } \textbf{1937.}$$

Height of nut:

$$\left.\begin{aligned} h &= d \text{ for rough work.} \\ h &= d - \tfrac{1}{16}'' \text{ for finished work.} \end{aligned}\right\} \quad \textbf{(221.)}\ \text{Art. } \textbf{1937.}$$

Thickness of washer, $t = .15\,d$. **(222.)** Art. **1937.**

Diameter of washer, $D_2 = 1\tfrac{1}{8}\,D_1$. **(223.)** Art. **1937.**

The above proportions for diameters D hold for both hexagonal and square nuts. The diameter across corners D_1 is as follows:

For hexagonal nuts:

$$D_1 = \begin{cases} 1.73\,d + .14'' \text{ for rough nuts.} \\ 1.73\,d + .07'' \text{ for finished nuts.} \end{cases} \quad \textbf{(224.)}\ \text{Art. } \textbf{1937.}$$

For square nuts:

$$D_1 = \begin{cases} 2.12\,d + .18'' \text{ for rough nuts.} \\ 2.12\,d + .09'' \text{ for finished nuts.} \end{cases} \quad \textbf{(225.)}\ \text{Art. } \textbf{1937.}$$

Height of head:

$$\left.\begin{aligned} h' &= \tfrac{2}{4}\,d + \tfrac{1}{16}'' \text{ for rough bolts.} \\ h' &= d - \tfrac{1}{16}'' \text{ for finished bolts.} \end{aligned}\right\} \quad \textbf{(226.)}\ \text{Art. } \textbf{1937.}$$

PROPORTIONS OF KEYS.

Let b = width of key in inches;
t = thickness of key in inches;
l = length of key in inches;
N = number of revolutions per minute;
H = horsepower transmitted by a pulley.

$$\left.\begin{aligned} b &= \frac{d}{7} + \frac{1}{4}''. \\ t &= \frac{d}{12} + \frac{3}{16}''. \end{aligned}\right\} \text{ For driven pulleys.} \quad \textbf{(234.)}\ \text{Art. } \textbf{1965.}$$

$b = .2\,d + .16''.$
$t = .1\,d + .16''.$ } For driving pulleys. **(235.)** Art. **1965.**

When d is less than $1\frac{3}{16}$ in., $b = \frac{d}{3}.$ $t = \frac{d}{5}.$ **(236.)** Art. **1965.**

When the pulley transmits but a small part of the power of a shaft, use the formula $d = 5\sqrt[3]{\frac{H}{N}}$ from which to obtain the value of d to use in formula **236.**

For sliding feather keys,

$$b = \frac{d}{8} + \frac{1}{16}''. \qquad t = \frac{3\,d}{16} + \frac{1}{16}''. \qquad \textbf{(237.)}\ \text{Art. } \textbf{1965.}$$

PROPORTIONS AND STRENGTH OF GEAR-WHEELS.

Let p = pressure acting at pitch line;
b = breadth of tooth;
n = number of revolutions per minute;
H = horsepower transmitted;
D = diameter of gear in inches;
C = the circular pitch;
P = diametral pitch;
S = allowable working stress in tooth;
v = velocity of a point in the pitch circle in feet per minute;
c = thickness of rim.

$$b\,C = 16.8\,\frac{p}{S} \qquad \textbf{(266.)}\ \text{Art. } \textbf{2024.}$$

If v is greater than 100 feet per minute make

$$S = \frac{9{,}600{,}000}{v + 2{,}160}. \qquad \textbf{(267.)}\ \text{Art. } \textbf{2024.}$$

To insure durability make

$$\frac{p\,n}{b} = 28{,}000, \text{ or } b = 4\tfrac{1}{2}\,\frac{H}{D}. \qquad \textbf{(268.)}\ \text{Art. } \textbf{2025.}$$

To calculate the dimensions of the arms:

Let z = number of arms in wheel;
a = width of arm, measured at center of wheel;
x = thickness of arm;
S_1 = allowable safe stress in arm;
R = radius of wheel in inches;
p = pressure between gears at pitch line.

For cross-shaped, T, or I sections:

$$\left.\begin{aligned} a &= \sqrt{\frac{R\,b}{z}}. \\ x &= \tfrac{1}{2}\,C. \end{aligned}\right\} \qquad \textbf{(270.)} \quad \text{Art. } \textbf{2030.}$$

For oval or elliptical sections:

$$\left.\begin{aligned} a &= 1.2\sqrt[3]{\frac{b\,C\,R}{z}}. \\ a &= 1.75\sqrt[3]{\frac{b\,R}{P\,z}}. \end{aligned}\right\} \qquad \textbf{(271.)} \quad \text{Art. } \textbf{2030.}$$

$$x = \frac{a}{2}.$$

For thickness of rim:

$c = \frac{1}{2}\,C$ for large gears and coarse pitch.
$c = .4\,C \times \frac{1}{8}''$ for gears with small pitch.

PROPORTIONS OF BEVEL GEARS.

Denote the mean circular pitch by C_m.

Then, $b\,C_m = 16.8\,\dfrac{p}{S}$. **(266*a*.)** Art. **2033.**

$$c = .4\,C_m + \tfrac{1}{8}'' = \frac{5}{4\,P} + \tfrac{1}{8}''. \quad \textbf{(269}a\textbf{.)} \quad \text{Art. } \textbf{2033.}$$

$$\left.\begin{aligned} a &= \sqrt{\frac{R\,b}{z}}. \\ x &= \tfrac{1}{2}\,C_m. \end{aligned}\right\} \qquad \textbf{(270}a\textbf{.)} \quad \text{Art. } \textbf{2033.}$$

$$\left.\begin{aligned} a &= 1.2\sqrt[3]{\frac{b\,C_m\,R}{z}}. \\ a &= 1.75\sqrt[3]{\frac{b\,R}{P_m\,z}}. \end{aligned}\right\} \qquad \textbf{(271}a\textbf{.)} \quad \text{Art. } \textbf{2033.}$$

RULES AND FORMULAS USED IN PRINCIPLES OF ELECTRICITY AND MAGNETISM.

OHM'S LAW.

The strength of an electric current in any circuit is directly proportional to the electromotive force developed in that circuit and inversely proportional to the resistance of the circuit; i. e., is equal to the quotient arising from dividing the electromotive force by the resistance.

Let E = electromotive force;
C = current;
R = resistance of circuit.

Then, $$C = \frac{E}{R},$$

$$E = CR, \text{ and } R = \frac{E}{C}. \quad \text{Art. } \mathbf{2253.}$$

STRENGTH OF CURRENT DETERMINED BY DECOMPOSITION OF WATER.

By universal agreement, 1 *ampere* is that strength of current which will decompose .00009324 gram or .0014388 grain of water in 1 second.

Rule.—*To find the strength of an electric current in amperes by the decomposition of water, divide the weight of the quantity of water decomposed by the time in seconds required to decompose it; if the mass of water is expressed in grams, divide the quotient by .00009324; but if expressed in grains, divide by .0014388.*

Let W = weight of water decomposed in grams;
w = weight of water decomposed in grains;
t = time in seconds required for decomposition;
C = current in amperes.

Then the strength of the current in amperes is given by the formulas:

$$C = \frac{W}{t \times .00009324}. \qquad \mathbf{(401.)}$$

$$C = \frac{w}{t \times .0014388}. \qquad \textbf{(402.)} \quad \text{Art. } \textbf{2276.}$$

Rule.—*To find the quantity of water which an electric current of a given strength can decompose in a given time, multiply the strength of the current in amperes by the time in seconds during which the current flows; if the quantity of water is to be expressed in grams, multiply the product by .00009324; but if in grains, multiply by .0014388.*

Let q = quantity of water in grams;
q' = quantity of water in grains;
t = time in seconds of current flow;
C = current in amperes.

Then the quantity of water which can be decomposed by a current of C amperes in t seconds is given by the formulas:

$$q = .00009324\ C\ t. \qquad \textbf{(403.)}$$

$$q' = .0014388\ C\ t. \qquad \textbf{(404.)} \quad \text{Art. } \textbf{2277.}$$

RELATION OF AMPERE AND COULOMB.

Let Q = quantity of electricity in *coulombs;*
C = strength of current in amperes;
t = time in seconds.

Then, $Q = C\ t.$ **(405.)** Art. **2281.**

By transposition, $C = \frac{Q}{t}$ and $t = \frac{Q}{C}$.

RESISTANCE OF CONDUCTORS.

The resistance of a given conductor is always constant at the same temperature, irrespective of the strength of current flowing through it or the electromotive force of the current. Art. **2291.**

The resistance of a given conductor increases as the length of the conductor increases; that is, the resistance of a conductor is directly proportional to its length. Art. **2293.**

Let r_1 = the original resistance;
r_2 = the required or changed resistance;
l_1 = the original length;
l_2 = the changed length.

Then, $r_1 : r_2 = l_1 : l_2$, or $r_2 = \frac{r_1 l_2}{l_1}$. **(406.)**
Art. **2292.**

The resistance of a conductor varies inversely as its sectional area.

Let r_1 = the original resistance;
r_2 = the required resistance;
a_1 = the original sectional area;
a_2 = the changed sectional area.

Then, $r_1 : r_2 :: a_2 : a_1$, or $r_2 = \frac{r_1 a_1}{a_2}$. **(407.)**
Art. **2296.**

The resistance of a conductor of circular cross-section is inversely proportional to the square of its diameter.

Let r_1 = the original resistance;
r_2 = the required resistance;
D = the original diameter;
d = the changed diameter.

Then, $r_1 : r_2 :: d^2 : D^2$, or $r_2 = \frac{r_1 D^2}{d^2}$. **(408.)**
Art. **2298.**

OHM'S LAW APPLIED TO CLOSED CIRCUITS.

Ohm's law expresses the relation between the three fundamental units of resistance, electrical pressure, and current. If any two of these values are known, the third is found by solving the simple equation of their relation. Before applying this law, however, the following four facts should be carefully noted:

I.—*The strength of a current* (C) *is the same in all parts of a closed circuit, except in the case of divided circuits.*

II.—*In the case of a divided circuit, the sum of the currents in the separate branches is always equal to the current in the main or undivided circuit.*

III.—*The resistance* (R) *is the total resistance of the circuit, that is, the sum of the resistances of the internal circuit and of the external circuit, or its equivalent.*

IV.—*The electromotive force* (E) *in a closed circuit is the total generated difference of potential in that circuit.*

The law may now be stated by the following rules and formulas:

Rule I.—*The strength in amperes of a current* (C) *flowing in a closed circuit, when the electromotive force* (E) *and the total resistance* (R) *are known, is found by dividing the electromotive force in volts by the total resistance in ohms; that is,*

$$\text{Current} = \frac{\text{electromotive force}}{\text{resistance}}, \text{ or } C = \frac{E}{R}. \qquad \textbf{(409.)}$$

Rule II.—*The total resistance* (R) *in ohms of a closed circuit, when the electromotive force* (E) *and the current* (C) *are known, is found by dividing the electromotive force in volts by the current in amperes; that is,*

$$\text{Resistance} = \frac{\text{electromotive force}}{\text{current}}, \text{ or } R = \frac{E}{C}. \qquad \textbf{(410.)}$$

Rule III.—*The total electromotive force* (E) *in volts developed in a closed circuit, when the current* (C) *and the total resistance* (R) *are known, is found by multiplying the current in amperes by the total resistance in ohms; that is,*

$$\text{Electromotive force} = \text{current} \times \text{resistance},$$

or $$E = CR. \qquad \textbf{(411.)} \quad \text{Art. } \textbf{2310.}$$

TOTAL AND AVAILABLE E. M. F. IN VOLTAIC CELL.

Let E = total generated E. M. F.;
E' = available E. M. F.;
C = current flowing when the circuit is closed;
r_i = internal resistance of the cell;
r_e = an external resistance.

Then $$E' = C r_e,$$

$$E = C(r_i + r_e),$$

and the drop or loss of potential is

$$E - E' = Cr_4. \qquad \text{Art. } \mathbf{2318.}$$

OHM'S LAW APPLIED TO DERIVED CIRCUITS.

Let r_1, r_2, r_3, etc. = the separate resistances of the several branches, respectively;

c_1, c_2, c_3, etc. = the currents in each branch, respectively;

C = the current in the main circuit;

R'' = joint resistance of two branches;

R''' = joint resistance of three branches.

For a derived circuit of two branches:

$$c_1 + c_2 = C,$$

$$c_1 : c_2 = \frac{1}{r_1} : \frac{1}{r_2}, \text{ or } \frac{c_1}{c_2} = \frac{r_2}{r_1}. \qquad \text{Art. } \mathbf{2323.}$$

$$\text{Joint conductivity} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2}.$$

$$R'' = \frac{r_1 r_2}{r_1 + r_2}. \qquad \mathbf{(412.)} \quad \text{Art. } \mathbf{2325.}$$

The joint resistance of two conductors in parallel is equal to the product of their separate resistances divided by the sum of their separate resistances.

For a derived circuit of three branches:

$$c_1 + c_2 + c_3 = C,$$

$$c_1 : c_2 : c_3 = \frac{1}{r_1} : \frac{1}{r_2} : \frac{1}{r_3}.$$

$$\text{Joint conductivity} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3}.$$

$$R''' = 1 \div \frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3} = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}. \qquad \mathbf{(413.)}$$

Art. **2326.**

The joint resistance of three or more conductors in parallel is equal to the reciprocal of their joint conductivity.

ELECTRICAL WORK.

Rule.—*To find the amount of electrical work accomplished in joules during a given time, multiply the quantity of electricity in coulombs which has passed in the circuit during that time by the loss or drop of potential.* Art. **2334.**

This rule may be expressed by the following formulas, for the three cases occurring in practical work:

Let J = electrical work in joules;
C = current in amperes;
t = time in seconds during which current flows;
E = E. M. F. of circuit;
R = resistance of circuit.

Then, $J = C^2 R t.$ **(414.)** Art. **2335.**

$J = C E t.$ **(415.)** Art. **2336.**

$J = \frac{E^2 t}{R}.$ **(416.)** Art. **2337.**

If F. P. denotes the electrical work in foot-pounds,

F. P. $= .7373 J.$ **(417.)** Art. **2338.**

JOULE'S LAW.

Let H = B. T. U. developed in the circuit;
C = current in amperes;
R = resistance in ohms;
t = time in seconds.

Then, $H = .0009477\ C^2 R t.$ **(418.)** Art. **2343.**

ELECTRICAL POWER.

In every electrical circuit the power in watts is equal to the product obtained by multiplying the current in amperes by the electromotive force in volts.

In every electrical circuit the power in watts is equal to the product obtained by multiplying the square of the current strength in amperes by the resistance of the circuit in ohms.

In every electrical circuit the power in watts is equal to the quotient obtained by dividing the square of the electromotive force in volts by the resistance in ohms.

Let W = total watts exerted in the circuit;
E = volts of electromotive force;
C = current in amperes;
R = resistance in ohms.

Then, $W = E\,C.$ **(419.)** Art. **2350.**

$W = C^2 R.$ **(420.)** Art. **2352.**

$$W = \frac{E^2}{R}.$$ **(421.)** Art. **2353.**

Let H. P. = horsepower.

Then, $$\text{H. P.} = \frac{W}{746}.$$ **(423.)** Art. **2356.**

$$\text{H. P.} = \frac{EC}{746}.$$ **(424.)** Art. **2356.**

$$\text{H. P.} = \frac{C^2 R}{746}.$$ **(425.)** Art. **2356.**

$$\text{H. P.} = \frac{E^2}{746\,R}.$$ **(426.)** Art. **2356.**

MAGNETIC DENSITY AND MAGNETIC LINES PER UNIT POLE.

Let N = total number of lines of force;
A = sectional area of magnetic circuit in square inches;
B = magnetic density per square inch.

Then, $$\mathsf{B} = \frac{N}{A}.$$ **(427.)** Art. **2377.**

$N = A\,\mathsf{B}.$ **(428.)** Art. **2378.**

Every magnet pole of unit strength has 12.5664 ($= 4\pi$) magnetic lines. Art. **2380.**

Unit density of magnetism is a density of 6.452 lines of force per square inch. Art. **2381.**

TO DETERMINE THE POLARITY OF A SOLENOID.

Rule.—*In looking at the end of the helix, if it is so wound that the current flows around in the direction of the hands of a watch, that end will be a south pole; if in the other direction, it will be a north pole.* Art. **2390.**

MAGNETOMOTIVE FORCE.

Let a = current in amperes;
t = number of turns;
$a\text{-}t$ = ampere-turns;
l = length of magnetic circuit;
H = intensity of magnetomotive force.

Then, magnetomotive force
$= 1.257 \times a\text{-}t$,
$= 3.192 \times a\text{-}t$ for English units. **(429.)** Art. **2391.**

$$\mathsf{H} = \frac{3.192 \times a\text{-}t}{l}. \qquad \textbf{(430.)} \quad \text{Art. } \textbf{2392.}$$

CALCULATION OF THE MAGNETIC CIRCUIT.

Let l = length of magnetic circuit in inches;
A = sectional area of circuit in square inches,
μ = permeability;
N = induction;
$a\text{-}t$ = ampere-turns;
R = reluctance of magnetic circuit;
R_1, R_2, etc. = reluctances of various substances in compound circuit.

Then, $$\mathsf{R} = \frac{l}{A\,\mu}. \qquad \textbf{(431.)} \quad \text{Art. } \textbf{2409.}$$

$$N = \frac{3.192 \times a\text{-}t}{\mathsf{R}}. \qquad \textbf{(432.)} \quad \text{Art. } \textbf{2409.}$$

$$a\text{-}t = \frac{N}{3.192} \times \mathsf{R}. \quad \text{Art. } \textbf{2409.}$$

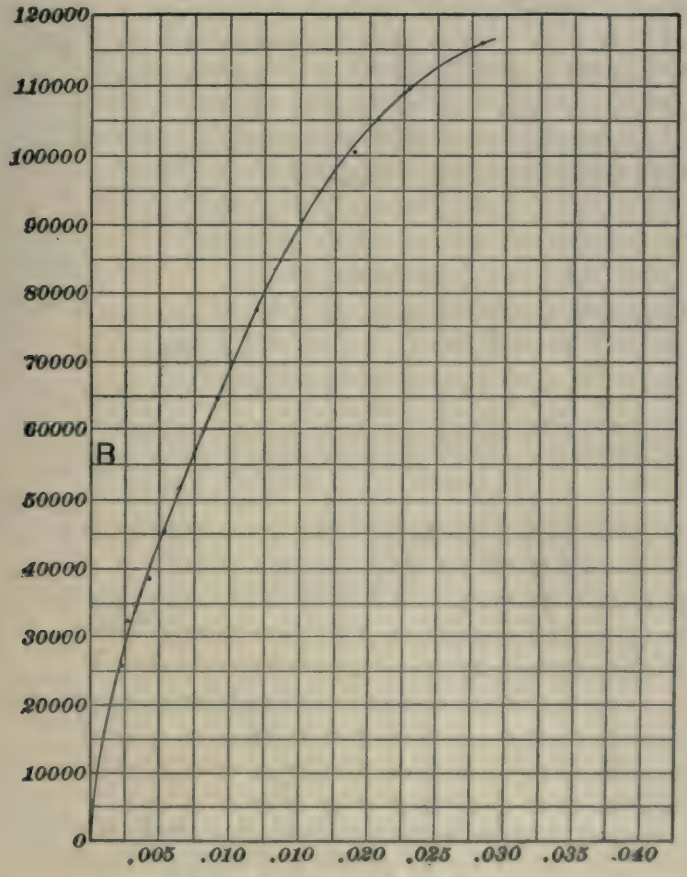
$$a\text{-}t = \frac{N}{3.192} \times (\mathsf{R}_1 + \mathsf{R}_2 + \text{etc.}). \qquad \textbf{(433.)} \quad \text{Art. } \textbf{2409.}$$

POWER EXPENDED BY HYSTERESIS.

Let w = power in watts expended per cubic inch per cycle;
v = volume in cubic inches;
n = cycles per second;
W = total watts expended

Then, $W = w v n.$ **(434.)** Art. **2414.**

Rule.—*To find the power expended by hysteresis in sheet iron at a given stage of magnetization, multiply the watts expended at that stage, as given by the curve, by the number*



of cubic inches of iron in the magnet and the number of cycles per second.

Various corresponding readings of **B** and w are plotted on a sheet of cross-section paper, and the various points are connected by a curved line. The ordinates represent the different densities **B**, and the abscissas the corresponding number of watts expended in one cubic inch of iron for one cycle per second. By referring to the curve, all the intermediate values of **B** and the corresponding watts expended can be determined.

MAGNETIC LEAKAGE.

If the total number of lines of force produced by the magnetizing coils and the useful number are known, the magnetic leakage can be expressed by a per cent. of the total number produced. Thus,

Let l = total number of lines of force;
l_u = useful number of lines of force;
l_s = stray lines of force;
p = per cent. leakage.

Then, $l_s = l - l_u$. **(435.)** Art. **2418.**

The percentage of leakage is found from the formula

$$p = \frac{100\, l_s}{l}. \qquad \textbf{(436.)} \quad \text{Art. } \textbf{2419.}$$

To find the total number of lines of force when the percentage of leakage and the number of useful lines of force are known, use the following formula:

$$l = \frac{100\, l_u}{100 - p}. \qquad \textbf{(437.)} \quad \text{Art. } \textbf{2420.}$$

CALCULATION FOR LIFTING MAGNET.

Let A = total area of contact surface;
B = density in lines of force per square inch;
P = total tractive force in pounds;
p = tractive force in pounds per square inch;
N = induction, or total number of lines of force.

Then, $P = \frac{\mathbf{B}^2 A}{72,134,000}$. **(438.)** Art. **2424.**

That is, *the tractive force of a magnet increases directly as the total area of the surface in contact with the armature, and as the square of the density of the lines of force in the magnetic circuit where it passes across that surface.*

$$\mathbf{B} = 8,493 \sqrt{\frac{P}{A}}. \qquad \textbf{(439.)} \text{ Art. } \textbf{2426.}$$

$$N = 72,134,000 \frac{P}{\mathbf{B}}. \qquad \textbf{(440.)} \text{ Art. } \textbf{2427.}$$

$$p = \frac{N^2}{72,134,000 A^2}. \qquad \textbf{(441.)} \text{ Art. } \textbf{2428.}$$

$$p = \frac{\mathbf{B}^2}{72,134,000}. \qquad \textbf{(442.)} \text{ Art. } \textbf{2429.}$$

$$A = \frac{N^2}{72,134,000 P}. \qquad \textbf{(443.)} \text{ Art. } \textbf{2430.}$$

To find the number of ampere-turns required to energize a magnet for a given traction when the permeability of the iron or steel used is known and the dimensions of the armature and magnet have been established:

Let P = tractive force of one contact surface; then, $2P$ is the total tractive force of the magnet;

l_1 and l_2 = the lengths of the magnetic circuit in magnet and armature, respectively;

A_1 and A_2 = sectional areas of magnetic circuit in magnet and armature, respectively;

μ_1 and μ_2 = permeabilities of the iron or steel used in the magnet and armature, respectively;

$\mathbf{B}$ = magnetic density at contact surface.

Then, the ampere-turns

$$a\text{-}t = 22,598,370 \times \frac{P}{\mathbf{B}} \times \left(\frac{l_1}{A_1 \times \mu_1} + \frac{l_2}{A_2 \times \mu_2} \right). \qquad \textbf{(444.)}$$

Art. **2431.**

Rule.—*In the case of an electromagnet intended to develop a given tractive power, the ampere-turns are equal to the*

tractive force of one contact surface multiplied by the reluctance of the circuit and by 22,598,370, and divided by the magnetic density at the contact surface.

To find the ampere-turns required to energize a magnet for a given tractive force when the armature and magnet are made of the same quality of iron or steel and the sectional area of the magnetic circuit is the same in the armature, magnet, and contact surfaces:

Let l = total length of magnetic circuit in inches;
P = tractive force at one surface;
μ = permeability of iron or steel used;
A = cross-sectional area of magnetic circuit;
N = total number of lines of force in the magnetic circuit.

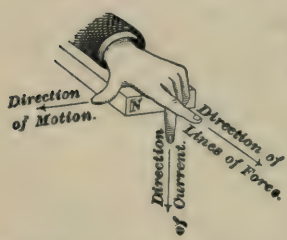
Then, $a\text{-}t = 2{,}661 \times \frac{l}{\mu} \times \sqrt{\frac{P}{A}}$. **(445.)** Art. **2432.**

As showing the relation between formulas **439** and **445**, the latter may be written:

$$a\text{-}t = \frac{8{,}493\sqrt{\frac{P}{A}}}{3.192} \times \frac{l}{\mu} = \frac{\mathsf{B}}{3.192} \times \frac{l}{\mu}. \quad \textbf{(446.)} \text{ Art. } \textbf{2432.}$$

ELECTROMAGNETIC INDUCTION.

To determine the direction of motion of a conductor carrying a current of electricity when placed in a magnetic field:



Rule.—*Place thumb, forefinger, and middle finger of the left hand each at right angles to the other two; if the forefinger shows the direction of the lines of force and the middle finger shows the direction of the current, then the thumb will show the direction of motion given to the conductor.* Art. **2439.**

To determine the direction of induced currents:

Rule.—*Place thumb, forefinger, and middle finger of the right hand each at right angles to the other two; if the forefinger shows the direction of the lines of force and the thumb shows the direction of motion of conductor, the middle finger will show the direction of the induced current.* Art. **2442.**

To determine the direction of induced currents in a closed coil:

Rule.—*If the effect of the movement is to diminish the number of lines of force that pass through the coil, the current will flow around in the conductor in the direction of the hands of a watch as viewed by a person looking along the magnetic field in the direction of the lines of force; but if the effect is to increase the number of lines of force that pass through the coil, the current will flow around in the opposite direction.* Art. **2446.**

DETERMINATION OF E. M. F.

One absolute unit of potential is generated in a conductor when it is cutting lines of force at the rate of *one line of force per second.*

By definition, *one volt* is equal to 100,000,000 (10^8) *absolute units;* consequently, in order to generate an electromotive force of one volt, the *rate of cutting* must be 10^8 lines of force per second. This can also be expressed algebraically.

Let E = the electromotive force in volts;

N = the total number of lines of force cut by the conductor;

t = time in seconds taken to cut the lines of force.

Then, $$E = \frac{N}{10^8 \times t}. \qquad (447.)$$ Art. **2449.**

That is, *the electromotive force in volts generated in a moving conductor is found by dividing the total number of lines of force cut by the conductor by the time taken and by 100,000,000.*

RULES AND FORMULAS USED IN ELECTRICAL MEASUREMENTS.

THE GALVANOMETER.

Let f = force in dynes exerted on magnet pole;
r = radius to which conductor is bent, centimeters;
t = number of turns in the coil;
A = current expressed in C. G. S. units;
H = horizontal component of the earth's field;
$m°$ = angle between magnet and direction of earth's field;
C = current in amperes;
Q = quantity of electricity in coulombs passing through the coils of the galvanometer;
d = deflection in divisions on scale (reflecting galvanometer);
K = galvanometer constant, which is determined for each individual instrument.

Then, $f = \frac{2\pi A t}{r}$. **(448.)** Art. **2461.**

For a tangent galvanometer,

$f = H \tan m°$. **(449.)** Art. **2463.**

$C = K \tan m°$. **(450.)** Art. **2468.**

For a sine galvanometer,

$f = H \sin m°$. **(451.)** Art. **2473.**

In the ballistic galvanometer,

$Q = K \sin \frac{m°}{2}$. **(452.)** Art. **2478.**

$Q = K d$. **(453.)** Art. **2478.**

GALVANOMETER SHUNTS.

Let C_g = current in galvanometer;
C_s = current in shunt;
C = total current;

R_g = resistance in galvanometer;
R_s = resistance in shunt;
n = ratio $\frac{R_g}{R_s}$.

Then, $C = C_g + C_s$.

$$C_g = \frac{C}{n+1}. \qquad (454.)\ \text{Art. } \mathbf{2488.}$$

MEASUREMENT OF CURRENT.

By decomposition of water:

Let w_1 = the original weight of apparatus;
w_2 = the weight after the current has passed;
t = time in seconds during which the current flows;
C = strength of current in amperes.

Then, if the weights are taken in *grams*,

$$C = \frac{w_1 - w_2}{.00009324\,t}. \qquad (455.)\ \text{Art. } \mathbf{2495.}$$

If the weights are in *grains*,

$$C = \frac{w_1 - w_2}{.0014388\,t}. \qquad (456.)\ \text{Art. } \mathbf{2495.}$$

Rule.—*To determine the strength of a current by decomposition of water, subtract from the original weight of the apparatus its weight after the current has passed through; divide this result, expressed in grams or grains, by the length of time the current was passing, in seconds, multiplied by the number of grams or grains of water which can be decomposed by 1 ampere in 1 second.*

By deposition of copper:

Let w_1 = the original weight of gain plate;
w_2 = the weight after the current has passed;
t = time in seconds during which the current flows;
C = strength of current in amperes.

Then, if the weights are in grams,

$$C = \frac{w_2 - w_1}{.0003286\,t}. \qquad (457.)\ \text{Art. } \mathbf{2501.}$$

If the weights are in grains,

$$C = \frac{w_2 - w_1}{.005068\, t}. \qquad \textbf{(458.)} \quad \text{Art. } \textbf{2501.}$$

Rule.—*In order to determine the strength of current by measurement of copper deposited, subtract the original weight of the gain plate, in grams, from the weight as found after the experiment; divide this result by the length of time the current was flowing in seconds multiplied by the number of grams of copper which can be deposited by 1 ampere in 1 second.*

EFFECT OF TEMPERATURE ON RESISTANCE.

The formulas representing the effects of the *increase* of temperature upon the conductivity of a substance may be written as follows:

Let r_1 = original resistance;
r_2 = resistance after rise of temperature;
a = temperature coefficient for each degree Centigrade;
b = temperature coefficient for each degree Fahrenheit:
$C°$ = rise of temperature degrees Centigrade (see table of Centigrade and Fahrenheit Degrees);
$F°$ = rise of temperature degrees Fahrenheit.

Then, $r_2 = r_1 (1 + a\, C°)$, **(460.)** Art. **2517.**

and $r_2 = r_1 (1 + b\, F°)$. **(461.)** Art. **2517.**

The formulas for the decrease of resistance with *decrease* of temperature are as follows:

Let r_1 = original resistance;
r_2 = resistance after lowering of temperature;
a = temperature coefficient for each degree Centigrade;
b = temperature coefficient for each degree Fahrenheit;
$C°$ = fall of temperature degrees Centigrade;
$F°$ = fall of temperature degrees Fahrenheit.

Then, $r_2 = \frac{r_1}{1 + a C^\circ}$. **(462.)** Art. **2518.**

$$r_2 = \frac{r_1}{1 + b F^\circ}. \qquad \textbf{(463.)} \text{ Art. } \textbf{2518.}$$

The values of a and b used in formulas **460** to **463** may be found in the table of Temperature Coefficients for Various Metals.

INSULATION RESISTANCE OF A LINE.

First method:

Let R = known resistance;
I = insulation resistance;
d = the angle of deflection when R is in circuit;
d_1 = the angle of deflection when I is in circuit.

Then, $I = \frac{R \tan d}{\tan d_1}$. **(464.)** Art. **2522.**

That is, *the insulation resistance of a line is equal to a given resistance multiplied by the quotient obtained by dividing the tangent of the angle of galvanometer deflection when that resistance is in circuit by the tangent of the angle of deflection when the circuit is through the line.*

Second method:

Let I = insulation resistance;
R = known adjustable resistance;
x = number of cells in battery;
d = deflection of galvanometer connected in series with one cell and known resistance;
d_1 = deflection of galvanometer when connected in series with all the cells and the insulation resistance.

Then, $I = \frac{R x d}{d_1}$. **(465.)** Art. **2530.**

Resistance measured by voltmeter:

Let r = resistance of voltmeter;
R = resistance to be measured;
d = deflection of voltmeter with R *not* in circuit;
d_1 = deflection of voltmeter with R in circuit.

Then, $R = r\left(\frac{d}{d_1} - 1\right)$. **(466.)** Art. **2546.**

FORMULAS USED IN BATTERIES.

ELECTROCHEMICAL CALCULATIONS.

z = electrochemical equivalent of any substance;
W = weight of substance liberated by electrochemical action in grams;
h = heat evolved per gram of substance in calories;
H = total heat evolved in calories;
E = electromotive force in volts;
Q = quantity in coulombs;
J = work in joules.

Then, $W = Qz$. **(467.)** Art. **2583.**

$H = Wh = Qzh$ **(468.)** Art. **2584.**

$J = 4.2\,Qzh$. **(469.)** Art. **2585.**

$J = EQ$. **(470.)** Art. **2586.**

$E = 4.2\,hz$. **(471.)** Art. **2587.**

E. M. F. OF THE LATIMER-CLARK CELL.

Let t = temperature in degrees Centigrade at which measurement is made;
E = electromotive force of cell.

Then, $E = 1.434\,[1 - .00077\,(t - 15)]$ volts. **(472.)** Art. **2693.**

RULES AND FORMULAS USED IN APPLIED ELECTRICITY.

ELECTROMETALLURGY.

I.—*The quantity of a metal liberated at a negative electrode in a given time is proportional to the strength of the current passing through the electrolyte.*

II.—*The quantity of a metal liberated at a negative electrode per second is equal to the strength of the current in amperes multiplied by the electrochemical equivalent.*

The electrochemical equivalents of metals may be obtained from column 6, table of Principal Chemical Elements.

GENERATION OF E. M. F. IN THE DYNAMO.

Let $\mathbf{B}$ = the density of the magnetic field, lines per square inch;

N = total number of lines of force cut by conductor;

L = the length of the conductor, inches;

M = velocity of conductor at right angles to lines of force, inches per second;

E = E. M. F. generated.

Then, $E = \dfrac{\mathbf{B}\,L\,M}{10^8}$. **(473.)** Art. **3018.**

Let p° = angle between path of conductor and direction of lines of force;

s° = angle between conductor and the direction of the path of one of its points;

r° = angle between conductor and direction of lines of force.

Then, $E = \dfrac{\mathbf{B}\,L\,M \sin p^\circ \sin s^\circ \sin r^\circ}{10^8}$. **(477.)** Art. **3024.**

If the conductor moves in a circular path of radius r and makes S revolutions per minute, r and s become each 90°, and formula **477** reduces to

$$E = \frac{2\pi r\,\mathbf{B}\,L\,S \sin p^\circ}{10^8 \times 60} = \frac{N\pi S \sin p^\circ}{10^8 \times 60}.$$

When E is at its maximum value, E_m, $p = 90°$ and sin $p = 1$; hence,

$$E_m = \frac{N \pi S}{10^8 \times 60}. \qquad \textbf{(479.)} \quad \text{Art. } \textbf{3046.}$$

Let c = total number of conductors on surface of armature;
N = total lines of force from one pole-piece cut by the conductors;
S = revolutions per minute of armature.

Then, $E = \frac{c N S}{60 \times 10^8}$. **(480.)** Art. **3104.**

In a two-circuit ring winding,

$$E = \frac{c p N S}{60 \times 10^8}, \qquad \textbf{(482.)} \quad \text{Art. } \textbf{3136.}$$

where p = number of pairs of poles.

If m windings are employed on the same armature (in multiple windings),

$$E = \frac{c N S}{60 \times 10^8 m}. \qquad \textbf{(485.)} \quad \text{Art. } \textbf{3168.}$$

For multiple-wound two-circuit windings,

$$E = \frac{c p N S}{60 \times 10^8 m}. \qquad \textbf{(486.)} \quad \text{Art. } \textbf{3168.}$$

THE FORCE TENDING TO MOVE A CONDUCTOR IN A MAGNETIC FIELD.

Let A = current in absolute units;
L = length of conductor in centimeters;
$\mathbf{B}$ = density of field, lines of force per sq. cm.;
f = force acting on conductor in dynes.

Then, $f = A \mathbf{B} L$. **(478.)** Art. **3028.**

FORMULAS USED IN ELECTRIC TRANSMISSION.

COPPER WIRE.

Let W = weight of copper wire per mile in pounds;
R = resistance of copper wire per mile in ohms;
d = diameter of wire in mils.

Then, $W = \frac{d^2}{62.5}$. **(490.)** Art. **3277.**

$$R = \frac{56,970}{d^2}. \quad \textbf{(491.)} \text{ Art. } \textbf{3277.}$$

To calculate the resistance of given length and diameter:

Let L = length in feet;
d = diameter in mils;
K = constant from the following table corresponding to the given temperature;
R_t = resistance.

Then, $R_t = \frac{L}{d^2} K$. **(492.)** Art. **3285.**

Resistance per Mil-Foot in Legal Ohms.	Temperature F.	Resistance per Mil-Foot in Legal Ohms.	Temperature F.	Resistance per Mil-Foot in Legal Ohms.	Temperature F.
10.00	50.47	10.60	78.01	11.10	100.40
10.10	55.15	10.70	82.47	11.20	104.36
10.20	59.79	10.80	86.90	11.30	108.64
10.30	64.40	10.90	91.31	11.40	112.90
10.40	68.97	11.00	95.69	11.50	117.14
10.50	73.51				

RESISTANCE OF IRON AND STEEL WIRE.

Let d = diameter of wire in mils;
R = resistance of wire per mile in ohms.

Then, for wrought-iron wire of good quality,

$$R = \frac{360,000}{d^2}. \quad \textbf{(493.)} \text{ Art. } \textbf{3288.}$$

For steel wire,

$$R = \frac{470,000}{d^2}. \quad \textbf{(494.)} \text{ Art. } \textbf{3288.}$$

PERCENTAGE CONDUCTIVITY.

Let R = the resistance per grain-foot of a pure metal wire at a temperature t, from the tables;
L = the length of the wire to be tested;
w = weight of the wire to be tested;
r' = the calculated resistance in ohms of a pure metal wire of the same weight, length, and temperature;
r'' = the observed resistance in ohms of the wire to be tested;
x = percentage conductivity.

Then, $r' = \dfrac{L\,R}{\dfrac{w}{L}} = \dfrac{L^2 R}{w}$. **(495.)** Art. **3300.**

$$x = 100\frac{r'}{r''}. \quad \textbf{(496.)} \quad \text{Art. } \textbf{3300.}$$

DROP OF POTENTIAL AND LINE LOSS.

Let e and e' = potentials at two points of a conductor;
$E = e - e'$ = drop of potential;
R = resistance of conductor between the two points;
C = current flowing between the two points;
W = energy loss.

Then, the application of Ohm's law gives

$$C = \frac{e - e'}{R} = \frac{E}{R}.$$

$E = CR.$ **(499.)** Art. **3303.**

$R = \dfrac{E}{C}.$ **(500.)** Art. **3303.**

$W = C^2 R.$ **(501.)** Art. **3306.**

When the loss in a circuit is given as a percentage, it is not to be taken as applying to the voltage at the receiving end of the line, but at the dynamo or switchboard.

Let V = voltage at dynamo;
V' = voltage at the receiving end of line;

p = per cent. of loss;
E = actual drop on line in volts.

Then, $E = \frac{100\,V'}{100 - p} - V' = \frac{p\,V'}{100 - p}$. **(502.)** Art. **3308.**

FORMULAS USED IN ELECTRIC RAILWAYS.

CONDUCTORS FOR TRANSMISSION OF POWER.

Let R = resistance of conductor in ohms;
D = length of conductor in feet;
C = current in amperes;
d = diameter of conductor in mils;
e = drop in volts.

Then, $R = \frac{10.79\,D}{d^2}$. **(511.)** Art. **3399.**

$d^2 = \frac{10.79\,C\,D}{e}$. **(512.)** Art. **3399**

$e = \frac{10.79\,C\,D}{d^2}$. **(513.)** Art. **3399.**

For a uniformly distributed load:

$d^2 = \frac{5.4\,C\,D}{e}$. **(514.)** Art. **3399.**

$e = \frac{5.4\,C\,D}{d^2}$. **(515.)** Art. **3399.**

Safe current for feeders:

$C = \sqrt{\frac{d^3}{1,300}}$. **(516.)** Art. **3400.**

POWER REQUIRED FOR ELECTRIC STREET-RAILWAY CARS.

Let f = force used for propulsion in pounds;
f' = force required for starting on a grade;
F = force required to give a car a given speed in a given time;

F' = force required to give a car a given speed in a given time on a grade;
p = per cent. grade;
W = weight of car in tons;
w = weight of car in pounds;
w' = weight on drivers in pounds;
s = speed of car in miles per hour;
t = time during which acceleration takes place, seconds;
H = total horsepower of motors;
h = perpendicular distance ascended per minute in feet;
D = horizontal distance traveled per minute in feet;
E = efficiency of motor;
G_s = per cent. grade at which slipping occurs;
G_r = maximum grades which a running car will ascend;
r = track resistance, pounds per ton weight of car;
a = ratio of adhesive force to weight on drivers.

Then, $f = 30\ W.$ **(522.)** Art. **3428.**

$f = r\ W.$ **(523.)** Art. **3428.**

$$H = \frac{h w + D f}{33{,}000\ \mathsf{E}}.$$ **(524.)** Art. **3429.**

$f' = (70 + 20p)\ W.$ **(525.)** Art. **3431.**

$$F = f + \frac{91.1\ W s}{t}.$$ **(527.)** Art. **3432.**

$$F' = f + \frac{91.1\ W s}{t} + 20\ W p.$$ **(529.)** Art. **3433.**

Average H. P. at starting of car $= .00133\ F' s.$ **(530.)** Art. **3434.**

When the whole weight of the car is on the drivers:

$$G_s = \frac{2{,}000\ a - 70}{20} \text{ per cent.}$$ **(531.)** Art. **3435.**

When $\frac{1}{x}$ of the weight is on the drivers,

$$G_s = \frac{\frac{2{,}000\ a}{x} - 70}{20} \text{ per cent.}$$ **(532.)** Art. **3435.**

When the car is running, only 30 pounds per ton is necessary for propulsion, and the limit of grade which may be ascended is, letting G_r = maximum grade which a running car will ascend,

$$G_r = \frac{\frac{2{,}000\,a}{x} - 30}{20} \text{ per cent.} \qquad \textbf{(533.)} \quad \text{Art. } \textbf{3436.}$$

RULES AND FORMULAS USED IN ELECTRIC LIGHTING.

POWER REQUIRED FOR INCANDESCENT LAMPS.

The **common candle-powers** for incandescent lamps are 8, 10, 16, 20, 32, 50, and 100. The 16 candle-power and 32 candle-power are most generally used.

The same incandescent lamp may be used on either alternating or direct current circuits.

Rule.—*The power required for the average incandescent lamp may be calculated by allowing about 3.44 watts per candle, so that a 16 candle-power lamp requires about 16 × 3.44 = 55 watts.*

Rule.—*In figuring the number of lamps which a certain engine horsepower can supply, it is customary to allow 10 Edison 16 candle-power lamps per indicated horsepower.*

The current required for 16 candle-power lamps of different voltages is given below, figured on a basis of 55 watts per lamp :

Electromotive Force in Volts.	Current in Amperes.
110	.5000
75	.7338
55	1.0000

CANDLE-POWER.

Let L = light from standard candle ;
D = distance between candle and screen ;
d = distance between lamp tested and screen;
l = light from the lamp tested.

Then, $$l = \frac{d^2 L}{D^2}.$$

Since L is taken as a standard candle, or *unity*, the candle-power of the lamp tested is

$$l = \frac{d^2}{D^2}. \qquad \textbf{(534.)} \quad \text{Art. } \textbf{3472.}$$

That is to say, *the intensity of the light received by an object varies inversely as the square of its distance from the source.*

The **mean spherical candle-power** is the average candle-power on the surface of a sphere with the light as its center.

Rule.—*The mean spherical candle-power of an arc lamp is equal to one-half the mean horizontal candle-power plus one-quarter the maximum candle-power.*

Rule.—*If the horizontal candle-power of an Edison incandescent lamp is 1, the mean spherical candle-power is .98. At an angle of 45° from the horizontal, the candle-power is 1.33. Similarly, the spherical intensity of a Maxim lamp is .74, and of a Swan or Lane-Fox lamp, .58.* Art. **3474.**

To determine whether an arc lamp is giving as much light as is being paid for.

Measure by means of a standard voltmeter the voltage at the terminals of the lamp and determine with an ammeter the current flowing. If the product is equal to or greater than 450 for a 2,000 candle-power lamp, or 300 for a 1,200 candle-power lamp, the quantity of light paid for at these ratings is actually given off or exceeded. Art. **3475.**

GENERAL FORMULAS RELATING TO ELECTRIC LAMPS.

Let E = efficiency of lamp;
C = current passing through lamp;
E = E. M. F. between terminals;
W = electrical power in watts;
R_h = resistance, hot.
E. H. P. = electrical horsepower.

Then, $$\mathsf{E} = \frac{\text{mean spherical candle-power}}{E\,C} \qquad \textbf{(535.)}$$ Art. **3476.**

$$R_h = \frac{E}{C}. \qquad \textbf{(536.)}$$ Art. **3476.**

$$W = E\,C = C^2\,R_h. \qquad \textbf{(537.)}$$ Art. **3476.**

The electrical horsepower per lamp:

$$\text{E. H. P.} = \frac{E\,C}{746} = \frac{C^2\,R_h}{746}. \qquad \textbf{(539.)}$$ Art. **3476.**

$$\text{Lamps per E. H. P.} = \frac{746}{E\,C} = \frac{746}{C^2\,R_h}. \qquad \textbf{(540.)}$$ Art. **3476.**

$$\text{Candles per E. H. P.} = \frac{\text{c. p.} \times 746}{E\,C}. \qquad \textbf{(541.)}$$ Art. **3476.**

$$\text{Heat units per second per candle} = \frac{.24\,E\,C}{\text{c. p.}}\ \text{calories.} \qquad \textbf{(542.)}$$ Art. **3476.**

$$\text{Watts per candle-power} = \frac{E\,C}{\text{c. p.}} = \frac{C^2\,R_h}{\text{c. p.}}. \qquad \textbf{(543.)}$$ Art. **3476.**

CALCULATIONS FOR INCANDESCENT LIGHTING CIRCUITS.

Let R_f = resistance per foot of the conductor;
E = drop in volts in the circuit;
N = number of 16 c. p. lamps;
F = distance in feet from dynamo or distributing board to lamps;
A = area of conductor in circular mils.

For 110-volt circuits, $R_f = \frac{E}{NF}$. **(547.)** Art. **3533.**

For 55-volt circuits, $R_f = \frac{E}{2\,NF}$. **(548.)** Art. **3534.**

For 110-volt circuits, $A = \frac{10.8\,NF}{E}$. **(549.)** Art. **3535.**

For 55-volt circuits, $A = \frac{21.6\,NF}{E}$. **(550.)** Art. **3535.**

If the drop E is required, formulas **547** to **550** may be transposed. Thus for 110-volt circuits,

$$E = R_f\,NF. \quad \textbf{(553.)} \text{ Art. } \textbf{3537.}$$

$$E = \frac{10.8\,NF}{A}, \quad \textbf{(554.)} \text{ Art. } \textbf{3537.}$$

and for 55-volt circuits,

$$E = 2\,R_f\,NF. \quad \textbf{(555.)} \text{ Art. } \textbf{3537.}$$

$$E = \frac{21.6\,NF}{A}. \quad \textbf{(556.)} \text{ Art. } \textbf{3537.}$$

For a three-wire system,

$$R_f = \frac{2\,E}{NF}. \quad \textbf{(557.)} \text{ Art. } \textbf{3538.}$$

$$A = \frac{5.4\,NF}{E}. \quad \textbf{(558.)} \text{ Art. } \textbf{3538.}$$

Rule.—*In determining the safe carrying capacity of conductors for the three-wire system, remember that, with the same number and kind of lamps, the conductor carries only one-half the current of a conductor installed on the multiple-arc system.*

Rule.—*No wire smaller than a No. 16 B. & S. gauge should be used in any dynamo circuit, nor any wire smaller than a No. 14 B. & S. in any inside wiring.*

FORMULAS USED IN DYNAMO-ELECTRIC MACHINE DESIGN.

(CONTINUOUS-CURRENT.)

HEATING IN ARMATURE AND FIELD SPOOLS.

Let t = temperature rise in degrees Fahrenheit;
w = watts expended in heat;
s = surface of armature in square inches;
v = peripheral velocity in feet per second.

Then, $$t = \frac{99\,w}{s(1+0.00018\,v)}.$$ **(559.)** Art. **3546.**

Let t = temperature rise of spool in degrees Fahrenheit;
w = watts expended in coils;
s = surface of spools (wire only) in square inches.

Then, $$t = \frac{80\,w}{s}.$$ **(560.)** Art. **3550.**

ARMATURE AMPERE-TURNS.

Let C = current flowing through armature;
c = number of conductors on periphery of armature;
At_a = total ampere-turns of armature;
At_g = field ampere-turns required to overcome reluctance of air-gaps;
At_b = back ampere-turns;
At_c = cross ampere-turns;
$\mathbf{B}'_g$ = density of lines of force, per square inch, in leading pole tip;
$\mathbf{B}''_g$ = density of lines of force in following pole tip;
$\mathbf{B}_g$ = induction at center line;
l_g = length of single air-gap in inches;
l = 180° less twice the angle of lead;
r = angle of lead of brushes.

Then, $$\mathbf{B}'_g = \frac{3.192\left(\frac{At_g}{2} - \frac{C\,c}{8}\right)}{l_g}.$$ **(566.)** Art. **3556.**

$$B''_g = \frac{3.192\left(\frac{At_g}{2} + \frac{Cc}{8}\right)}{l_g}. \qquad (567.)\ \text{Art. } \mathbf{3557.}$$

$$B_g = \frac{3.192\,At_g}{2\,l_g} = \frac{1.596\,At_g}{l_g}. \qquad (568.)\ \text{Art. } \mathbf{3558.}$$

Effect of back ampere-turns:

$$At_b = \frac{C\,r\,c}{360}. \qquad (569.)\ \text{Art. } \mathbf{3561.}$$

Cross ampere-turns:

$$At_c = \frac{C}{2} \times \frac{l}{360} \times c = \frac{C\,l\,c}{720}. \qquad (570.)\ \text{Art. } \mathbf{3562.}$$

Limiting value of air-gap density:

$$B'_g = \frac{3.192\left(\frac{At_g}{2} - \frac{C\,l\,c}{1{,}440}\right)}{l_g}. \qquad (573.)\ \text{Art. } \mathbf{3564.}$$

Limiting values of cross and armature ampere-turns:

For a bipolar drum winding,

$$At_c = \tfrac{2}{3}\,At_g, \qquad (574.)\ \text{Art. } \mathbf{3568.}$$

$$At_a = \frac{120\,At_g}{l}. \qquad (577.)\ \text{Art. } \mathbf{3569.}$$

For a bipolar ring winding,

$$At_c = \tfrac{1}{2}\,At_g. \qquad (578.)\ \text{Art. } \mathbf{3570.}$$

$$At_a = \frac{90\,At_g}{l}. \qquad (579.)\ \text{Art. } \mathbf{3571.}$$

Limiting volume of current:

Let C' = current flowing in each conductor;
V = volume of current flowing across armature.

Then, $V = C'c.$ (580.) Art. **3573.**

$V = \tfrac{2}{3}\,At_a.$ (581.) Art. **3574.**

For a drum winding,

$$V = \frac{240\ At_g}{l}. \qquad \textbf{(582.)} \quad \text{Art. } \textbf{3574.}$$

For a ring winding,

$$V = \frac{180\ At_g}{l}. \qquad \textbf{(583.)} \quad \text{Art. } \textbf{3575.}$$

Formulas **582** and **583** hold not only for bipolar machines but also for those of the multipolar type.

ARMATURE AMPERE-TURNS; MULTIPOLAR DYNAMO.

In addition to the symbols previously given, let P = number of poles, and l = angle embraced by pole-piece.

$$At_c = \frac{C'\,l\,c}{360}. \qquad \textbf{(584.)} \quad \text{Art. } \textbf{3577.}$$

$$At_b = C' \times \frac{2\,r}{360} \times c = \frac{C'\,r\,c}{180}. \qquad \textbf{(585.)} \quad \text{Art. } \textbf{3578.}$$

$$At_a = \frac{V}{P}. \qquad \textbf{(586.)} \quad \text{Art. } \textbf{3579.}$$

For a multiple-circuit winding,

$$At_a = \frac{C\,c}{P^2}. \qquad \textbf{(587.)} \quad \text{Art. } \textbf{3580.}$$

For a two-circuit winding,

$$At_a = \frac{C\,c}{2\,P}. \qquad \textbf{(588.)} \quad \text{Art. } \textbf{3581.}$$

LENGTH OF AIR-GAP.

Let l = length of single air-gap;
w = width of armature slot.

Then, for densities approximating 32,000 lines of force per square inch,

$$l > \frac{w}{3}. \qquad \textbf{(589.)} \quad \text{Art. } \textbf{3589.}$$

For densities approximating 50,000 lines of force per square inch.

$$l = \frac{w}{2}. \qquad \textbf{(590.)} \quad \text{Art. } \textbf{3590.}$$

Also, for high densities,

$$l > \frac{w}{2}. \qquad \textbf{(591.)} \quad \text{Art. } \textbf{3591.}$$

SIZE OF WIRE FOR SHUNT WINDING.

Let A = area of cross-section in circular mils;
l_m = mean length in feet of one turn of wire;
At = ampere-turns for the magnetic circuit;
E_b = E. M. F. at the brushes;
e = drop of potential through field regulator.

Then, $$A = \frac{12 \times l_m \times At}{E_b - e}. \qquad \textbf{(592.)} \quad \text{Art. } \textbf{3624.}$$

RELATION OF SHUNT RESISTANCE TO ARMATURE RESISTANCE.

Let E = electrical efficiency;
r_a = resistance of armature;
r_s = resistance of shunt.

Then, $$\mathsf{E} = \frac{1}{1 + 2\sqrt{\frac{r_a}{r_s}}}. \qquad \textbf{(593.)} \quad \text{Art. } \textbf{3633.}$$

PROPORTION OF SHUNT AND SERIES WINDINGS.

Rule.—*The energy loss, in watts, of the shunt and series windings of a compound dynamo should be proportioned to the respective ampere-turns carried by each coil.*

DIVISION OF SHUNT WINDING.

Let A = total winding area in cross-section;
A_1 = required area for larger wire;
A_2 = required area for smaller wire;
R_1 = specific resistance of larger wire;
R_2 = specific resistance of smaller wire;
R = sum of the specific resistances of the two wires.

Then, $\frac{A_1}{A} = \frac{R_2}{R}$, whence $A_1 = \frac{A \times R_2}{R}$. **(594.)**

$\frac{A_2}{A} = \frac{R_1}{R}$, and $A_2 = \frac{A \times R_1}{R}$. **(595.)**

Art. **3648.**

ELECTRICAL EFFICIENCY.

The electrical efficiency is the ratio of the useful output to the total output, and is therefore usually high. This ratio may be expressed as a per cent. as follows:

$$\mathbf{E} = \frac{100\, e\, C}{E\, C_a}, \qquad \textbf{(596.)} \quad \text{Art. } \textbf{3668.}$$

where e = E. M. F. at brushes;
E = total E. M. F. generated;
C = current available for outside circuit;
C_a = total current in armature.

COMMERCIAL EFFICIENCY.

Let I = the input of a dynamo, or the power applied at the pulley;
O = the output;
$\mathbf{E}$ = the per cent. commercial efficiency;
L = per cent. loss.

Then, $\mathbf{E} = \frac{100\, O}{I}$. **(597.)** Art. **3671.**

$L = \frac{100\,(I - O)}{I}$. **(598.)** Art. **3672.**

$I = \frac{100\, O}{\mathbf{E}}$. **(599.)** Art. **3673.**

$O = \frac{I\, \mathbf{E}}{100}$ **(600.)** Art. **3674.**

FORMULAS USED IN MOTOR DESIGN.

COUNTER E. M. F. OF MOTOR.

Let E = E. M. F. between mains;
E_m = counter E. M. F. of motor;
C = current flowing through armature;
R_a = resistance of armature.

Then, $E = E_m + C R_a$. **(601.)** Art. **3783.**

TORQUE.

Let C_m = current flowing through armature;
C = total number of conductors on armature;
N = total magnetic flux;
T' = total torque exerted on armature in foot-pounds.

Then, $C_m = \frac{E - E_m}{R_a}$. **(603.)** Art. **3786.**

$$T' = .00117 \frac{N C C_m}{10^6}. \quad \textbf{(605.)} \text{ Art. } \textbf{3787.}$$

SPEED OF SHUNT MOTORS.

Let n = number of revolutions of motor per second;
C = total number of conductors on armature;
N = total magnetic flux;
E_m = counter E. M. F.

Then, $E_m = \frac{n C N}{10^8}$; or,

$$n = \frac{E_m 10^8}{N C}. \quad \textbf{(606.)} \text{ Art. } \textbf{3794.}$$

SERIES MOTOR CIRCUIT.

Let E = E. M. F. between mains;
E_m = counter E. M. F.;
C = current;
R_a = resistance of armature;
R_f = resistance of field.

Then, $E = E_m + C R_a + C R_f$. **(607.)** Art. **3797.**

OUTPUT AND EFFICIENCY OF MOTORS.

Let E = E. M. F. between mains;
E_m = counter E. M. F.;
C = current flowing through armature;
c = current through shunt field.

For a Series Motor:—

Total energy supplied from mains $= CE$. **(608.)** Art. **3817.**

Energy developed in armature $= CE_m$. **(609.)** Art. **3817.**

Electrical efficiency $= \frac{CE_m}{CE} = \frac{E_m}{E}$. **(610.)** Art. **3817.**

For a Shunt Motor:—

Total energy supplied $= CE + cE$. **(611.)** Art. **3817.**

Energy developed $= CE_m$. Art. **3817.**

Electrical efficiency $= \frac{CE_m}{E(C+c)}$. **(612.)** Art. **3817.**

FORMULAS USED IN THEORY OF ALTERNATING-CURRENT APPARATUS.

FREQUENCY AND PERIOD.

Let n = frequency, or number of cycles per second;
t = period, or time of one cycle.

Then, $n = \frac{1}{t}$. **(613.)** Art. **3844.**

SELF-INDUCTION.

Let L = coefficient of self-induction;
N = number of lines corresponding to a current of one ampere;
T = number of turns.

Then, $L = \frac{N\,T}{10^8}$ **(614.)** Art. **3875.**

$$\text{henrys} = \frac{\text{magnetic flux} \times \text{turns}}{\text{current} \times 10^8}. \quad \textbf{(615.)} \quad \text{Art. } \textbf{3875.}$$

CIRCUITS CONTAINING RESISTANCE AND SELF-INDUCTION.

Average and Effective Induced E. M. F.—Let $\bar{C}$ = effective current flowing through the circuit. Then, using the notation of the preceding formulas, the average induced E. M. F. is

$$E \text{ av.} = \frac{4\,N\,T\,n}{10^8}; \quad \textbf{(616.)} \quad \text{Art. } \textbf{3882.}$$

and the effective induced E. M. F. is

$$\bar{E} = \frac{4.44\,N\,T\,n}{10^8}. \quad \textbf{(617.)} \quad \text{Art. } \textbf{3882.}$$

$$L = \frac{N\,T}{\bar{C}\sqrt{2} \times 10^8}. \quad \textbf{(618.)} \quad \text{Art. } \textbf{3884.}$$

Combining formulas **617** and **618,**

$$\bar{E} = 2\pi n L \bar{C}. \quad \textbf{(620.)} \quad \text{Art. } \textbf{3884.}$$

Impressed E. M. F.—

Let $\bar{E}$ = the impressed E. M. F.;
R = resistance of circuit.

Then, $\bar{E}^2 = \bar{C}^2 R^2 + 4\pi^2 n^2 L^2 \bar{C}^2$, **(621.)** Art. **3889.**

and $\bar{E} = \bar{C}\sqrt{R^2 + (2\pi n L)^2}$. **(623.)** Art. **3889.**

Denoting by the term *impedance* the quantity $\sqrt{R^2 + (2\pi n L)^2}$,

$$\bar{C} = \frac{\bar{E}}{\sqrt{R^2 + (2\pi n L)^2}}, \quad \textbf{(624.)} \quad \text{Art. } \textbf{3891.}$$

or $$\text{current} = \frac{\text{E. M. F.}}{\text{impedance}}.$$

Angle of Lag.—

Let Φ = angle of lag.

Then, $\tan \Phi = \frac{2\pi n L}{R}$. Art. **3892.**

CAPACITIES.

Let J_1 and J_2 be the two separate capacities, and J_1 the combined capacity.

If the capacities are connected in parallel,

$$J = J_1 + J_2.$$

If connected in series,

$$\frac{1}{J} = \frac{1}{J_1} + \frac{1}{J_2};$$

or, $J = \frac{J_1 J_2}{J_1 + J_2}$. **(626.)** Art. **3896.**

CIRCUITS CONTAINING RESISTANCE AND CAPACITY.

Effective E. M. F. to Overcome Capacity or Condenser E. M. F.—

Let E = maximum E. M. F. applied at terminals;
J = capacity in farads;
n = frequency;
C = maximum current;
$\bar{C}$ = effective current.

Denoting by $\bar{E}$ the effective E. M. F. required to overcome capacity,

$C = 2\pi n J E.$ **(628.)** Art. **3905.**

$E = \frac{C}{2\pi n J}$. **(629.)** Art. **3905.**

$\bar{E} = \frac{\bar{C}}{2\pi n J}$. **(630.)** Art. **3905.**

Impressed E. M. F.—

Denoting now the impressed E. M. F. by $\bar{E}$ and the resistance of the circuit by R,

$$\bar{E} = \bar{C}\sqrt{R^2 + \left(\frac{1}{2\pi n J}\right)^2} \qquad \textbf{(633.)} \text{ Art. } \textbf{3910.}$$

$$\bar{C} = \frac{\bar{E}}{\sqrt{R^2 + \left(\frac{1}{2\pi n J}\right)^2}}. \qquad \textbf{(634.)} \text{ Art. } \textbf{3910.}$$

Angle Between Current and E. M. F.—

Let Φ = angle by which the current leads the E. M. F.

Then, $\tan \Phi = \frac{1}{2\pi n J R}$. **(631.)** Art. **3908.**

CIRCUITS CONTAINING RESISTANCE, SELF-INDUCTION, AND CAPACITY.

The symbols have the same meanings as in the preceding formulas. $\bar{E}$ denotes the effective impressed E. M. F. and $\bar{C}$ the effective current.

$$\bar{E} = \bar{C}\sqrt{R^2 + \left(2\pi n L - \frac{1}{2\pi n J}\right)^2}. \qquad \textbf{(636.)} \text{ Art. } \textbf{3913.}$$

$$\bar{C} = \frac{\bar{E}}{\sqrt{R^2 + \left(2\pi n L - \frac{1}{2\pi n J}\right)^2}}. \qquad \textbf{(637.)} \text{ Art. } \textbf{3913.}$$

$$\tan \Phi = \frac{2\pi n L - \frac{1}{2\pi n J}}{R}. \qquad \textbf{(638.)} \text{ Art. } \textbf{3914.}$$

Frequency for Maximum Current.—

If the self-induction is neutralized by the capacity,

$$2\pi n L = \frac{1}{2\pi n J},$$

whence $n = \frac{1}{2\pi}\sqrt{\frac{1}{JL}}$. **(639.)** Art. **3916.**

POWER DELIVERED TO ALTERNATING-CURRENT CIRCUIT.

Using the preceding symbols,

average power $= \bar{E}\,\bar{C}\cos\Phi$. **(640.)** Art. **3925.**

POWER MEASUREMENTS.

Three-Voltmeter Method.—

Let $\bar{E}_2$ = pressure across circuit;
$\bar{E}_1$ = pressure across non-inductive resistance;
$\bar{E}$ = pressure across both combined;
$\bar{C}$ = effective current;
R = non-inductive resistance;
W = power in watts.

Then, $W = \frac{1}{2R}(\bar{E}^2 - \bar{E}_1{}^2 - \bar{E}_2{}^2)$. **(641.)** Art. **3948.**

$$W = \frac{\bar{C}}{2\bar{E}_1}(\bar{E}^2 - \bar{E}_1{}^2 - \bar{E}_2{}^2.) \qquad \textbf{(642.)}\ \text{Art. } \textbf{3948.}$$

Three-Ammeter Method.—

The non-inductive resistance is connected in parallel with the circuit under test.

Let $\bar{C}$ = current in main circuit;
$\bar{C}_1$ = current in resistance;
$\bar{C}_2$ = current in circuit in which the power expended is to be measured.

Then, $W = \frac{R}{2}(\bar{C}^2 - \bar{C}_1{}^2 - \bar{C}_2{}^2)$. **(643.)** Art. **3949.**

$$W = \frac{\bar{E}}{2\bar{C}_1}(\bar{C}^2 - \bar{C}_1{}^2 - \bar{C}_2{}^2). \qquad \textbf{(644.)}\ \text{Art. } \textbf{3949.}$$

FREQUENCY, SPEED, AND NUMBER OF POLES OF SINGLE-PHASE ALTERNATOR.

Let n = frequency;
P = number of poles;
$p = \frac{P}{2}$ = number of pairs of poles;
s = number of times per second coil is moved past poles, or number of revolutions per second.

Then, $n = \frac{P}{2} s.$ **(645.)** Art. **3950.**

$n = p s.$ **(646.)** Art. **3956.**

$s = \frac{n}{p}.$ **(647.)** Art. **3956.**

E. M. F. GENERATED BY ALTERNATORS.

Let s = revolutions per second;
p = number of pairs of poles;
N = number of lines flowing from one pole;
T = number of turns in series;
$\overline{E}$ = effective E. M. F.

Then, $\overline{E} = \frac{4.44\, N\, T\, n}{10^8}.$ **(648.)** Art. **3960.**

CURRENT, E. M. F., AND OUTPUT OF THREE-PHASE ALTERNATORS.

Let $\overline{E}$ = line voltage;
$\overline{\epsilon}$ = volts generated per phase;
$\overline{C}$ = line current;
$\overline{c}$ = current per phase or current in windings;
W = total output in watts.

For a Y-connected Winding:—

$\overline{E} = 2\ \overline{\epsilon} \cos 60° = \overline{\epsilon}\sqrt{3}.$ **(649.)** Art. **3992.**

$\overline{\epsilon} = \frac{\overline{E}}{\sqrt{3}}.$ **(650.)** Art. **3992.**

$$\bar{C} = \bar{c}. \qquad \textbf{(651.)} \quad \text{Art. } \textbf{3992.}$$

$$W = \bar{C}\,\bar{E}\sqrt{3}. \qquad \textbf{(652.)} \quad \text{Art. } \textbf{3992.}$$

For a Δ Winding:—

$$\bar{E} = \bar{\varepsilon}. \qquad \textbf{(653.)} \quad \text{Art. } \textbf{3994.}$$

$$\bar{C} = \bar{c}\sqrt{3}. \qquad \textbf{(654.)} \quad \text{Art. } \textbf{3994.}$$

$$\bar{c} = \frac{\bar{C}}{\sqrt{3}}. \qquad \textbf{(655.)} \quad \text{Art. } \textbf{3994.}$$

$$W = \bar{C}\,\bar{E}\sqrt{3}. \qquad \textbf{(652.)} \quad \text{Art. } \textbf{3994.}$$

TRANSFORMERS.

Let T_p = number of turns in primary coil;
T_s = number of turns in secondary coil;
$\bar{E}_p$ = E. M. F. of line;
$\bar{E}_s$ = E. M. F. set up in secondary coil;
$\bar{C}_p$ = current in primary coil;
$\bar{C}_s$ = current in secondary coil.

Then, $$\frac{\bar{E}_p}{\bar{E}_s} = \frac{T_p}{T_s}; \qquad \textbf{(656.)} \quad \text{Art. } \textbf{4004.}$$

or, $$\bar{E}_s = \bar{E}_p \frac{T_s}{T_p}. \qquad \textbf{(657.)} \quad \text{Art. } \textbf{4004.}$$

$$\bar{C}_p = \bar{C}_s \frac{\bar{E}_s}{\bar{E}_p} = \bar{C}_s \frac{T_s}{T_p}. \qquad \textbf{(658.)} \quad \text{Art. } \textbf{4009.}$$

Single-Phase Alternating Transformer.—

Let $\bar{E}$ = alternating voltage;
V = direct voltage.

Then, $\bar{E} = .707\,V.$ **(660.)** Art. **4051.**

Three-Phase Alternating Transformer.—

With the notation of formula **660,**

$$\bar{E} = .612\,V. \qquad \textbf{(661.)} \quad \text{Art. } \textbf{4053.}$$

FORMULAS USED IN DESIGN OF ALTERNATING-CURRENT APPARATUS.

HYSTERESIS LOSS.

Let V = volume of iron in cubic centimeters;
$\mathbf{B}$ = maximum magnetic density at which iron is worked;
n = number of cycles through which the magnetism is carried per second;
W_H = energy in watts expended in hysteresis;
k = a constant depending upon the magnetic qualities of the iron under consideration.

Then, $W_H = \frac{k\,n\,V\,\mathbf{B}^{1.6}}{10^7}$. **(662.)** Art. **4068.**

The value of the constant k may be taken at .002 to .0025 for soft transformer iron, and as .0033 for armature iron.

E. M. F. FOR VARIOUS ARMATURE WINDINGS.

Single-Phase Windings.—

Let $\overline{E}$ = effective E. M. F.;
T = total number of turns *connected in series* on the armature;
N = total flux from one pole;
n = frequency;
k = a constant depending upon the style of winding used.

Then, $\overline{E} = \frac{4.44\,k\,n\,N\,T}{10^8}$. **(668.)** Art. **4089.**

The values of k are as follows:

Single-coil, or concentrated, winding	1.000
Two-coil winding	.707
Three-coil winding	.667
Uniformly-distributed winding	.636

Polyphase Windings.—

Let $\bar{E}$ = E. M. F. generated *per phase* in a polyphase armature;

T = number of turns connected in series *per phase;*

N = flux from one pole;

n = frequency;

k = constant depending upon the arrangement of the winding.

Then,

$$\bar{E} = \frac{4.44\, k\, n\, N\, T}{10^8}. \qquad \textbf{(670.)} \quad \text{Art. } \textbf{4090.}$$

The values of k are as follows:

Winding with one group of conductors per pole per phase	1.000
Two-phase winding, uniformly distributed	.900
Three-phase winding, uniformly distributed	.950
Three-phase winding with two groups of conductors per pole per phase	.965

Formula **670** applies also to induction-motor windings.

RESISTANCE OF A CONDUCTOR.

Let L = length of wire in feet;

m = area of cross-section in circular mils;

R = resistance of a given length of the conductor;

k = resistance of one mil-foot of the wire.

Then, $$R = \frac{k\, L}{m}. \qquad \textbf{(680.)} \quad \text{Art. } \textbf{4139.}$$

For copper wire at ordinary temperature, $k = 10.4$ to 10.6; hence,

$$\text{(cold) } R = \frac{10.4\, L}{m}. \qquad \textbf{(681.)} \quad \text{Art. } \textbf{4139.}$$

The resistance per mil-foot at the working temperature is about 11.5 ohms; hence,

$$\text{(hot) } R = \frac{11.5\, L}{m}. \qquad \textbf{(682.)} \quad \text{Art. } \textbf{4139.}$$

CALCULATION OF SEPARATELY-EXCITED WINDING.

Let E = E. M. F. generated by exciter;
e = drop of voltage in rheostat;
R = resistance of a pair of spools;
t = number of turns on a pair of spools;
l_m = mean length of a turn in feet;
m = cross-section of field wire in circular mils;
C = current in field;
P = number of poles;
$p = \frac{P}{2}$ = number of pairs of poles.

Then, $C = \frac{2(E-e)}{PR}$. **(687.)** Art. **4177.**

$$R = \frac{11.5\, l_m\, t}{m}. \qquad \textbf{(688.)} \text{ Art. } \textbf{4177.}$$

$$C = \frac{2(E-e)\,m}{11.5\, P l_m\, t} = .174\frac{(E-e)\,m}{P l_m\, t}. \qquad \textbf{(689.)} \text{ Art. } \textbf{4177.}$$

$$m = \frac{11.5\, p\, l_m\, t\, C}{E-e}. \qquad \textbf{(690.)} \text{ Art. } \textbf{4177.}$$

PULLEY AND BELT OF GENERATOR.

Let s = belt speed in feet per minute;
r = number of revolutions per minute;
d = diameter of pulley in inches;
w = output in watts;
b = width of belt in inches.

Then, $d = \frac{12\, s}{\pi\, r}$. **(691.)** Art. **4200.**

$$b = \frac{.7\, w}{s}. \qquad \textbf{(692.)} \text{ Art. } \textbf{4200.}$$

MAGNETIC FLUX AND LENGTH OF FIELD OF INDUCTION MOTORS.

Let N = flux from one pole;
l = length of field parallel to the shaft;
p = number of pairs of poles;
d_f = inside diameter of field;
$\mathbf{B}$ = maximum magnetic density in the air-gap.

Then, $$N = \frac{2\,l\,d_f\,\mathbf{B}}{2\,p} = \frac{l\,d_f\,\mathbf{B}}{p}.$$

$$l = \frac{p\,N}{d_f\,\mathbf{B}}. \qquad \textbf{(704.)} \quad \text{Art. } \textbf{4257.}$$

CURRENT IN SECONDARY WINDING.

Let C_s = secondary current;
C_p = primary current;
r = ratio of primary to secondary conductors;
k = power factor.

Then, $$C_s = \frac{C_p\,r}{k}. \qquad \textbf{(705.)} \quad \text{Art. } \textbf{4261.}$$

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